A. Özpineci

Physics Department, METU

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Quantum Chromo Dynamics (QCD)

ozpineci@metu.edu.tr

Outline

- A Brief History of Particle Physics
- The Standard Model and QCD
- Some Lattice Calculations
- Overview of Sum Rules
- Conclusions

HISTORY

http://particleadventure.org/particleadventure/other/history/smt.html

- 1964

  Murray Gell-Mann and George Zweig tentatively put forth the idea of quarks.

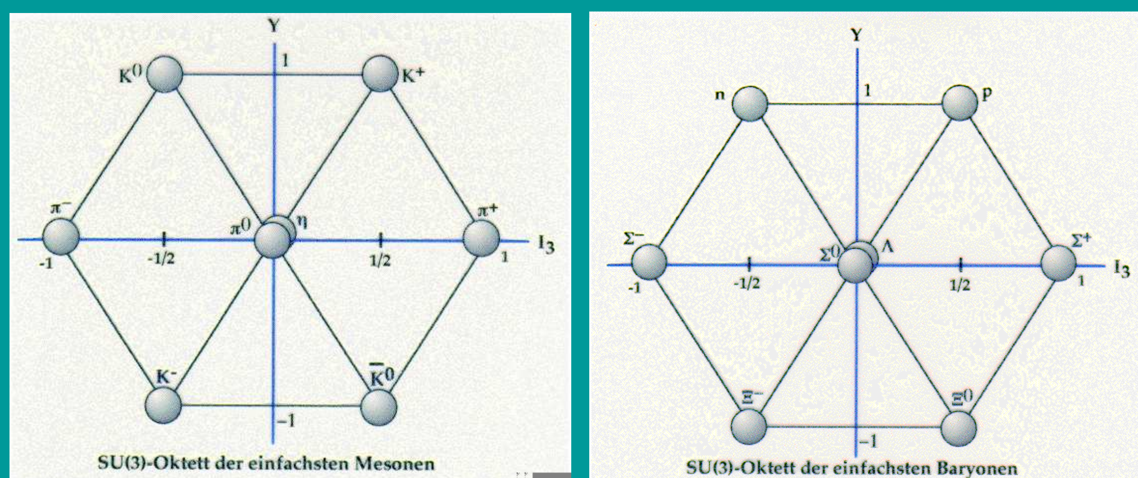


Figure 1: V. Linke, Institut fur Theoretische Physik , Freie Universitat Berlin
http://www-old.physik.fu-berlin.de/quarkstext.html

- 1964

  Since leptons had a certain pattern, several papers suggested a fourth quark carrying another flavor to give a similar repeated pattern for the quarks, now seen as the generations of matter. Very few physicists took this suggestion seriously at the time. Sheldon Glashow and James Bjorken coin the term ”charm” for the fourth (c) quark.

- 1965

  O.W. Greenberg, M.Y. Han, and Yoichiro Nambu introduce the quark property of color charge. All observed hadrons are color neutral.

- ...1966...

  The quark model is accepted rather slowly because quarks hadn't been observed.

- 1967

  Steven Weinberg and Abdus Salam separately propose a theory that unifies electromagnetic and weak interactions into the electroweak interaction. Their theory requires the existence of a neutral, weakly interacting boson (now called the Z0) that mediates a weak interaction that had not been observed at that time. They also predict an additional massive boson called the Higgs Boson that has not yet been observed.

- 1968-69

  At the Stanford Linear Accelerator, in an experiment in which electrons are scattered off protons, the electrons appear to be bouncing off small hard cores inside the proton. James Bjorken and Richard Feynman analyze this data in terms of a model of constituent particles inside the proton.

- 1973

  Donald Perkins, spurred by a prediction of the Standard Model, re-analyzes some old data from CERN and finds indications of weak interactions with no charge exchange (those due to a Z0 exchange.)

- 1973

  A quantum field theory of strong interaction is formulated. This theory of quarks and gluons (now part of the Standard Model) is similar in structure to quantum electrodynamics (QED), but since strong interaction deals with color charge this theory is called quantum chromodynamics (QCD). Quarks are determined to be real particles, carrying a color charge. Gluons are massless quanta of the strong-interaction field. This strong interaction theory was first suggested by Harald Fritzsch and Murray Gell-Mann.

- 1973

  David Politzer, David Gross, and Frank Wilczek discover that the color theory of the strong interaction has a special property, now called "asymptotic freedom." The property is necessary to describe the 1968-69 data on the substrate of the proton.

- D. J. Gross, "Twenty Five Years of Asymptotic Freedom," hep-th/9809060

  > The plan of the attack was twofold. First, I qould prove thath "ultraviolet stability," the vanishing of the effective coupling at short distances, later called asymptotic freedom, was necessary to explain scaling. Second, I would show that there existed no asymtotically free field theories. The latter was to be expected.

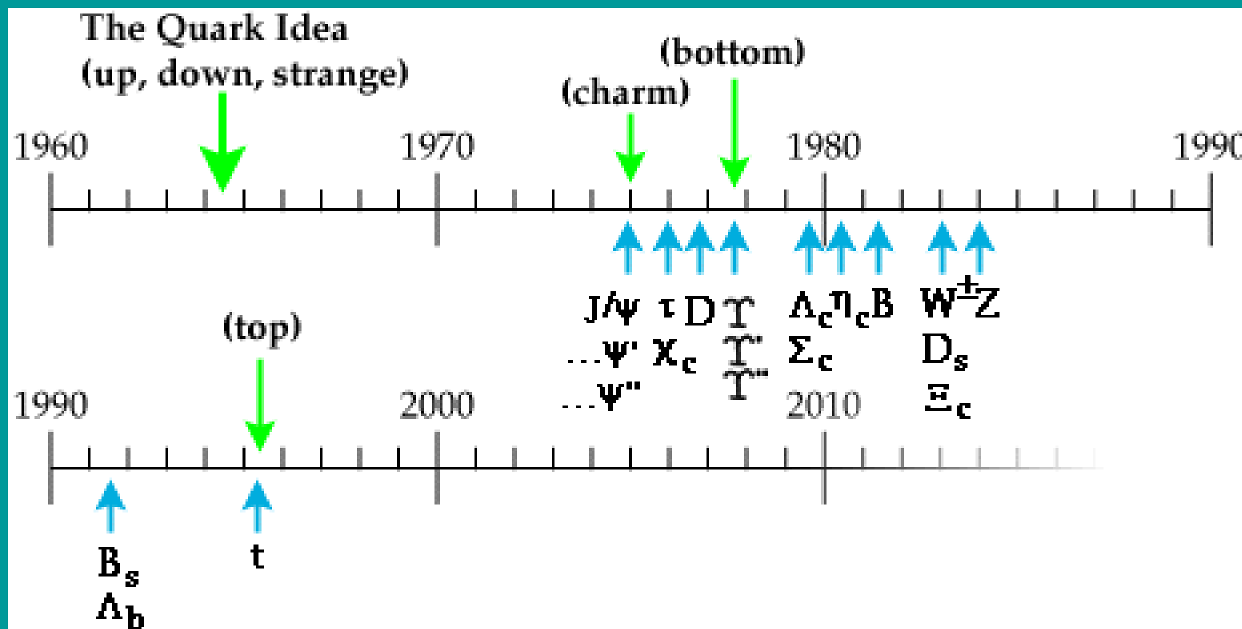


Figure 2: http://www-donut.fnal.gov/web_pages/standardmodelpg/TheStandardModel.html

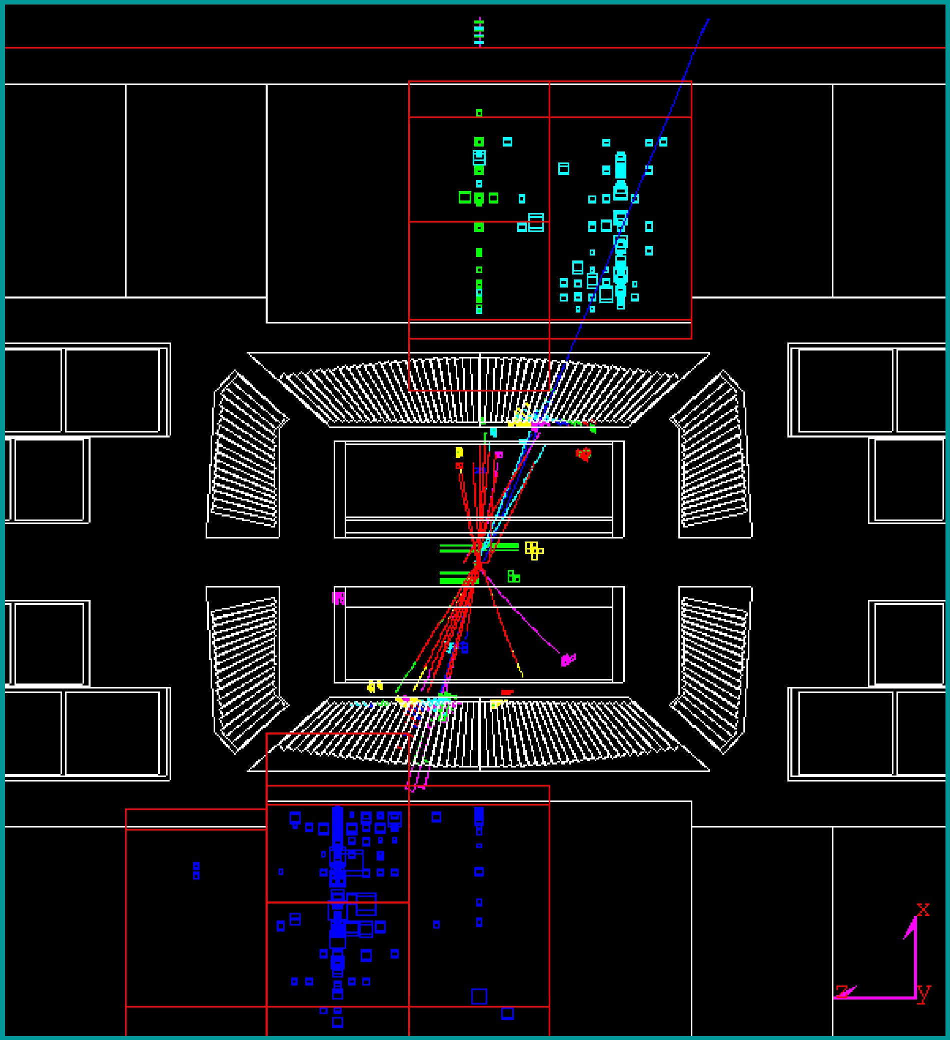


Figure 3: Date: 9 July 1996 at 14:13,
http://l3.web.cern.ch/l3/scan_program/160GeV/events/R656706E4683xzhcal.gif

The Standard Model

- Standard Model is a renormalizable local gauge field theory based on the gauge group $SU(3) \times SU(2) \times U(1)$ and with the particle content:

  quark sector:
  $$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, u_R, d_R, c_R, s_R, t_R, b_R$$

  lepton sector:
  $$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L, e_R, \mu_R, \tau_R$$

  gauge sector: $\gamma, g, Z, W^\pm$

- The Lagrangian of the standard model is:

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}\mathbf{A}_{\mu\nu}\mathbf{A}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu} \\
&+ \bar{R}i\not{\mathcal{D}}R + \bar{L}i\not{\mathcal{D}}L - K(\bar{L}R\phi + \phi^\dagger\bar{R}L) \\
&+ (\mathcal{D}_\mu\phi)^\dagger\mathcal{D}^\mu\phi - V(\phi^\dagger\phi) \qquad (1)
\end{aligned}
$$

where $V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$,
$i\mathcal{D}_\mu = i\partial_\mu + g_1\frac{Y}{2}B_\mu + \frac{g_2}{2}\mathbf{A}_\mu + \frac{g_3}{2}\mathbf{G}_\mu$

Quantum Chromo Dynamics: QCD

- QCD is a local gauge field theory based on the gauge group $SU(3)$, with 6 quark fields, each transforming in the fundamental representation of the gauge group.
- The Lagrangian of QCD is

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i \not{\mathcal{D}} - m_q) q \tag{2}$$

where $i\mathcal{D}_\mu = i\partial_\mu + g_s G^a_\mu \frac{\lambda^a}{2}$ and $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^a_{bc} G^b_\mu G^c_\nu$

- This Lagrangian is not the most general Lagrangian that one can write. The term

$$\theta \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta}$$

can also be added.

- Experimentally, θ is very small: the strong CP problem.

- What is the spectrum of QCD, i.e. its bound states, the particles that we observe in nature?
- Is QCD really confining? That is, are the colored particles absent in the spectrum?
- What are the properties of the bound states?
- How can we do calculations in QCD? Can we use perturbation theory?

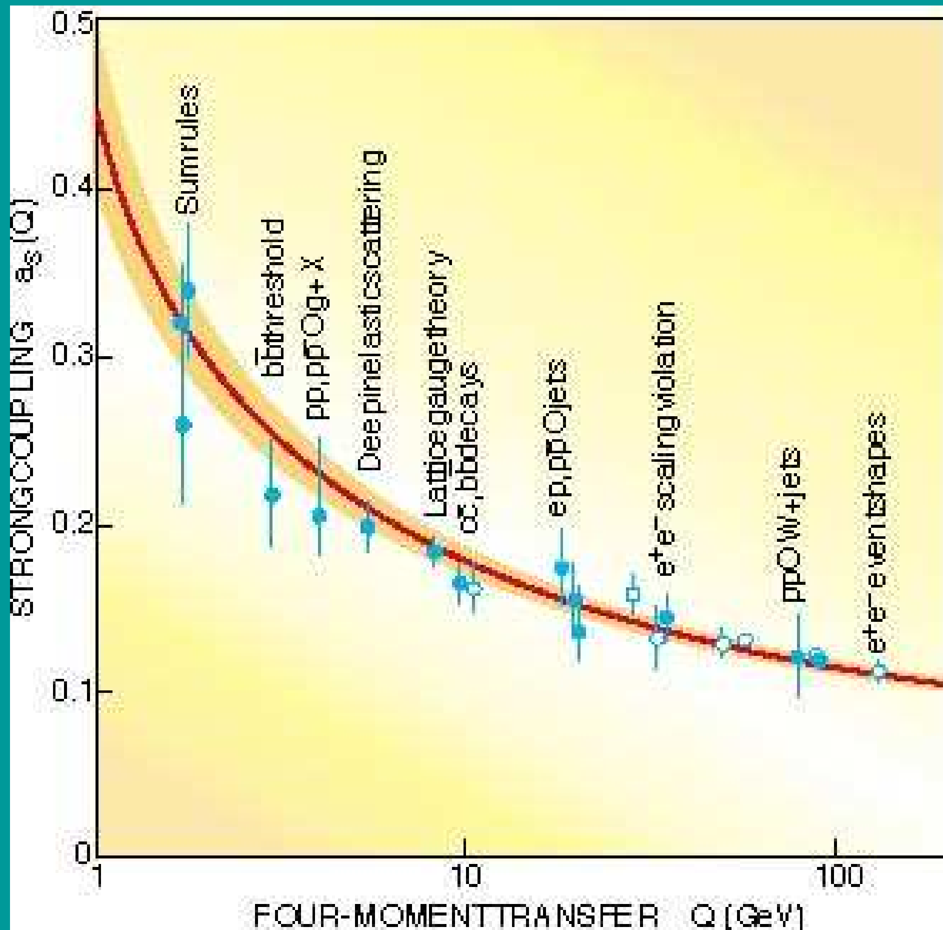


Figure 4: Running coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$

F. Wilczek, "QCD Made Simple", Physics Today, Vol. 53 p.22, http://www.aip.org/pt/vol-53/iss-8/p22.html#fig4

- We can use perturbation theory at high momentum transfer since the coupling constant is small due to asymptotic freedom.
- At lower energies, we need to use models or non-perturbative methods.
- Two methods based on the QCD Lagrangian: Lattice Calculations and QCD sum rules

- In order to find the spectrum, one studies the correlation functions of the form:

$$\Pi = i \int d^4 p e^{ipx} \langle 0 | \mathcal{T} j_1(x) j_1^\dagger(0) | 0 \rangle \tag{3}$$

- The poles tell us the masses of the bound states and the residues give us information on the structure of the bound state.
- Lattice Calculations calculate the correlation function numerically by discretization the space time.
- Requires tremendous amounts of computing power.

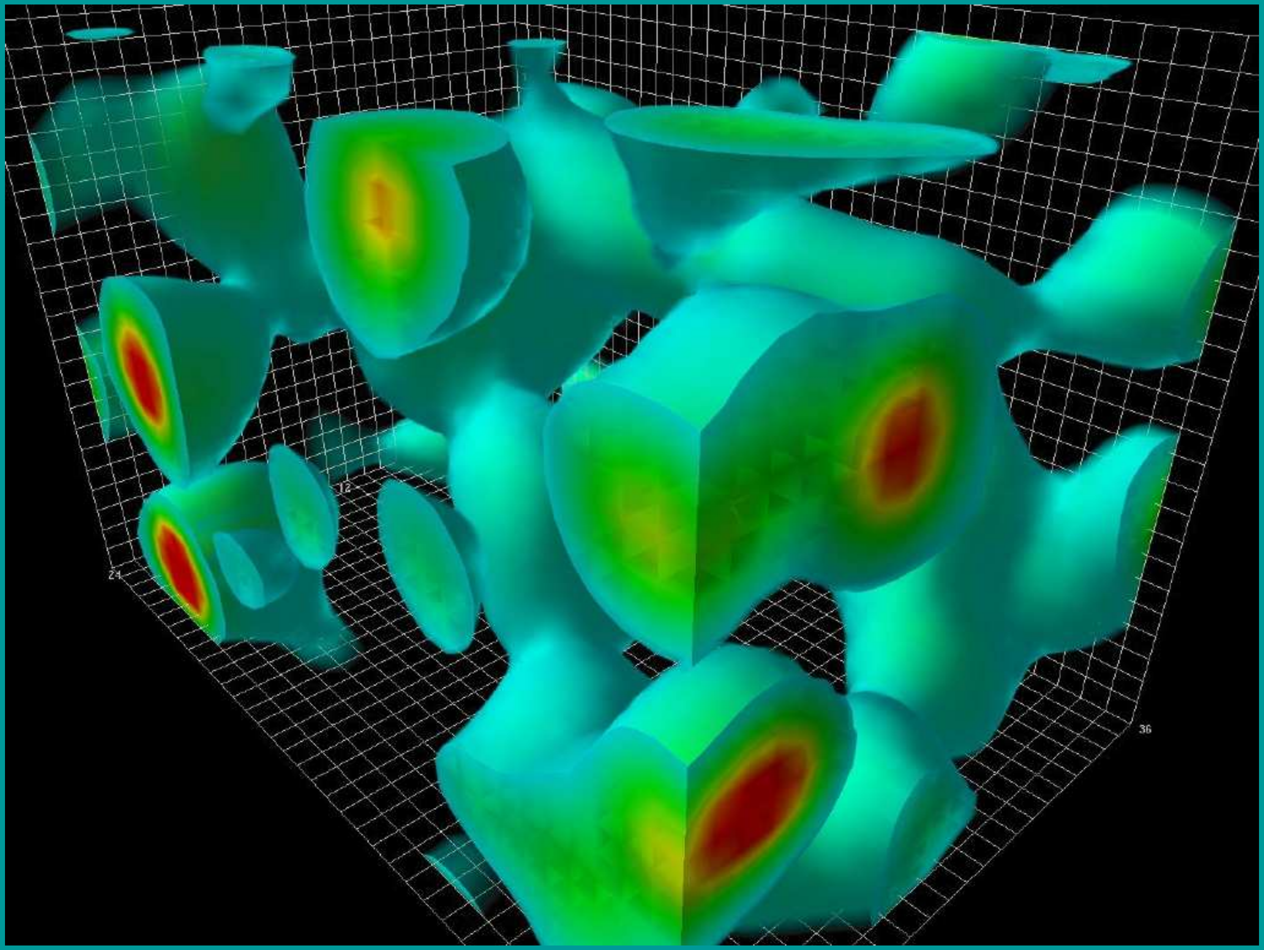


Figure 5: Action Density in the QCD Vacuum
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html

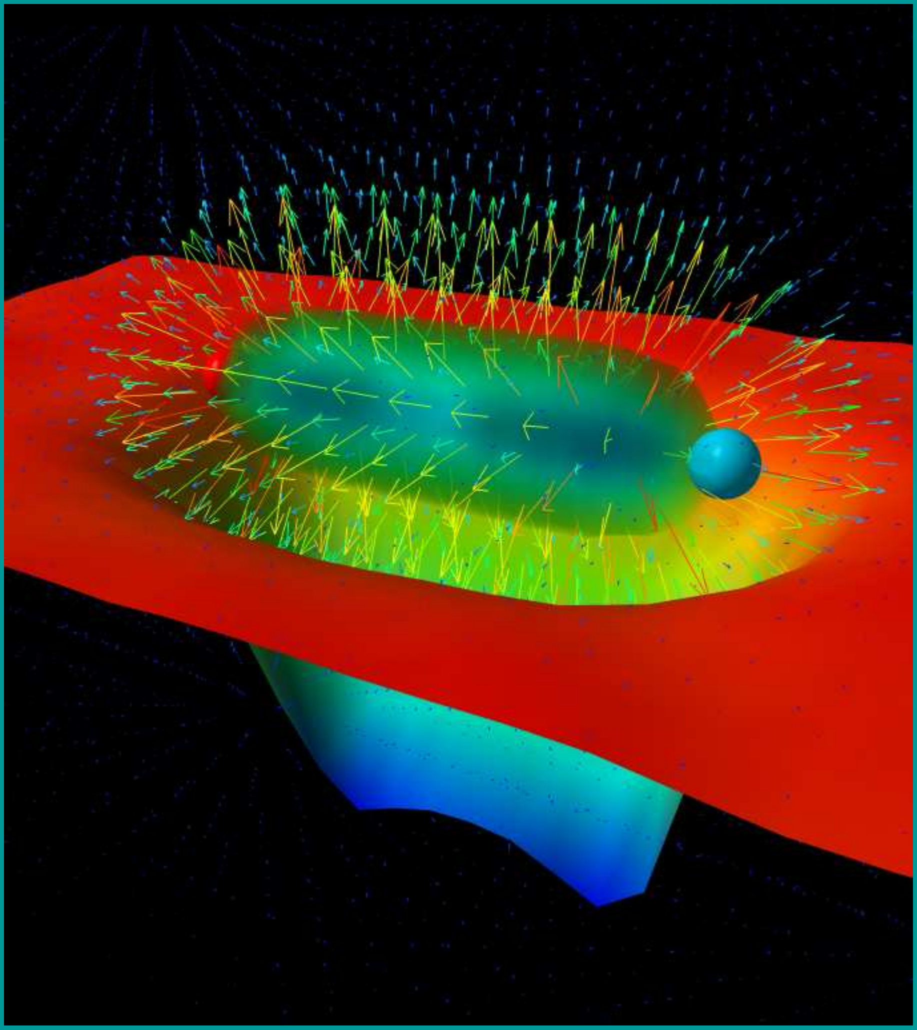


Figure 6: Flux Tube in a meson
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html

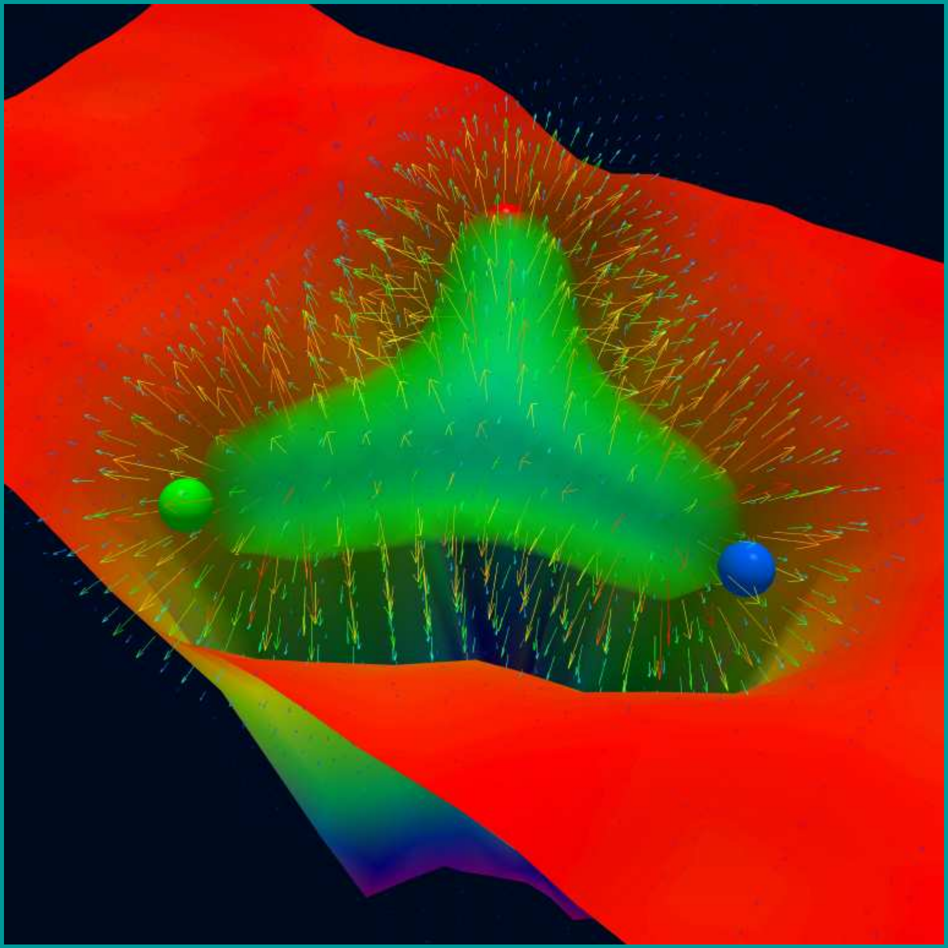


Figure 7: Flux Tubes in a Hadron
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html

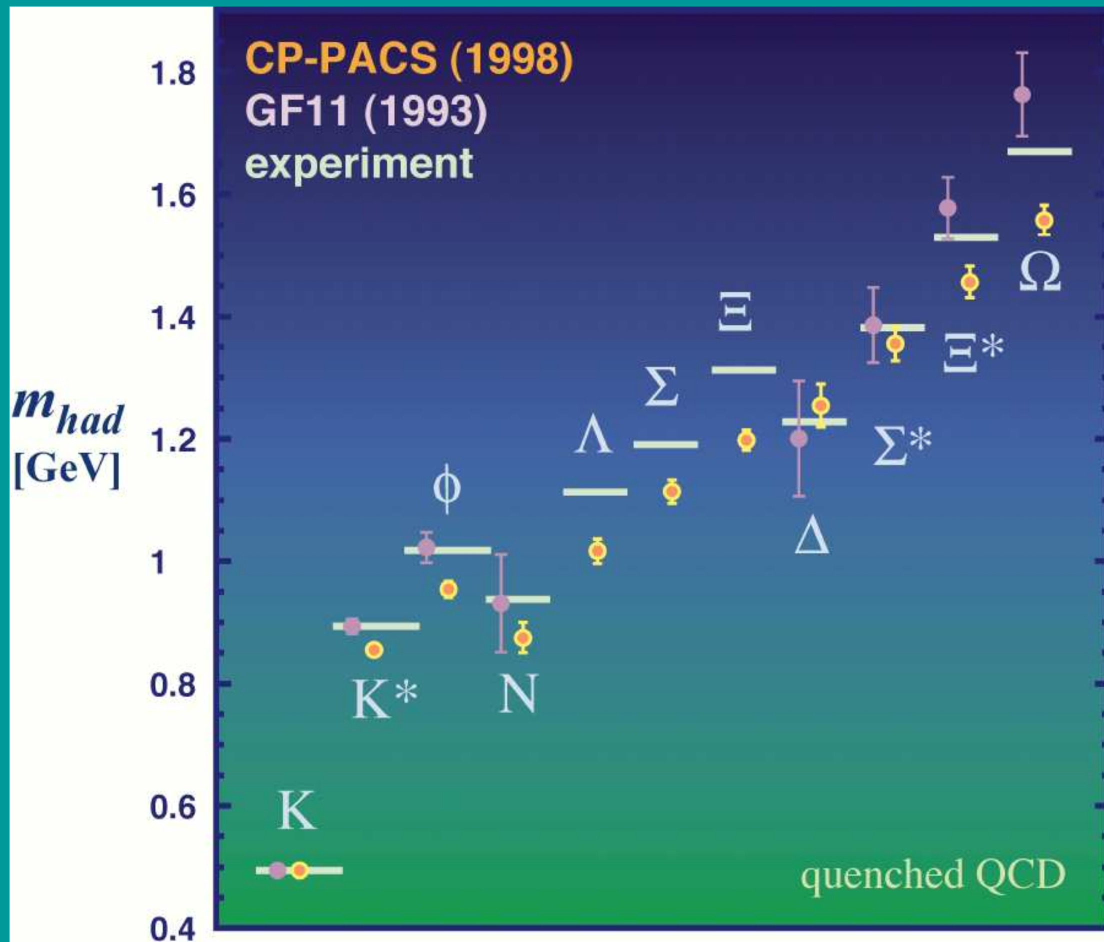


Figure 8: QCD spectrum predicted by Lattice Calculations
K. Kanaya, http://www.ccs.tsukuba.ac.jp/people/kanaya/Research/Lattice/index-e.html

- Limitations of Lattice Calculation:

  Requires enormous computational power

  Can not give limiting behavior

  For the time being, can not simulate very light and very heavy quarks due to finite lattice size and finite lattice spacing.

  Does not give a deeper understanding

- QCD Sum rules (M. A. Shifman, V. I. Vainshtein, V. I. Zakharov, *Nucl. Phys.* **B147** (1979) 385.) is an analytical method which relates the properties of the hadrons to the parameter of QCD Lagrangian and the structure of the QCD vacuum.
- The poles of the correlation function are located at $p^2 > 0$, physical states of positive mass.

- In the $p^2 << 0$ region, the main contribution to the correlation function comes from regions for which $x \sim 0$.
-

$$\mathcal{T} j_1(x) j_1^\dagger(0) = \sum_d C_d(x^2) O_d \qquad (4)$$

where O_d are local operators, $O_0 = 1$, $O_3 = \bar{q}\Gamma q$, $O_4 = G_{\mu\nu}G_{\alpha\beta}$, etc.

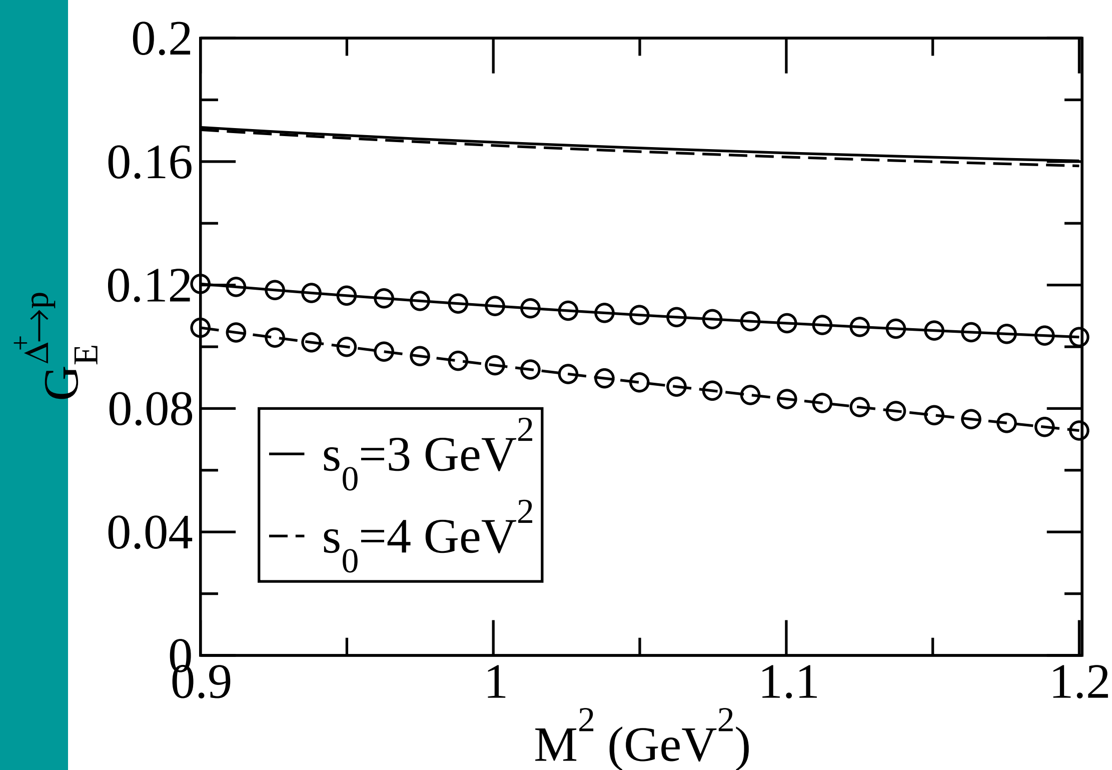
- In perturbation theory, $\langle 0|O_d|0\rangle = 0$ for $d \neq 0$
- Due to the non-trivial nature of QCD vacuum, these vacuum matrix elements are non-zero in the QCD vacuum. They are treated as phenomenological input parameters.

$$\Pi = \sum_d C_d(p^2)\langle\Omega|O_d|\Omega\rangle \tag{5}$$

- In practice, one only keeps the first few vacuum condensates and fits them to a few experimental data. And then uses these values to predict the properties of other hadrons.

- The two expressions are matched using spectral representation and analytic continuation in the complex p^2 plane.
- If there are contribution in the $P^2 = -p^2 > 0$ domain which are exponentially suppressed, e^{-Pr} such as the contributions from the instantons, these can be neglected for $P^2 > 0$, but after analytical continuation, they become oscillatory and no longer exponentially suppressed.

Process	G_E	$G_E^{[11]}$	G_M	$G_M^{[11]}$	$\mathcal{R}_{EM}(\%)$	$\mathcal{R}_{EM}^{[11]}(\%)$
$\Delta^+ \to p$	0.17 ± 0.05	-0.04(11)	2.5 ± 1.3	2.01(33)	-6.8	3(8)
$\Delta^0 \to n$	-0.17 ± 0.05	0.04(11)	-2.5 ± 1.3	-2.01(33)	-6.8	3(8)
$\Sigma^{*+} \to \Sigma^+$	-0.08 ± 0.02	-0.06(8)	2.1 ± 0.85	2.13(16)	-3.8	5(6)
$\Sigma^{*0} \to \Sigma^0$	-0.034 ± 0.007	-0.02(4)	0.89 ± 0.38	0.87(7)	-3.8	4(6)
$\Sigma^{*-} \to \Sigma^-$	-0.010 ± 0.004	0.020(10)	-0.31 ± 0.10	-0.38(4)	-3.2	8(4)
$\Xi^{*0} \to \Xi^0$	-0.09 ± 0.02	0.03(4)	-2.2 ± 0.74	-2.26(14)	-4.1	2.4(27)
$\Xi^{*-} \to \Xi^-$	0.011 ± 0.003	-0.018(7)	0.31 ± 0.11	0.38(3)	-3.5	7.4(30)


0.2
0.16
0.12
0.08
0.04
0
0.9
1
1.1
1.2
M^2 (GeV^2)
$G_E^{\Delta^+ \to p}$
s_0=3 GeV^2
s_0=4 GeV^2

Conclusion:

NO CONCLUSION