Magnetic Moments and Pion Couplings of Ξ_Q Baryons in Light Cone QCD Sum Rules

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Outline

1. Introduction
2. QCD Sum Rules
   - Introduction to QCD Sum Rules
   - Currents for Ξ_Q
3. Results for Ξ_Q
   - Mass Postdictions for the Ξ_Q
   - Magnetic Moment Predictions for Ξ_Q
   - $g_{\pi\Xi}$ Coupling Constant Predictions(PRELIMINARY)

- In quark model, there should exist baryons with a heavy charm or bottom quark.
- Ξ_Q baryons contain qsQ quarks.
- They are expected to decay weakly.
- Ξ_b contains one quark from each family.

- In recent years, significant progress have been made in spectroscopy of heavy baryons.
- In 2007, D0 and CDF claimed to have observed Ξ_b^- directly.
- Ξ_b^- and Ξ_c had been observed indirectly by DELPHI Collaboration in 2004 in the channels $\Xi_b^- \to \Xi^- \ell^- \bar{\nu} X$ and $\Xi_c^0 \to \Xi_c^- \pi^+$
- Experimentally, charmed baryons can also be produced by B meson decays with a branching fraction of about 5%

- Especially Ξ_b baryons are a new laboratory for studying b decays.
- The Ξ_Q contains information about the heavy quark spin
- In this work, QCD Sum Rules is used to study their properties.

Outline

- In QCD Sum Rules, hadronic properties are expressed in terms of the properties of vacuum.
- One starts with a correlation function of the form:

$$\Pi = i \int d^4 p e^{ipx} \langle \mathcal{M} | \mathcal{T} \eta_1(x) \eta_2(0) | \Omega \rangle \tag{1}$$

  where $\mathcal{M}$ is the QCD vacuum for mass, or can be a hadronic state if one is interested in coupling constants or form factors.
- η_i are composite operators made up of quark fields that have the same quantum numbers as the hadrons under consideration.
- The correlation functions can be expressed in terms of the properties of hadrons and also interms of the properties of the vacuum.

- By inserting complete sets of hadronic states, the correlation function can be written as:
  - if $\mathcal{M} = \Omega$ and $\eta_1 = \eta_2 = \eta$:

$$\Pi = \sum_h \frac{\langle \Omega | \eta | h(p) \rangle \langle h(p) | \eta | \Omega \rangle}{p^2 - m_h^2} \tag{2}$$

  - if $\mathcal{M}(q)$ is another hadron,

$$\Pi = \sum_h \frac{\langle \mathcal{M}(q) | \eta_1 | h(p+q) \rangle \langle h(p+q) | \eta_2 | \Omega \rangle}{(p+q)^2 - m_h^2} \tag{3}$$

  or by inserting another complete set

$$\Pi = \sum_{h_1, h_2} \frac{\langle \Omega | \eta_1 | h_1(p) \rangle \langle h_1(p) \mathcal{M}(q) | h_2(p+q) \rangle \langle h_2(p+q) | \eta_2 | \Omega \rangle}{(p^2 - m_{h_1}^2)((p+q)^2 - m_{h_2}^2)} \tag{4}$$

- The matrix elements $\langle\Omega|\eta|h(p)\rangle = \lambda u(p)$ where $u(p)$ is the wavefunction (a spinor in our case) and λ is called the corresponding residue.
- The matrix elements $\langle\mathcal{M}(q)|\eta|h(p+q)\rangle$ and $\langle\mathcal{M}(q)h_1(p)|h_2(p+q)\rangle$ can be expressed in terms of coupling constants or form factors.

- The correlation function can also be calculated in the deep Euclidean region using OPE:

$$\mathcal{T}\eta_1(x)\eta_2(0) = \sum_d C_d(x^2)\mathcal{O}_d(x) \tag{5}$$

- In the case of mass sum rules or traditional sum rules, $\mathcal{O}_d(x)$ are local operators. After fourier transform, the correlation function becomes:

$$\Pi = \sum_d C_d(p^2)\langle\Omega|\mathcal{O}_d|\Omega\rangle \tag{6}$$

where $\langle\Omega|\mathcal{O}_d|\Omega\rangle$ are called the vacuum condansates.

- Some of the well known vacuum condansates include:

$$\begin{aligned} d &= 3 & \langle\bar{q}q\rangle \\ d &= 4 & \langle g_s^2 G^2\rangle \\ d &= 5 & \langle\bar{q}G\sigma q\rangle \end{aligned} \tag{7}$$

- In case of light cone sum rules, matrix elements of the form $\langle \mathcal{M}(q) | \mathcal{O}_d(x) | \Omega \rangle$ are needed.
- The matrix elements are expanded around $x^2 \simeq 0$ in terms of distribution amplitudes. The distribution amplitudes describes the parton content of $\mathcal{M}$

- Two expressions for the correlation function is matched using spectral representation.

$$\Pi(p^2) = \int ds \frac{\rho(s)}{s - p^2} + \text{polynomials in } p^2 \tag{8}$$

- To subtract the contributions of higher states and continuum, quark hadron duality is assumed:

$$\rho^{hadron}(s) = \rho^{OPE}(s) \ \text{ for } s > s_0 \tag{9}$$

- Contribution of higher states and continuum are further suppressed by Borel transformation:

$$B_{M^2}(p^2)^n \to 0 \quad \text{for } n > 0$$

$$B_{M^2}\left(\frac{1}{(m^2-p^2)^n}\right) = \frac{1}{\Gamma(n)} \frac{e^{\frac{-m^2}{M^2}}}{(M^2)^{n-1}} \tag{10}$$

- The sum rules is obtained through

$$\Pi^{\text{lowest lying}}(M^2) = \int_0^{s_0} ds \rho(s) e^{-\frac{s}{M^2}} \tag{11}$$

- There are two auxiliary parameters of the sum rules: M^2 and s_0

Outline

- Any operator η can be used in the sum rules as long as $\langle \Omega | \eta | \Xi \rangle \neq 0$
- The commonly used current for the light Ξ baryon is:

$$\eta_\Xi = -\epsilon^{abc} \left[(s^{aT} C u^b) \gamma_5 s^c + t (s^{aT} C \gamma_5 u^b) s^c \right] \tag{12}$$

- For the heavy Ξ_Q, we use

$$\begin{aligned} \eta_\Xi &= -\epsilon^{abc} \left[(s^{aT} C u^b) \gamma_5 Q^c + t (s^{aT} C \gamma_5 u^b) Q^c \right] \\ &\quad -\alpha \epsilon^{abc} \left[(Q^{aT} C u^b) \gamma_5 s^c + t (Q^{aT} C \gamma_5 u^b) s^c \right] \end{aligned} \tag{13}$$

- $\alpha = 0$ corresponds to light quarks forming a diquark which orbits around the heavy quark
- $\alpha = 1$ has an appealing $Q \leftrightarrow s$ symmetry

Outline

Mass Postdictions for the Ξ_Q

- The correlation function can be written as

$$\Pi(p^2) = \sum_h \lambda_h^2 \frac{\not{p} + m_h}{p^2 - m_h^2} = \Pi_1(p^2)\ \not{p} + \Pi_2(p^2) \tag{14}$$

where $\langle \Omega | \eta_{\Xi_Q} | h \rangle = \lambda_h u_h(p)$

- After borel trasformation

$$\Pi_1^{subtracted}(M^2) = \lambda_{\Xi_Q}^2 e^{\frac{-m_{\Xi_Q}^2}{M^2}} \tag{15}$$

- Mass of the Ξ_Q baryons can be extracted using:

$$m_{\Xi}^2 = M^4 \frac{\partial}{\partial M^2} \ln \Pi_1^{subtracted} \tag{16}$$

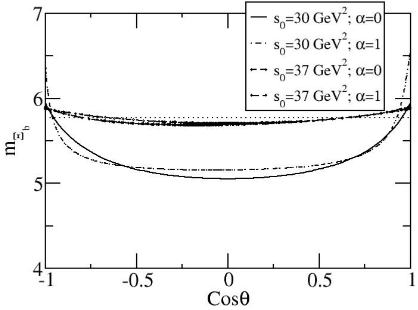


Figure: Variation of the mass of Ξ_b with respect to $\cos\theta$ where $t = \tan\theta$

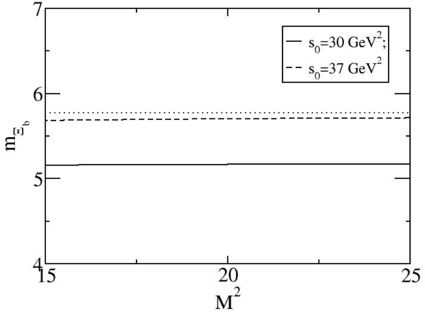


Figure: Variation of m_{Ξ_b} with respect to M^2

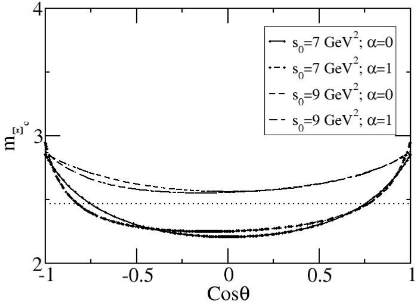


Figure: Variation of m_{Ξ_c} with respect to $\cos\theta$

Outline

- The correlation function can be expressed as:

$$\begin{aligned}\Pi &= -\lambda_Q^2 \epsilon^\mu \frac{\not{p}_f + m_{\Xi_Q}}{p_f^2 - m_{\Xi_Q}^2}\left[(f_1+f_2)\gamma_\mu + \frac{(p_i-p_f)_\mu}{2m_{\Xi_Q}} f_2\right]\frac{\not{p}_i + m_{\Xi_Q}}{p_i^2 - m_{\Xi_Q}^2} \\ &= -\mu_{\Xi_Q}\frac{\lambda_Q^2}{(p_i^2 - m_{\Xi_Q}^2)(p_f^2 - m_{\Xi_Q}^2)}\,\not{p}_f \not{\epsilon} \not{p}_i + \cdots \end{aligned} \tag{17}$$

where $\mu = f_1 + f_2$

- The magnetic moment is μ evaluated at $q^2 = 0$

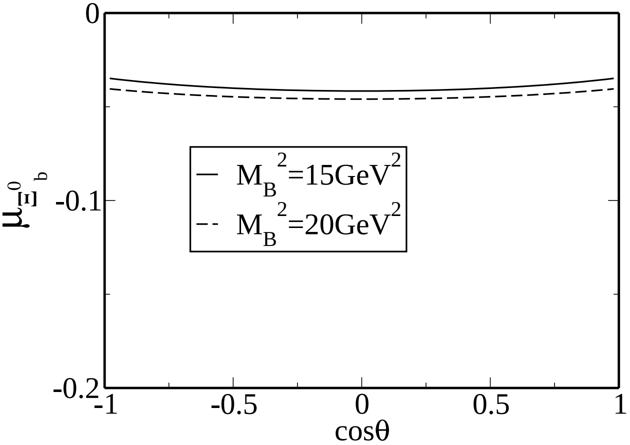


Figure: Variation of μ_{Ξ_b} with respect to $\cos\theta$ at $\alpha = 0$

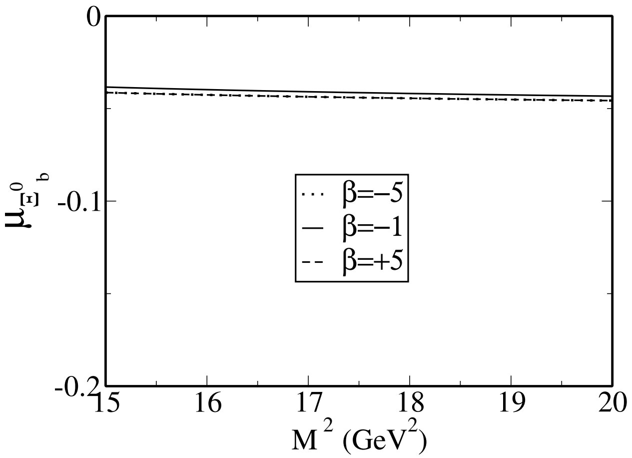


Figure: Variation of μ_{Ξ_b} with respect to M^2 at $\alpha = 0$

	$\mu_{\Xi_b^0}$	$\mu_{\Xi_b^-}$	$\mu_{\Xi_c^0}$	$\mu_{\Xi_c^+}$
Our results	-0.045 ± 0.005	-0.08 ± 0.02	0.35 ± 0.05	0.50 ± 0.05
RQM	-0.06	-0.06	0.39	0.41
NQM	-0.06	-0.06	0.37	0.37
B. Patel, *et al.*	-	-	$-1.02 \div -1.06$	$0.45 \div 0.48$
M. Savage	-	-	0.32	0.42
D. O. Riska	-	-	0.38	0.38
Y. Oh, *et al*	-	-	0.28	0.28
C. S. An	-	-	$0.28 \div 0.34$	$0.39 \div 0.46$

Table: Results for the magnetic moments of Ξ_Q baryons in different approaches.

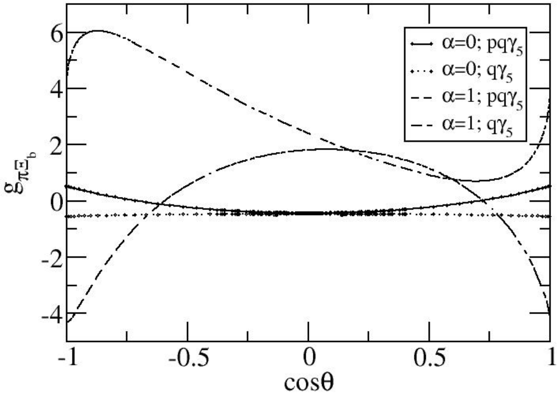
Outline

$g_{\pi\Xi}$ Coupling Constant Predictions

- The hadronic representation of the correlation function:

$$\Pi(p^2,(p+q)^2) = i\frac{\lambda^2_{\Xi_Q} g_{\pi\Xi_Q}}{(p^2-m^2_{\Xi_q})((p+q)^2-m^2_{\Xi_Q})} \times(-\not{p}\,\not{q}\gamma_5 - M_1\not{q}\gamma_5) \tag{18}$$

- Two possible sum rules from the structures $\not{p}\,\not{q}\gamma_5$ and $\not{q}\gamma_5$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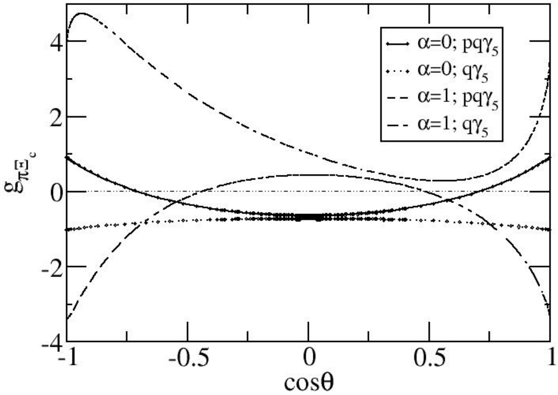
α=0; qγ5
α=1; pqγ5
α=1; qγ5
6
4
2
0
-2
-4
-1
-0.5
0
0.5
1
cosθ
gπΞb


α=0; pqγ5
α=0; qγ5
α=1; pqγ5
α=1; qγ5
4
2
0
-2
-4
-1
-0.5
0
0.5
1
cosθ

Summary

- Currents have been constructed for Ξ_Q baryons
- Experimentally observed masses can be reproduced.
- Magnetic Moments are predicted
- Couplings to the π are sensitive to the form of the current.