Meson-Baryon Couplings, SU(3) Symmetry and QCD Sum rules

T. M. ALIEV,[a] A. OZPINECI,[a] M. SAVCI,[a], AND V. S. ZAMIRALOV[b]

[a]Department of Physics, Middle East Technical University, Ankara, Turkey,
[b] Institute of NuclearPhysics, M. V. Lomonosov, MSU, Moscow, Russia

HadronPhysics2
Study of Strongly Interacting Matter

ABSTRACT

$SU(3)_f$ symmetry allows one to relate various meson-baryon couplings to each other. Using this symmetry, octet-baryon octet-meson couplings can all be expressed in terms of two parameters only. But in nature $SU(3)_f$ symmetry is broken due to the large strange quark mass. In this work, using light cone QCD sum rules, we show that even if one considers $SU(3)_f$ violation, meson baryon coupling constants can be written in terms of a few functions of the quark properties. [1, 2]

1 $SU(3)_f$ Symmetry

In the limit $m_u = m_d = m_s$, QCD has an exact $SU(3)_f$ flavour symmetry. This symmetry allows us to group the mesons and baryons into flavour multiplets.
For the light baryons, since they are made of three quarks, they can be grouped into the $SU(3)_f$ multiplets as

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

and for the light mesons we have the decomposition

$$\bar{3} \otimes 3 = 1 \oplus 8.$$

In this work, we are interested in the 8 dimensional representation of the baryons and mesons in the corresponding decomposition.

1.1 $SU(3)_f$ Multiplets

1.1.1 Octet Baryons and Octet Pseudoscalar Mesons

$$B^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$M^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

1.1.2 Vector Mesons

Due to tha large s quark mass, the $SU(3)_f$ symmetry is broken down to $SU(2)$ isospin symmetry. This leads to a mixing of the singlet and the M^3_3 mesons. In the pseudoscalar case, the $\eta - \eta'$ mixing is ignored as it is known to be small. In the case of the vector mesons, the mixing between the singlet, denoted ω_0, and M^3_3, denoted ω_8, is non-negligible. Hence the vector mesons are grouped into a nonet:

$$V^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.$$

where, experimentally it is known that $\omega \simeq \bar{s}s$ and $\rho \simeq \frac{1}{\sqrt{2}}\bar{u}u + \bar{d}d$

1.2 F and D constants

The most general interaction Lagrangian is:

$$\mathcal{L} = \sqrt{2}FTr\bar{B}[B,M] + \sqrt{2}DTr\bar{B}\{B,M\} - \frac{1}{\sqrt{2}}(F+D)Tr\bar{B}BTrM$$

The coefficient of the third term is chosen to make sure that the coupling of the ϕ meson to the nucleon is zero. This coupling is OZI suppressed.

References

[1] T. M. Aliev, A. Ozpineci, M. Savci, V. S. Zamiralov, arXiv:0905.4664.

[2] T. M. Aliev, A. Ozpineci, S. B. Yakovlev, V. S. Zamiralov, Phys. Rev. **D74** (2006) 116001

[3] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385

[4] H. Kim, T. Doi, M. Oka and S.H. Lee, Nucl. Phys. **A678** (2000) 295

[5] M. E. Bracco, F. S. Navarra, M. Nielsen, Phys. Lett. **B454** (1999) 346

[6] S. Choe, Phys. Rev. **C62** (2000) 025204

[7] T. Doi, Y. Kondo, and M. Oka, Phys. Rept. **398** (2004) 253

[8] R. Lawall, *et al.* Eur. Phys. J. **A24** (2005) 275

[9] R. A. Arndt, Z.-J. Li, L. D. Roper, R. L. Workman, Phys. Rev. Lett. **65** (1990) 157

[10] S. L. Zhu, Phys. Rev. C **59**, 435 (1999).

[11] Zhi-Gang Wang, Phys. Rev. D **75**, 054020 (2007).

[12] G. Erkol, R. G. E. Timmermans and Th. A. Rijken, Phys. Rev. C **74**, 045201 (2006).

2 Light Cone QCD Sum Rules

QCD rum rules is a non-perturbative method in which hadronic properties are related to the properties of the vacuum [3]. In order to calculate the baryon-meson coupling constants, one starts by considering the following correlation function:

$$\Pi^{B_1\to B_2 M} = i\int d^4x e^{ipx}\langle M(q)|\mathcal{T}\eta_{B_2}(x)\eta_{\bar{B}_1}(0)|0\rangle$$

where η_B is an interpolating current with the quantum numbers of the B baryon. By inserting a complete set of hadronic states, this correlation function can be written as:

$$\Pi^{B_1\to B_2 M} = \sum_{h,h'}\frac{\langle 0|\eta_{B_2}|h'(p)\rangle}{p^2 - m^2_{h'}}\langle M(q)h'(p)|h(p+q)\rangle \times \frac{\langle h(p+q)|\eta_{B_1}|0\rangle}{(p+q)^2 - m^2_h}$$

The matrix element $\langle M(q)h'(p)|h(p+q)\rangle$ can be expressed in terms of the coupling constants. By calculating the residue at poles of the correlation function, one can obtain the mass and the coupling constants.

The interpolating currents are chosen as:

$$\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[\left(u^{aT}Cs^b\right)\gamma_5 d^c + t\left(u^{aT}C\gamma_5 s^b\right)d^c \right.\\
&\quad \left. + \left(d^{aT}Cs^b\right)\gamma_5 u^c + t\left(d^{aT}C\gamma_5 s^b\right)u^c\right]\\
\eta^{\Sigma^+} &= -\frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d\to u)\,, \quad \eta^{\Sigma^-} = -\frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u\to d)\\
\eta^p &= -\eta^{\Sigma^+}(s\to d)\,, \quad \eta^n = -\eta^{\Sigma^-}(s\to u)\\
\eta^{\Xi^0} &= -\eta^n(d\to s)\,, \quad \eta^{\Xi^-} = -\eta^p(u\to s)
\end{aligned}$$

and for the Λ baryon:

$$\begin{aligned}
2\eta_{\Sigma^0}(d\leftrightarrow s) + \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda\\
2\eta_{\Sigma^0}(u\leftrightarrow s) - \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda
\end{aligned}$$

3 Relations Between Correlation Functions

For convention, express the quark content of π^0 (ρ^0) pseudoscalar (vector) meson as:

$$\pi^0(\rho^0) = g_{\pi(\rho)\bar{u}u}\bar{u}u + g_{\pi(\rho)\bar{d}d}\bar{d}d + g_{\pi(\rho)\bar{s}s}\bar{s}s$$

This allows us to write the $\Sigma^0 \to \Sigma^0\pi^0(\rho^0)$ correlation function as:

$$\begin{aligned}
\Pi^{\Sigma^0\to\Sigma^0\pi^0(\rho^0)} &= g_{\pi(\rho)\bar{u}u}\Pi_1(u,d,s) + g_{\pi(\rho)\bar{d}d}\Pi'_1(u,d,s)\\
&\quad + g_{\pi(\rho)\bar{s}s}\Pi_2(u,d,s)
\end{aligned}$$

Note that since the Σ^0 current is symmetric under the $u \leftrightarrow d$, $\Pi'_1(u,d,s) = \Pi_1(d,u,s)$.

3.1 Relations Involving π^0

Consider the coupling constant of π^0 to Σ^+. Since, Σ^+ contains only the u and s quarks, upto OZI suppressed contributions, one can write:

$$\Pi^{\Sigma^+\to\Sigma^+\pi^0} = g_{\pi\bar{u}u}\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle$$

The current for Σ^+ can be obtained from that of Σ^0 by $\Sigma^0(d\to u) = -\sqrt{2}\Sigma^+$ and hence

$$\langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle(d\to u) = 2\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle'.$$

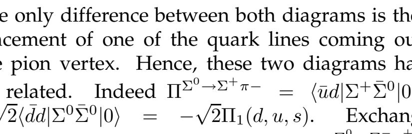
The prime on the matrix element means that it considers only one contraction of the identical quarks, and the emission of the $\bar{u}u$ component of the π^0 from only one of the quark lines. To obtain all the contribution, it has to be multiplied by $2\times 2 = 4$.

$$\begin{aligned}
\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle &= 4\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle' =\\
&= 2\langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle(d\to u) = 2\Pi_1(u,u,s)
\end{aligned}$$

Hence $\Pi^{\Sigma^+\to\Sigma^+\pi^0} = \sqrt{2}\Pi_1(u,u,s)$
Most of the other couplings of the other correlation functions $\Pi^{B\to BM}$, where M is a meson with hidden flavor, can be obtained similarly.

3.2 Relations for Mesons of Open Flavor

As an example consider the $\Pi^{\Sigma^0\to\Sigma^+\pi^-}$ and $\Pi^{\Sigma^0\to\Sigma^0\pi^0}$ couplings.



The only difference between both diagrams is the replacement of one of the quark lines coming out of the pion vertex. Hence, these two diagrams has to be related. Indeed $\Pi^{\Sigma^0\to\Sigma^+\pi-} = \langle\bar{u}d|\Sigma^+\bar{\Sigma}^0|0\rangle = -\sqrt{2}\langle\bar{d}d|\Sigma^0\bar{\Sigma}^0|0\rangle = -\sqrt{2}\Pi_1(d,u,s)$. Exchanging u and d quarks, one also obtains $\Pi^{\Sigma^0\to\Sigma^-\pi^+} = \langle\bar{d}u|\Sigma^-\bar{\Sigma}^0|0\rangle = \sqrt{2}\Pi_1(u,d,s)$.

3.3 Relations Involving Λ Baryon

Using $2\Sigma^0(d \leftrightarrow s) = -\sqrt{3}\Lambda - \Sigma^0$ and the result $\Pi^{\Sigma^0\to\Sigma^-\pi^+} = \sqrt{2}\Pi_1(u,d,s)$, we have $2\sqrt{2}\Pi_1(u,s,d) = \sqrt{3}\Pi^{\Lambda\to\Xi^-K^+} + \Pi^{\Sigma^0\to\Xi^-K^+}$. Since, under exchanges of quarks, Λ current always appears with Σ^0 current, it is not possible to seperate correlation function involving Σ^0 and those involving Λ. For this reason, one more function is needed:

$$\Pi_3(u,d,s) = -\Pi^{\Sigma^0\to\Xi^-K^+} = -\langle\bar{s}u|\Xi^-\bar{\Sigma}^0|0\rangle$$

Note that $\Pi^{\Xi^-\to\Sigma^0K^-} = \langle\bar{u}s|\Sigma^0\bar{\Xi}^-|0\rangle = -\Pi_3(u,s,d)$. Finally, the symmetric part of Π_3 can also be related to Π_1 and Π_2 through

$$\Pi_3(u,d,s) + \Pi_3(u,s,d) = \sqrt{2}\left[\Pi_1(u,d,s) + \Pi_1(u,s,d) - \Pi_2(s,d,u)\right].$$

These three functions are enough to describe all octet baryon-pseudoscalar octet meson couplings.
The couplings of the vector mesons to the baryons involve two different couplings: the electric like coupling and the magnetic like coupling. For each of these couplings, three functions are needed.

4 Results and Conclusions

4.1 Pseudoscalar Couplings

Channel	Gen. Cur.	$t=-1$	$SU(3)_f$	QSR*	QSR†[6]	Exp.
$\Lambda\to nK$	-13 ± 3	-9.5 ± 1	-13.6	-2.37[5]	-2.49	-13.5 [8]
$\Lambda\to\Sigma^+\pi^-$	10 ± 3	12 ± 1	9.58			
$\Lambda\to\Xi^0K^0$	4.5 ± 2	-2.5 ± 0.5	4.04			
$n\to p\pi^-$	21 ± 4	20 ± 2	18.95			21.2 [9]
$n\to\Sigma^0K^0$	-3.2 ± 2.2	-9.5 ± 0.5	-3.2	-0.025[5]	-0.40	-4.25 [8]
$p\to\Lambda K^+$	-13 ± 3	-10 ± 1	-13.6	-2.37 [5]	-2.49	-13.5 [8]
$p\to p\pi^0$	14 ± 4	15 ± 1	13.4	13.5 [4]		14.9 [9]
$p\to\Sigma^+K^0$	4 ± 3	14 ± 1	4.52			
$\Sigma^0\to nK^0$	-4 ± 3	-9.5 ± 1	-3.2	-0.025 [5]	-0.40	-4.25 [8]
$\Sigma^0\to\Lambda\pi^0$	11 ± 3	12 ± 1.5	9.58	6.9[7]		
$\Sigma^0\to\Xi^0K^0$	-13 ± 3	-13.5 ± 1	-13.4			
$\Sigma^-\to nK^-$	5 ± 3	15 ± 2	3.2			
$\Sigma^+\to\Lambda\pi^+$	10 ± 3.5	12.5 ± 1	9.58			
$\Sigma^+\to\Sigma^0\pi^+$	-9 ± 2	-7.5 ± 0.7	-10.2	-11.9 [4]		
$\Xi^0\to\Lambda K^0$	4.5 ± 1	-2.6 ± 0.3	4.04			
$\Xi^0\to\Sigma^0K^0$	-12.5 ± 3	-13.5 ± 1	-13.4			
$\Xi^0\to\Sigma^+K^-$	18 ± 4	19 ± 2	18.95			
$\Xi^0\to\Xi^0\pi^0$	10 ± 2	0.3 ± 0.6	-3.2	-1.60[4]		

The column labeled $SU(3)_f$ is the best fit result of the general current predictions to the $SU(3)_f$ expressions. The best fit values correspond to $(F,\ D) = (5.3\pm2.6,\ 7.0\pm4.4)$

4.2 Electric Type Vector Couplings

$$\langle B_2(p_2)V(q)|B_1(p_1)\rangle = \bar{u}(p_2)\left[f_1\gamma_\mu - f_2\frac{i}{m_1+m_2}\sigma_{\mu\nu}q^\nu\right]u(p_1)\varepsilon^\mu$$

where f_1 is the electric type coupling and $f_1 + f_2$ is the magnetic type coupling.

channel	General current Result	General current $SU(3)_f$	Ioffe current Result	Ioffe current $SU(3)_f$	QSR [10]	QSR [11]	QSR [12]
$p\to p\rho^0$	-2.5 ± 1.1	-1.7	-5.9 ± 1.3	-6.4	2.5 ± 0.2	2.4 ± 0.6	3.2 ± 0.9
$p\to p\omega$	-8.9 ± 1.5	-10.3	-8.2 ± 0.4	-9.6	18 ± 8	7.2 ± 1.8	—
$\Xi^0\to\Xi^0\rho^0$	-4.2 ± 2.1	-4.3	-2.0 ± 0.2	-1.6	—	2.4 ± 0.6	1.5 ± 1.1
$\Sigma^0\to\Lambda\rho^0$	1.9 ± 0.7	1.5	-3.0 ± 0.5	-2.8	—	—	—
$\Lambda\to\Sigma^+\rho^-$	1.9 ± 0.7	1.5	-2.8 ± 0.6	-2.8	—	—	—
$\Sigma^+\to\Sigma^0\rho^+$	7.2 ± 1.2	6.0	8.5 ± 0.8	8.0	—	—	—
$\Sigma^+\to\Lambda\rho^+$	2.0 ± 0.6	1.5	-2.8 ± 0.6	-2.8	—	—	—
$p\to\Lambda K^{*+}$	5.1 ± 1.8	4.4	7.4 ± 0.8	8.3	—	—	—
$\Sigma^-\to nK^{*-}$	6.6 ± 1.8	6.1	1.7 ± 0.4	2.3	—	—	—
$\Xi^0\to\Sigma^+K^{*-}$	-2.3 ± 1.7	-2.4	-10.0 ± 1.8	-9.1	—	—	—
$\Xi^-\to\Lambda K^{*-}$	-5.9 ± 0.7	-5.8	-6.2 ± 0.4	-5.5	—	—	—
$\Sigma^0\to\Xi^0K^{*0}$	1.6 ± 1.0	1.7	7.1 ± 1.3	6.4	—	—	—
$\Lambda\to\Xi^0K^{*0}$	-6.0 ± 0.7	-5.9	-6.2 ± 0.2	-5.5	—	—	—
$n\to\Sigma^0K^{*0}$	-4.0 ± 0.7	-4.3	-1.5 ± 0.3	-1.6	—	—	—
$\Lambda\to\Lambda\omega$	-7.1 ± 1.1	-7.7	-4.8 ± 0.2	-4.8	—	4.8 ± 1.2	—
$\Xi^0\to\Xi^0\phi$	-9.5 ± 2.5	-8.5	-13.5 ± 1.6	-11.3	—	—	—
$\Lambda\to\Lambda\phi$	-5.3 ± 1.5	-3.6	-8.0 ± 1.0	-6.8	—	—	—
$\Sigma^0\to\Sigma^0\phi$	-6.0 ± 0.8	-6.1	-0.25 ± 0.50	-2.3	—	—	—

Electric like couplings of the vector mesons to the baryon octet. The best fits correspond to $(F,\ D) = (-3.0\pm0.5,\ 1.3\pm06)$ for the general current and to $(F,\ D) = (-4.2\pm0.6,\ -2.7\pm1.0)$ for the Ioffe current.

4.3 Magnetic Type Vector Couplings

channel	General current Result	General current $SU(3)_f$	Ioffe current Result	Ioffe current $SU(3)_f$	QSR [10]	QSR [11]	QSR [12]
$p\to p\rho^0$	19.7 ± 2.8	21.4	22.7 ± 1.3	24.7	21.6 ± 6.6	10.1 ± 3.7	36.8 ± 13
$p\to p\omega$	14.5 ± 2.6	15.0	21.2 ± 1.2	25.7	32.4 ± 14.4	5.0 ± 1.2	—
$\Xi^0\to\Xi^0\rho^0$	-2.8 ± 1.6	-3.2	-0.24 ± 0.24	0.5	—	-3.6 ± 1.6	-5.3 ± 3.3
$\Sigma^0\to\Lambda\rho^0$	13.8 ± 2.7	14.2	15.1 ± 0.9	14.0	—	—	—
$\Lambda\to\Sigma^+\rho^-$	14.3 ± 2.9	14.2	15.1 ± 0.8	14.0	—	—	—
$\Sigma^+\to\Sigma^0\rho^+$	-17.8 ± 2.2	-18.2	-27.9 ± 1.8	-25.2	—	7.1 ± 1.0	53.5 ± 19
$\Sigma^+\to\Lambda\rho^+$	14.3 ± 2.9	14.2	15.1 ± 0.8	14.0	—	—	—
$p\to\Lambda K^{*+}$	-22.9 ± 4.2	-22.9	-27.3 ± 1.5	-28.8	—	—	—
$\Sigma^-\to nK^{*-}$	3.8 ± 2.8	4.5	-0.79 ± 0.05	-0.7	—	—	—
$\Xi^0\to\Sigma^+K^{*-}$	33.8 ± 4.9	30.3	41.3 ± 2.4	34.9	—	—	—
$\Xi^-\to\Lambda K^{*-}$	11.6 ± 2.9	8.7	17.9 ± 1.0	14.8	—	—	—
$\Sigma^0\to\Xi^0K^{*0}$	-24.6 ± 4.8	-21.4	-29.2 ± 1.7	-24.7	—	—	—
$\Lambda\to\Xi^0K^{*0}$	11.1 ± 2.6	8.7	15.0 ± 1.0	14.8	—	—	—
$n\to\Sigma^0K^{*0}$	-2.8 ± 1.8	-3.2	0.56 ± 0.04	0.5	—	—	—
$\Lambda\to\Lambda\omega$	1.6 ± 0.6	1.8	7.1 ± 0.5	9.1	—	-5.7 ± 1.0	—
$\Xi^0\to\Xi^0\phi$	22.8 ± 6.4	25.7	37.7 ± 2.5	35.6	—	—	—
$\Lambda\to\Lambda\phi$	19.3 ± 5.0	18.7	22.0 ± 1.4	23.5	—	—	—
$\Sigma^0\to\Sigma^0\phi$	-3.5 ± 2.5	4.5	0.81 ± 0.05	0.7	—	—	—

The magnetic type couplings of the vector mesons to the octet baryons. The best fits correspond to $(F,\ D) = (9.2\pm1.4,\ 12.4\pm1.4)$ for the general current and to $(F,\ D) = (12.7\pm1.8,\ 12.2\pm0.8)$ for the Ioffe current.