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Strong Coupling Constants of the Octet Baryons to K and π Mesons in Light Cone QCD Sum Rules [a]:

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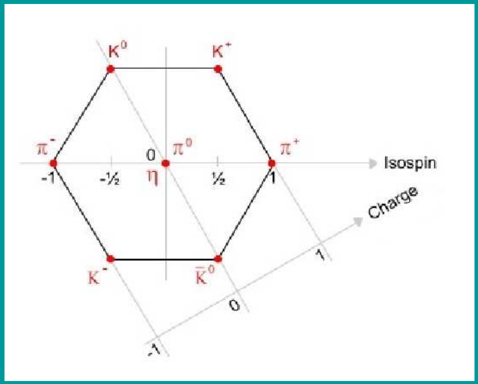
[a] T. M. Aliev, A. Ozpineci, S.B. Yakovlev, V. S. Zamiralov, Phys. Rev. **D74** (2006) 116001, hep-ph/0609026

Outline

- Introduction
- $SU(3)_f$ Limit
- Light Cone QCD Sum Rules
- Relations Between Correlation Functions
- General Analysis and Conclusion

Introduction

- QCD has an $SU(3)_f$ symmetry when $m_u = m_d = m_s$
- Mesons: $3 \otimes \bar{3} = 1 \oplus 8$
  Baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$



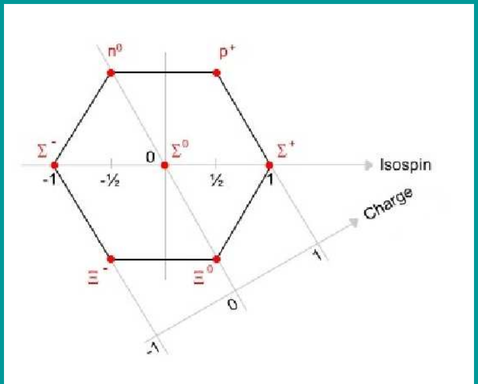


Figure 1: Octet of pseudoscalar mesons and spin-1/2 baryons

• $SU(3)_f$ Limit

- In the $SU(3)_f$ limit, the baryons can be grouped in the octet representation as:

$$B^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- The Mesons as:

$$P^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

- The most general $SU(3)_f$ interactions can be described by two terms:
  - $\mathrm{Tr}\bar{B}\left\{P,B\right\}$
  - $\mathrm{Tr}\bar{B}\left[P,B\right]$
- The most general interaction lagrangian:

$$\mathcal{L} = \sqrt{2}\left(D\mathrm{Tr}\bar{B}\left\{P,B\right\} + F\mathrm{Tr}\bar{B}\left[P,B\right]\right)$$

- The coupling constants can be read off as:

$$\begin{aligned} g_{\pi^0 pp} &= F+D,\ g_{\pi^0\Sigma^+\Sigma^+} = 2F,\ g_{\pi^-\Sigma^+\Sigma^0} = -2F \\ g_{\pi^0\Xi^0\Xi^0} &= F-D,\ g_{\pi^+\Xi^+\Xi^0} = -\sqrt{2}(F-D),\ \text{etc.} \end{aligned}$$

Light Cone QCD Sum Rules

- One starts with a correlation function of the form

$$\Pi = i \int d^4x e^{ipx} \langle \mathcal{M}(q) | \mathcal{T} \eta_{B_1}(x) \bar{\eta}_{B_2}(0) | 0 \rangle$$

- Inserting a complete set of hadronic states

$$\Pi^{phen} = \sum_{h,h'} \frac{\langle 0 | \eta_{B_1} | B_1^h \rangle}{p_1^{h2} - m_1^{h2}} \langle \mathcal{M} B_1^h | B_2^{h'} \rangle \frac{\langle B_2^{h'} | \eta_{B_2} | 0 \rangle}{p_2^{h'2} - m_2^{h'2}}$$

- Extraction more precise near the poles: $p_1^2 \simeq m_1^2 > 0$ and $p_2^2 \simeq m_2^2 > 0$

$$\langle 0|\eta_{B_i}|B_i(p,s)\rangle = \lambda_{B_i} u(p,s)$$

$$\langle B_1(p_1)\mathcal{M}(q)|B_2(p_2)\rangle = g_{B_2 B_1 \mathcal{M}}\bar{u}(p_1) i\gamma_5 u(p_2)$$

$$\begin{aligned} & \Pi^{B_2\to B_1\mathcal{M}}(p_1^2,p_2^2) \\ = \quad & i\frac{g_{B_2B_1\mathcal{M}}\lambda_{B_1}\lambda_{B_2}}{(p_1^2-M_1^2)(p_2^2-M_2^2)}\Big(-\not p \not q\gamma_5 - M_1 \not q\gamma_5 \\ + \quad & (M_2-M_1)\not p\gamma_5 + (M_1M_2-p^2)\gamma_5\Big) + \cdots \end{aligned}$$

- When $p_1^2 \to -\infty$ and $p_2^2 \to -\infty$, the main contribution is from small distances and times.
- Perturbative techniques can be applied:

$$\mathcal{T}\eta_{B_1}(x)\bar{\eta}_{B_2}(0) \simeq \sum_t C_t(x^2) : \mathcal{O}_t(x) :$$

  where $: \cdots :$ stand for the normal product, and the first operators are of the form $\bar{q}(x)\Gamma q(0)$.
- If the operators $\mathcal{O}_t$ are expanded around $x \sim 0$, one obtains the OPE.

- The matrix elements $\langle \mathcal{M}| : \mathcal{O}_t(x) : |0\rangle$ are expanded around $x^2 \sim 0$ (hence the name light cone).

$$\langle \mathcal{M}(q)| : \bar{q}(x)\gamma_\mu\gamma_5 q(0) : |0\rangle =$$
$$-if_{\mathcal{M}} q_\mu \int_0^1 du e^{i\bar{u}qx} \left(\varphi_{\mathcal{M}}(u) + \frac{1}{16} m_{\mathcal{M}}^2 x^2 \mathbb{A}(u)\right)$$

- With these inputs, one can calculate Π when $p_1^2 << 0$ and $p_2^2 << 0$.

- To extract the coupling constants, one needs to analytically continue to the $p_1^2 > 0$ and $p_2^2 > 0$ domain.
- This is achieved by spectral representation of the coefficient functions

$$\Pi(p_1^2, p_2^2) = \int ds_1 ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + \cdots$$

where $\cdots$ stand for polynomials in p_1^2 or p_2^2.

Relations Between Correlation Functions

- To obtain the correlation function, first suitable currents have to be chosen:

$$
\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[\left(u^{aT}Cs^b\right)\gamma_5 d^c + t\left(u^{aT}C\gamma_5 s^b\right)d^c\right. \\
&- \left.\left(s^{aT}Cd^b\right)\gamma_5 u^c - t\left(s^{aT}C\gamma_5 d^b\right)u^c\right] \\
\eta^{\Sigma^+} &= -\frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d\to u)\ , \quad \eta^{\Sigma^-} = \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u\to d) \\
\eta^p &= \eta^{\Sigma^+}(s\to d)\ , \quad \eta^n = \eta^{\Sigma^-}(s\to u) \\
\eta^{\Xi^0} &= \eta^n(d\to s)\ , \quad \eta^{\Xi^-} = \eta^p(u\to s) \\
\eta^{\Lambda} &= -\sqrt{\frac{1}{6}}\epsilon^{abc}\left[2\left(u^{aT}Cd^b\right)\gamma_5 s^c + 2t\left(u^{aT}C\gamma_5 d^b\right)s^c + \left(u^{aT}Cs^b\right)\gamma_5 d^c\right. \\
&+ \left. t\left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c + t\left(s^{aT}C\gamma_5 d^b\right)u^c\right]
\end{aligned}
$$

- The Λ current can also be obtained from that of the Σ^0 by:

$$
\begin{aligned}
2\eta_{\Sigma^0}(d \leftrightarrow s) + \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda \\
2\eta_{\Sigma^0}(u \leftrightarrow s) - \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda
\end{aligned}
$$

- Convention: Let $\pi^0 = g_{\pi uu}\bar{u}u + g_{\pi dd}\bar{d}d + g_{\pi ss}\bar{s}s$
- Denote $\Pi^{B_2 \to B_1 \mathcal{M}} = \langle \mathcal{M}|B_1\bar{B}_2|0\rangle$ and $\Pi^{\Sigma^0\to\Sigma^0\pi^0} = g_{\pi uu}\langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle + g_{\pi dd}\langle\bar{d}d|\Sigma^0\bar{\Sigma}^0|0\rangle + g_{\pi ss}\langle\bar{s}s|\Sigma^0\bar{\Sigma}^0|0\rangle$
- Let $\Pi_1(u,d,s) = \langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle$, $\Pi_2(u,d,s) = \langle\bar{s}s|\Sigma^0\bar{\Sigma}^0|0\rangle$
- $\langle\bar{d}d|\Sigma^0\bar{\Sigma}^0|0\rangle = \Pi_1(d,u,s)$ since $\Sigma^0(u \leftrightarrow d) = \Sigma^0$

- Consider $\Pi^{\Sigma^+\to\Sigma^+\pi^0} = g_{\pi\bar{u}u}\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle + g_{\pi\bar{s}s}\langle\bar{s}s|\Sigma^+\bar{\Sigma}^+|0\rangle$
- Note that $\Sigma^0(d\to u) = -\sqrt{2}\Sigma^+$
- Hence $\langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle(d\to u) = 2\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle'$
- $\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle = 4\langle\bar{u}u|\Sigma^+\bar{\Sigma}^+|0\rangle'$
  $= 2\langle\bar{u}u|\Sigma^0\bar{\Sigma}^0|0\rangle(d\to u) = 2\Pi_1(u,u,s)$
- Hence $\Pi^{\Sigma^+\to\Sigma^+\pi^0} = \sqrt{2}\Pi_1(u,u,s)$

- What about charged mesons? Consider $\Pi^{\Sigma^0\to\Sigma^+\pi^-}$

- Hence it is natural to expect that $\langle\bar{d}d|\Sigma^0\bar{\Sigma}^0|0\rangle$ to be proportional to $\langle\bar{u}d|\Sigma^+\bar{\Sigma}^0|0\rangle$

- Indeed
  $\Pi^{\Sigma^0\to\Sigma^+\pi-} = \langle\bar{u}d|\Sigma^+\bar{\Sigma}^0|0\rangle = -\sqrt{2}\langle\bar{d}d|\Sigma^0\bar{\Sigma}^0|0\rangle = -\sqrt{2}\Pi_1(d,u,s)$

- Exchanging u and d quarks, one also obtains
  $\Pi^{\Sigma^0\to\Sigma^-\pi^+} = \langle\bar{d}u|\Sigma^-\bar{\Sigma}^0|0\rangle = \sqrt{2}\Pi_1(u,d,s)$

- Similarly

  $$\Pi^{\Xi^0\to\Xi^-\pi^+} = \langle\bar{d}u|\Xi^-\Xi^0|0\rangle = -\sqrt{2}\langle\bar{u}u|\Xi^0\bar{\Xi}^0|0\rangle = -\Pi_2(s,s,u)$$

- Exchanging u and d, one obtains $\Pi^{\Xi^-\to\Xi^+\pi^-} = -\Pi_2(s,s,d)$

- What about Λ?
- Using $2\Sigma^0(d \leftrightarrow s) = -\sqrt{3}\Lambda - \Sigma^0$ and the result $\Pi^{\Sigma^0\to\Sigma^-\pi^+} = \sqrt{2}\Pi_1(u,d,s)$, we have $2\sqrt{2}\Pi_1(u,s,d) = \sqrt{3}\Pi^{\Lambda\to\Xi^-K^+} + \Pi^{\Sigma^0\to\Xi^-K^+}$
- How to seperate the two correlation functions? Define $\Pi_3(u,d,s) = -\Pi^{\Sigma^0\to\Xi^-K^+} = -\langle\bar{s}u|\Xi^-\bar{\Sigma}^0|0\rangle$ and $\Pi_4(u,d,s) = -\Pi^{\Xi^-\to\Sigma^0K^-} = -\langle\bar{u}s|\Sigma^0\bar{\Xi}^-|0\rangle$

- How are the functions related to the F and D constants in the $SU(3)_f$ limit?
- In the $SU(3)_f$ limit, $\Pi_1 \propto \sqrt{2}F$, $\Pi_2 \propto \sqrt{2}(F-D)$, and $\Pi_4 = \Pi_3 \propto -(F+D)$

Explicit Expressions:

$$
\begin{aligned}
& \Pi_1(u,d,s) = \\
& \frac{f_{\mathcal{M}}}{64\pi^2} M^4 \left(m_s(1-t)^2 - m_d(1-t^2)\right) i_2(\phi_{\mathcal{M}}) \\
- \quad & \frac{\mu_{\mathcal{M}}}{64\pi^2} M^4 \left(1-\tilde{\mu}_{\mathcal{M}}^2\right)(1-t^2) i_2(\phi_\sigma) \\
- \quad & \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2 M^2 \left(m_s(1-t)^2 + 3m_d(1-t^2)\right) i_1(\mathcal{V},1) \left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right) \\
+ \quad & \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2 \left(m_s(1-t)^2 - m_d(1-t^2)\right) i_1(\mathcal{V}_\perp,1) \left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right) \\
+ \quad & \frac{f_{\mathcal{M}}}{768\pi^2} \langle g_s^2 GG\rangle \left(m_s(1-t)^2 - m_d(1-t^2)\right) i_2(\phi_{\mathcal{M}}) \left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right) \\
- \quad & \frac{m_0^2 + 2M^2}{3456 M^6} \mu_{\mathcal{M}} \left(1-\tilde{\mu}_{\mathcal{M}}^2\right) \langle g_s^2 GG\rangle \left(m_d\langle\bar{s}s\rangle + m_s\langle\bar{d}d\rangle\right)(3+2t+3t^2) i_2(\phi_\sigma)
\end{aligned}
$$

$$
\begin{aligned}
- \quad & \frac{m_0^2}{648 M^2}\left(1-\tilde{\mu}_{\mathcal{M}}^2\right)\mu_{\mathcal{M}}(m_s\langle\bar{d}d\rangle + m_d\langle\bar{s}s\rangle)(5+4t+5t^2) i_2(\phi_\sigma) \\
- \quad & \frac{f_{\mathcal{M}}}{3072\pi^2 M^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2\left(m_s(1-t)^2 - m_d(1-t^2)\right)\left(4 i_1(\mathcal{A}_\parallel, 1-2v) - i_2(\mathbb{A})\right) \\
+ \quad & \frac{f_{\mathcal{M}}}{768\pi^2 M^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2\left(m_s(1-t)^2 + m_d(1-t^2)\right) i_1(\mathcal{V}_\parallel, 1) \\
- \quad & \frac{f_{\mathcal{M}}}{1152\pi^2}\langle g_s^2 GG\rangle\left(m_s(1-t)^2 - m_d(1-t^2)\right) i_2(\varphi_{\mathcal{M}}) \\
- \quad & \frac{f_{\mathcal{M}}}{128\pi^2} m_{\mathcal{M}}^2 M^2\left(m_s(1-t)^2 - m_d(1-t^2)\right) i_2(\mathbb{A}) \\
- \quad & \frac{f_{\mathcal{M}}}{24} M^2\left(\langle\bar{s}s\rangle(1-t)^2 - \langle\bar{d}d\rangle(1-t^2)\right) i_1(\varphi_{\mathcal{M}}) \\
+ \quad & \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2 M^2\left(m_s(1-t)^2 - m_d(1-t^2)\right) i_1(\mathcal{A}_\parallel, 1-2v) \\
- \quad & \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2\left(m_s(1-t)^2 + 2m_d(1-t^2)\right) i_1(\mathcal{V}_\parallel, 1)
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2 \left(m_s (1-t)^2 - m_d (1-t^2) \right) i_1(\mathcal{V}_\perp, 1) \\
+ \quad & \frac{\mu_{\mathcal{M}}}{144} \left(1 - \tilde{\mu}_{\mathcal{M}}^2\right) \left[\langle \bar{d} d \rangle \left(-3 m_d (1-t^2) - 2 m_s (3 + 2t + 3t^2) \right) \right. \\
- \quad & \left. \langle \bar{s} s \rangle \left(3 m_s (1-t^2) + m_d (6 + 4t + 6t^2) \right) \right] i_2(\phi_\sigma) \\
+ \quad & \frac{f_{\mathcal{M}}}{432} m_0^2 \left(3 \langle \bar{s} s \rangle (1-t)^2 - 2 \langle \bar{d} d \rangle (1-t)^2 \right) i_2(\varphi_{\mathcal{M}}) \\
+ \quad & \frac{f_{\mathcal{M}}}{96} m_{\mathcal{M}}^2 \left(\langle \bar{s} s \rangle (1-t)^2 - \langle \bar{d} d \rangle (1-t^2) \right) i_2(\mathbb{A}) \\
- \quad & \frac{\mu_{\mathcal{M}}}{48} (1-t^2) (\langle \bar{d} d \rangle m_d - \langle \bar{s} s \rangle m_s) i_1'(\mathcal{T}, 1-2v) \\
- \quad & \frac{f_{\mathcal{M}}}{24} m_{\mathcal{M}}^2 \left(\langle \bar{s} s \rangle (1-t)^2 - \langle \bar{d} d \rangle (1-t^2) \right) i_1(\mathcal{A}_\parallel, 1-2v) \\
+ \quad & \frac{f_{\mathcal{M}}}{24} m_{\mathcal{M}}^2 \left(\langle \bar{d} d \rangle (1-t^2) + \langle \bar{s} s \rangle (1-t)^2 \right) i_1(\mathcal{V}_\parallel, 1)
\end{aligned}
$$

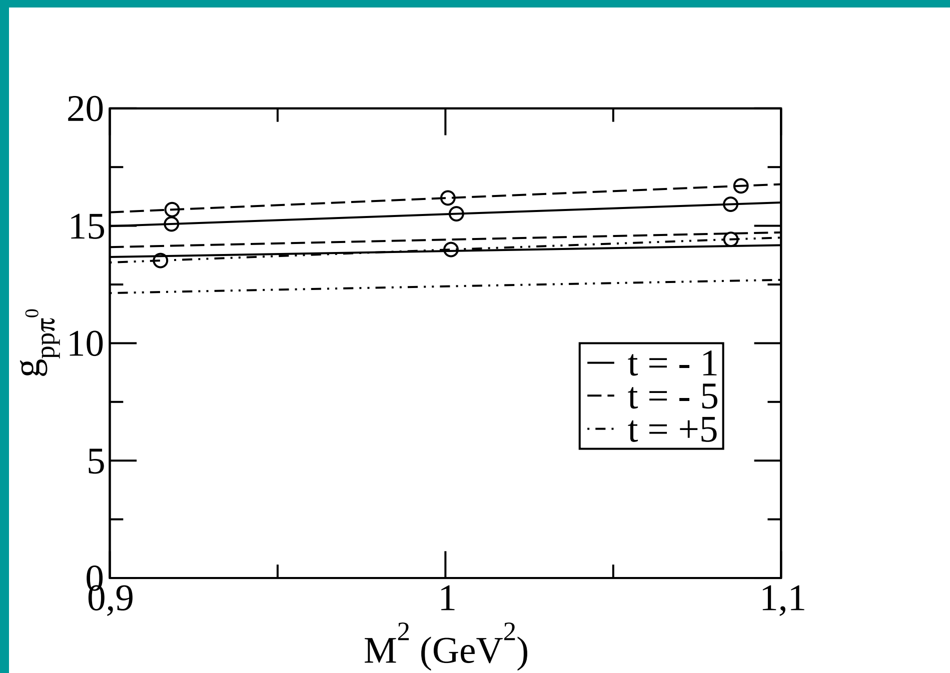


Figure 2: The M^2 dependence of the $g_{pp\pi}$ coupling constant.

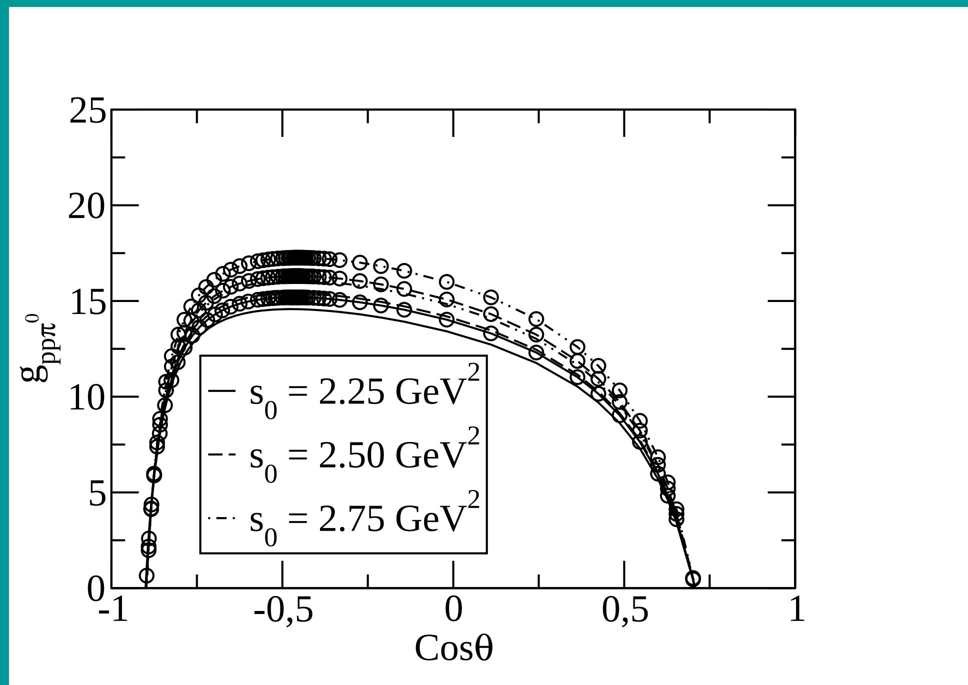


Figure 3: The θ dependence of the $g_{pp\pi}$ where $\tan\theta = t$

Channel	Gen. Current	$t=-1$	$SU(3)_f$	QSR*	QSR†	Exp.
$\Lambda \to nK$	-13 ± 3	-9.5 ± 1	-14.3	-2.37	-2.49	-13.5
$\Lambda \to \Sigma^+\pi^-$	10 ± 3	12 ± 1	10.0			
$\Lambda \to \Xi^0 K^0$	4.5 ± 2	-2.5 ± 0.5	4.25			
$n \to p\pi^-$	21 ± 4	20 ± 2	19.8			21.2
$n \to \Sigma^0 K^0$	-3.2 ± 2.2	-9.5 ± 0.5	-3.3	-0.025	-0.40	-4.25
$p \to \Lambda K^+$	-13 ± 3	-10 ± 1	-14.25	-2.37	-2.49	-13.5
$p \to p\pi^0$	14 ± 4	15 ± 1	Input	13.5		14.9
$p \to \Sigma^+ K^0$	4 ± 3	14 ± 1	5.75			
$\Sigma^0 \to nK^0$	-4 ± 3	-9.5 ± 1	-3.32	-0.025	-0.40	-4.25
$\Sigma^0 \to \Lambda\pi^0$	11 ± 3	12 ± 1.5	10.0	6.9		
$\Sigma^0 \to \Xi^0 K^0$	-13 ± 3	-13.5 ± 1	-14			
$\Sigma^- \to nK^-$	5 ± 3	15 ± 2	4.7			
$\Sigma^+ \to \Lambda\pi^+$	10 ± 3.5	12.5 ± 1	Input			
$\Sigma^+ \to \Sigma^0\pi^+$	-9 ± 2	-7.5 ± 0.7	-10.7	-11.9		
$\Xi^0 \to \Lambda K^0$	4.5 ± 1	-2.6 ± 0.3	4.25			
$\Xi^0 \to \Sigma^0 K^0$	-12.5 ± 3	-13.5 ± 1	-14			
$\Xi^0 \to \Sigma^+ K^-$	18 ± 4	19 ± 2	19.8			
$\Xi^0 \to \Xi^0\pi^0$	10 ± 2	0.3 ± 0.6	-3.32	-1.60		

Conclusion

- All the strong coupling constants are expressed in terms of four analytic functions
- The predicted coupling constants respect the $SU(3)_f$ symmetry
- There is good agreement with the experimental results.