Radiative Decays of Decuplet to Octet Baryons in Light Cone QCD Sum Rules (LCQSR)

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Outline

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- Sum Rules
- Light Cone QCD Sum Rules for radiative Decuplet to Octet Transition
- Numerical Analysis
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Introduction

- Electromagnetic interactions of hadrons provide unique insight into the structure of hadrons.
- That the nucleon could be deformed was proposed more than 20 years ago [1]
- It is still an intensive theoretical and experimental area of activity.
- The process $\Delta \to N\gamma$ can give us information on this aspect also.
- If the wave functions of both the initial and final states are spherical, the quadrupole moments for this decay should vanish.
- Recent experimental results show that the quadrupole moments are non zero.
- The spin-parity selection rules allow for magnetic dipole (M1), and electric (E2) or coulomb (C2) quadrupole moments.
- The moments have been studied using various models.

- In the naive (spherical) quark model of the nucleon, Δ is a pure spin flip (M1) transition, and $E2 = C2 = 0$
- Experimentally, indeed M1 dominates over the other moments.
- In other refined models, small $E2$ and $C2$ moments are predicted.
- In "QCD inspired" quark model, one introduces a tensor forces which introduce a d-state admixture to the nucleon.
- Stronger contributions are expected from pion clouds.

QCD Sum Rules

- Electromagnetic decays of baryons also constitute an important test for non perturbative methods.
- To study these processes, a reliable non perturbative method is needed.
- One of these methods is the Light Cone QCD sum rules approach.
- In QCD sum rules approach, properties of hadrons are expressed in terms of the vacuum properties through non zero condensates in the vacuum
- One starts by studying a correlation function of the form:

$$\Pi(p,q) = i \int d^4x e^{ipx} \langle \gamma(q) | \mathcal{T} \eta_{\mathcal{O}}(x) \bar{\eta}_{\mathcal{D}\mu}(0) | 0 \rangle \tag{1}$$

- For $p^2 > 0$, the correlation function is calculated in terms of hadronic parameters.
- In the deep Euclidean region, $p^2 << 0$, the correlation function is calculated using the OPE in terms of QCD degrees of freedom.
- Sum rules are obtained by matching the two representation using spectral representation.

Light Cone QCD Sum Rules for radiative Decuplet to Octet Transition

Hadronic Representation

- For $p^2 > 0$, two complete sets of hadronic states can be inserted to get:

$$\Pi(p,q) = \frac{\langle 0|\eta_{\mathcal{O}}|Octet(p)\rangle}{p^2 - m^2} \langle Octet|Decuplet\rangle_\gamma \frac{\langle Decuplet(p+q)|\eta_{\mathcal{D}\mu}|0\rangle}{(p+q)^2 - M^2} + \cdots \tag{2}$$

where $\cdots$ stand for the contribution of higher states and continuum.

- The matrix elements of the currents between the single baryon state and the vacuum are defined as

$$\begin{aligned} \langle 0|\eta_{\mathcal{O}}|Octet(p)\rangle &= \lambda_{\mathcal{O}} u(p,s) \\ \langle Decuplet(p+q)|\eta_{\mathcal{D}\mu}|0\rangle &= \lambda_{\mathcal{D}} u_\mu(p+q,s') \end{aligned} \tag{3}$$

where λ's are the residues and u_μ is the Rarita-Schwinger spinor.

- The electromagnetic vertex can be parametrized in terms of three form factors as[2]:

$$\begin{aligned}\langle Octet|Decuplet\rangle_\gamma &= eu(p,s)\left\{G_1\left(q_\rho \not\varepsilon - \varepsilon_\rho \not q\right)\gamma_5\right. \\ &+ G_2\left((P\varepsilon)q_\rho - (Pq)\varepsilon_\rho\right)\gamma_5 \\ &+ \left. G_3\left((q\varepsilon)q_\rho - q^2\varepsilon_\rho\right)\gamma_5\right\} u^\rho(p+q)\end{aligned} \tag{4}$$

  where $P = \frac{1}{2}\left(p + (p+q)\right)$ and ε is the photon polarization vector.

- In our case, we will be considering a real photon($q^2 = 0$ and $\varepsilon.q = 0$), hence G_3 will not contribute.

- For experimental studies, it is desirable to use form factors which diagonalize the cross section.
- Linear combinations of G_i give us the magnetic dipole, G_M, electric quadrupole, G_E and Coulomb quadrupole, G_C form factors[2]:

$$\begin{aligned}
G_M &= \left[\left((3M+m)(M+m)-q^2\right)\frac{G_1}{M}+\left(M^2-m^2\right)G_2+2q^2G_3\right]\frac{m}{3(M+m)} \\
G_E &= \left[\left(M^2-m^2+q^2\right)\frac{G_1}{M}+\left(M^2-m^2\right)G_2+2q^2G_3\right]\frac{m}{3(M+m)} \\
G_C &= \left[2MG_1+\frac{1}{2}\left(3M^2+m^2-q^2\right)G_2+\left(M^2-m^2+q^2\right)G_3\right]\frac{2m}{3(M+m)}
\end{aligned} \tag{5}$$

- The helicity amplitudes are given as:

$$\begin{aligned}
A_{1/2} &= -\frac{\sqrt{3}}{4m}\left(M^2-m^2\right)\left(\frac{m}{E}\right)^{\frac{1}{2}}\frac{1}{\sqrt{2q}}\left(G_M-3G_E\right)\frac{e}{2m} \\
A_{3/2} &= -\frac{3}{4m}\left(M^2-m^2\right)\left(\frac{m}{E}\right)^{\frac{1}{2}}\frac{1}{\sqrt{2q}}\left(G_M+G_E\right)\frac{e}{2m}
\end{aligned} \tag{6}$$

- Using the matrix elements for the currents and the vertex, and

$$\sum_s u_\alpha(p,s)\bar{u}_\beta(p,s) = -\left(\not{p}+M\right)\left\{g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2p_\alpha p_\beta}{3M^2} + \frac{p_\alpha\gamma_\beta - p_\beta\gamma_\alpha}{3M}\right\} \tag{7}$$

one can obtain an expression for the correlation function in terms of the form factors.

- Schematically, we have

$$\begin{aligned} T_\mu &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}\left[G_2(\varepsilon p)q_\mu \not{q}\not{p}\gamma_5 + \right. \\ &+ M\left(2G_1 - G_2(M-m)\right)(\varepsilon p)q_\mu\gamma_5 \\ &- \left.\left(2G_1 - G_2(M-m)\right)(\varepsilon p)q_\mu \not{p}\gamma_5 - \left(2G_1 + G_2 m\right)(\varepsilon p)q_\mu \not{q}\gamma_5\right] + \text{other structures with } \not{\varepsilon} \text{ at} \\ &\quad \text{the beginning and } \gamma_\mu,\ (p+q)_\mu \text{ or } \varepsilon_\mu \text{ at the end} \end{aligned} \tag{8}$$

where we have chosen the ordering $\not{\varepsilon}\not{q}\not{p}\gamma_\mu$

- The reason for choosing this ordering and the structure $\propto q_\mu$ is that, spin-1/2 particles do not contribute to this structure.

$$\langle 0|\eta_{\frac{3}{2}\mu}|\frac{1}{2}(p+q)\rangle = \left(A'(p+q)_\mu + B'\gamma_\mu\right)\gamma_5 u(p) \qquad (9)$$

- Using the matrix element, one can not create a structure $\propto q_\mu$ with this ordering.
- In our study, we choose the structures $(\varepsilon p)q_\mu\gamma_5$ and $q_\mu \not{\varepsilon}\gamma_5$.
- The coefficients of these structures in the hadronic representation are:

For the $(\varepsilon p)q_\mu\gamma_5$ structure:

$$\Pi_2 = e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}M\left(2G_1+G_2(m-M)\right) \qquad (10)$$

For the $q_\mu \not{\varepsilon}\gamma_5$ structure:

$$\Pi_4 = -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}\left(2G_1+G_2 m\right) \qquad (11)$$

- Both can be written in the form

$$\Pi_i = \int_0^\infty ds_1 ds_2 \frac{\rho_i^{phen}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)} + \cdots \tag{12}$$

where $\cdots$ are polynomials in p^2 and $(p+q)^2$, and

$$\begin{aligned}
\rho_2^{phen}(s_1, s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}} M \left(2G_1 + G_2(m - M)\right) \delta(s_1 - m^2)\delta(s_2 - M^2) + \cdots \\
\rho_4^{phen}(s_1, s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}} \left(2G_1 + G_2 m\right) \delta(s_1 - m^2)\delta(s_2 - M^2) + \cdots
\end{aligned} \tag{13}$$

where $\cdots$ represent the contributions of the excited states and continuum.

QCD Representation

- For $p^2 << 0$ and $(p+q)^2 << 0$, the main contributions come from small distances, hence one can use the OPE.
- One must choose appropriate currents.

  For the Decuplet:

$$
\begin{aligned}
\eta_\mu^{\Sigma^{*0}} &= \sqrt{\frac{2}{3}}\epsilon^{abc}\left[\left(u^{aT}C\gamma_\mu d^b\right)s^c + \left(d^{aT}C\gamma_\mu s^b\right)u^c + \left(s^{aT}C\gamma_\mu u^b\right)d^c\right] \\
\eta_\mu^{\Sigma^{*+}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(d \to u) \\
\eta_\mu^{\Sigma^{*-}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(u \to d) \\
\eta_\mu^{\Delta^{*+}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(s \to u) \\
\eta_\mu^{\Delta^{*0}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(s \to d) \\
\eta_\mu^{\Xi^{*0}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(d \to s) \\
\eta_\mu^{\Xi^{*-}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(u \to s)
\end{aligned}
\tag{14}
$$

For the Octet:

$$
\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[-\left(u^{aT}Cs^b\right)\gamma_5 d^c + \left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c - \left(s^{aT}C\gamma_5 d^b\right)u^c\right] \\
\eta^{\Sigma^+} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d \to u) \\
\eta^{\Sigma^-} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u \to d) \\
\eta^{p} &= -\sqrt{2}\eta^{\Sigma^0}(s \to u) \\
\eta^{n} &= -\sqrt{2}\eta^{\Sigma^0}(s \to d) \\
\eta^{\Xi^0} &= -\sqrt{2}\eta^{\Sigma^0}(d \to s) \\
\eta^{\Xi^-} &= -\sqrt{2}\eta^{\Sigma^0}(u \to s) \\
\eta^{\Lambda} &= \sqrt{\frac{1}{6}}\epsilon^{abc}\left[2\left(u^{aT}Cd^b\right)\gamma_5 s^c - 2\left(u^{aT}C\gamma_5 d^b\right)s^c + \left(u^{aT}Cs^b\right)\gamma_5 d^c\right. \\
&- \left.\left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c - \left(s^{aT}C\gamma_5 d^b\right)u^c\right] \qquad (15)
\end{aligned}
$$

- Accept for the correlation function for $\Sigma^{*0} \to \Lambda$, the others can be obtained from the correlation function for $\Sigma^{*0} \to \Sigma^0$:

$$
\begin{aligned}
\Pi^{\Sigma^{*+}\to\Sigma^+} &= \Pi^{\Sigma^{*0}\to\Sigma^0}(d \to u) \\
\Pi^{\Sigma^{*-}\to\Sigma^-} &= \Pi^{\Sigma^{*0}\to\Sigma^0}(u \to d) \\
\Pi^{\Delta^+\to p} &= -2\Pi^{\Sigma^{*0}\to\Sigma^0}(s \to u) \\
\Pi^{\Delta^0\to n} &= -2\Pi^{\Sigma^{*0}\to\Sigma^0}(s \to d) \\
\Pi^{\Xi^{*0}\to\Xi^0} &= -2\Pi^{\Sigma^{*0}\to\Sigma^0}(d \to s) \\
\Pi^{\Xi^{*-}\to\Xi^-} &= -2\Pi^{\Sigma^{*0}\to\Sigma^0}(u \to s)
\end{aligned}
\tag{16}
$$

- It is also possible to obtain the correlation function for $\Sigma^{*0} \to \Lambda$ from the correlation function for $\Sigma^{*0} \to \Sigma^0$ [3, 4].

- Note that:

$$2\eta^{\Sigma^0}(d \leftrightarrow s) = -\sqrt{3}\eta^{\Lambda} - \eta^{\Sigma^0} \tag{17}$$

- and hence

$$-\sqrt{3}\Pi^{\Sigma^{*0}\to\Lambda} = 2\Pi^{\Sigma^{*0}\to\Sigma^0}(d \leftrightarrow s) + \Pi^{\Sigma^{*0}\to\Sigma^0} \tag{18}$$

- Using Wick Theorem, one can express the correlation function in terms of diagrams like the ones in Fig. 1

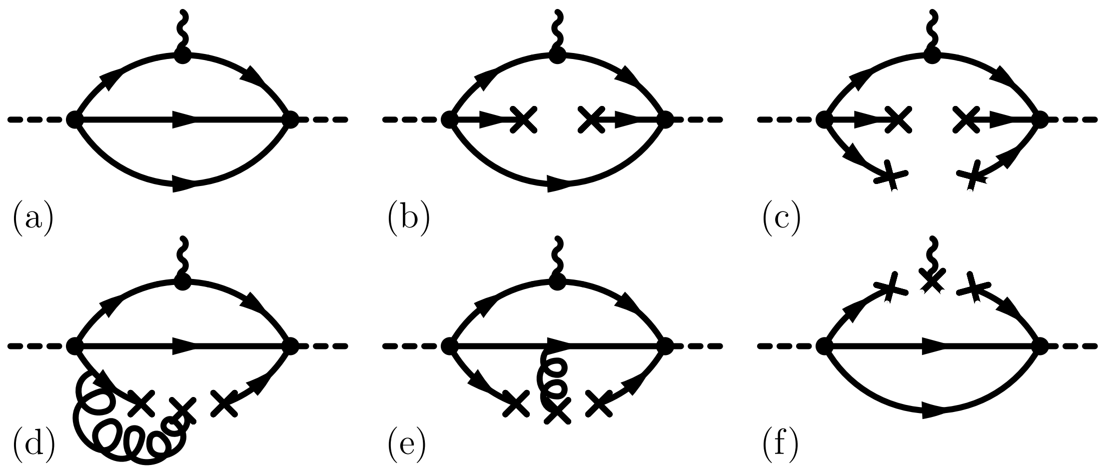

Figure 1: Some of the Feynman diagrams contributing to the correlation function

- The propagator for the light quarks are:

$$
\begin{aligned}
S_q(x) \quad = \quad & \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i\frac{m_q}{6}\not{x}\right) \\
& - i g_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma_{\mu\nu} - u x^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
& \left. - i\frac{m_q}{32\pi^2} G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\right] \qquad (19)
\end{aligned}
$$

where Λ is the energy cut off separating perturbative and non perturbative regimes.

- The emission of the photon can be both perturbative or non perturbative:
- To calculate the perturbative emission one uses the free quark propagator and the quark-photon vertex factor $-ie\gamma_\mu$.
- The non perturbative emission is described by matrix elements of the form $\langle\gamma(q)|\bar{q}(x)\Gamma q(0)|0\rangle$
- These matrix elements are expanded around the light cone $x^2 = 0$ and can be expressed in terms of photon wave functions: [5]:

$$
\begin{aligned}
&\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 due^{i\bar{u}qx}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right) \\
&-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 due^{i\bar{u}qx}h_\gamma(u) \\
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)g_s \tilde{G}_{\mu\nu}(vx)q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)g_s \tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha\gamma_5 q(0)|0\rangle = \cdots \\
&\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle = \cdots
\end{aligned} \tag{20}
$$

- With these ingredients, the correlation function can be calculated in terms of the photon wave functions, condensates and QCD parameters.

- The correlation function can be written is the spectral representation:

$$\Pi_i = \int_0^\infty \frac{\rho_i^{OPE}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)} + \cdots \tag{21}$$

  where $\cdots$ are polynomials in p^2 or $(p+q)^2$.

- For the spectral densities, on the phenomenological side we have:

$$
\begin{aligned}
\rho_2^{phen}(s_1,s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)\delta(s_1-m^2)\delta(s_2-M^2)+\cdots \\
\rho_4^{phen}(s_1,s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)\delta(s_1-m^2)\delta(s_2-M^2)+\cdots
\end{aligned}
\tag{22}
$$

  and we also have calculated the spectral densities using QCD parameters: $\rho_2^{OPE}(s_1,s_2)$ and $\rho_4^{OPE}(s_1,s_2)$.

- The two problems that are to be solved in order two obtain the sum rules:

  The contributions of the higher states and the continuum are not known.

  The are unknown polynomials in the spectral representations of the correlation function

- To model the contributions of the higher states and the continuum, quark-hadron duality is used, i.e.

$$\begin{aligned}
\rho_2^{phen}(s_1,s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)\delta(s_1-m^2)\delta(s_2-M^2)+\rho_2^{OPE}\theta(s_1-s_0)\theta(s_2-s_0) \\
\rho_4^{phen}(s_1,s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)\delta(s_1-m^2)\delta(s_2-M^2)+\rho_4^{OPE}\theta(s_1-s_0)\theta(s_2-s_0) \qquad (23)
\end{aligned}$$

- To eliminate the unknown polynomials, the results are Borel transformed with respect to $p_1^2=p^2$ and $p_2^2=(p+q)^2$:

$$\begin{aligned}
\frac{1}{(p_i^2-m_i^2)^n} &\rightarrow \frac{(-1)^n}{\Gamma(n)}\frac{1}{M_i^{2(n-1)}}e^{-\frac{m^2}{M^2}} \\
p_i^{2n} &\rightarrow 0 \qquad (24)
\end{aligned}$$

  where M_i^2 are called the Borel parameters.

- Borel transformation also suppresses the contributions of the higher states and the continuum.

- The sum rules are obtained by equating both sides of the Borel transformed expressions:

$$
\begin{aligned}
e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \int_0^{s_0}\rho_2^{OPE}(s_1,s_2)e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}} \\
-e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \int_0^{s_0}\rho_4^{OPE}(s_1,s_2)e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}
\end{aligned} \tag{25}
$$

- For the sum rules we obtain:

$$
\begin{aligned}
\sqrt{3}\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\Sigma_2^{\Sigma^{*0}\to\Sigma^0}e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \left(e_d+e_u-2e_s\right)\frac{M^6}{32\pi^4}E_2(x)+\cdots \\
\sqrt{3}\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\Sigma_4^{\Sigma^{*0}\to\Sigma^0}e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \left(e_u m_u+e_d m_d-2e_s m_s\right)\frac{11M^4}{384\pi^2}E_1(x)+\cdots
\end{aligned} \tag{26}
$$

  where $\frac{1}{M^2}=\frac{1}{M_1^2}+\frac{1}{M_2^2}$, M_i^2 are the Borel parameters, $x=\frac{s_0}{M^2}$, s_0 is the continuum threshold, the functions $E_n(x)$ are used to subtract the contributions of the higher states and continuum and are defined as:

$$
E_n(x)=1-e^{-x}\sum_{i=0}^{n}\frac{x^i}{i!} \tag{27}
$$

- In our numerical analysis, we set $M_1^2=M_2^2=2M^2$ and $u_0=\frac{M_1^2}{M_1^2+M_2^2}=\frac{1}{2}$

- An interesting limit to consider is the $SU(3)_f$ symmetry limit: i.e. the limit $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ and $m_u = m_d = m_s = m_q$

- In this limit, we get

$$\Pi^{\Sigma^{*0} \to \Sigma^0} = (e_u + e_d - 2e_s)\mathcal{C} = \mathcal{C} \tag{28}$$

- Setting $s \to u$ and multiplying by -2, we get

$$\Pi^{\Delta^+ \to p} = -2(e_d - e_u)\mathcal{C} = 2\mathcal{C} = 2\Pi^{\Sigma^{*0} \to \Sigma^0} \tag{29}$$

- Similarly

$$2\Pi^{\Sigma^{*0} \to \Sigma^0} = \Pi^{\Delta^+ \to p} = -\Pi^{\Delta^0 \to n} = \Pi^{\Sigma^{*+} \to \Sigma^+} = -\Pi^{\Xi^{*0} \to \Xi^0} \tag{30}$$

and

$$\Pi^{\Sigma^{*-} \to \Sigma^-} = \Pi^{\Xi^{*-} \to \Xi^-} = 0 \tag{31}$$

- The last ingredients needed to obtain a prediction for Σ_2 and Σ_4, are the residues $\lambda_{\mathcal{O}}$ and $\lambda_{\mathcal{D}}$ which can be obtained using the mass sum rules.

- One considers the correlator

$$\Pi = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \eta(x) \bar{\eta}(0) | 0 \rangle \tag{32}$$

- For the octet, on the phenomenological side it reduces to

$$\Pi = \lambda_{\mathcal{O}}^2 \frac{\not{p} + m}{p^2 - m^2} + \cdots \tag{33}$$

- For the decuplet

$$\Pi_{\mu\nu} = -\lambda_{\mathcal{D}}^2 \frac{\not{p} + M}{p^2 - M^2} \left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu \gamma_\nu - \frac{2 p_\mu p_\nu}{3M^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3M} \right) \tag{34}$$

- Note that the mass sum rules do not give us the sign of λ, hence LCQSR does not predict the sign of Σ_2 and Σ_4 separately, it only predicts their relative sign.

Numerical Analysis

For the numerical values of the input parameters, the following values are used:
$\langle \bar{u}u \rangle (1\ GeV) = \langle \bar{d}d \rangle (1\ GeV) = -(0.243)^3\ GeV^3$, $\langle \bar{s}s \rangle (1\ GeV) = 0.8 \langle \bar{u}u \rangle (1\ GeV)$,
$m_0^2(1\ GeV) = 0.8$, $\chi(1\ GeV) = -4.4\ GeV^{-2}$, $\Lambda = 300\ MeV$ and $f_{3\gamma} = -0.0039\ GeV^2$.

- M^2 is a completely arbitrary parameter, and the predictions should be independent of its numerical value.
- An upper bound for M^2 is determined by requiring that the contributions of the higher states and the continuum are below a certain limit.
- A lower bound is obtained by requiring that the contributions of the term containing the highest power of $\frac{1}{M^2}$ is less then a limit.

Structure	$s_0(GeV^2)$	$M^2(GeV^2)$	continuum (%)	highest power of $\frac{1}{M^2}$(%)
$\Sigma_2^{\Delta\to N}$	5.0	1.5-1.7	40	40
$\Sigma_4^{\Delta\to N}$	4.5	1.3-1.5	20	20
$\Sigma_2^{\Sigma^*\to\Sigma}$	6.5	1.6-1.8	30	30
$\Sigma_4^{\Sigma^*\to\Sigma}$	5.0	1.4-1.6	20	20
$\Sigma_2^{\Xi^*\to\Xi}$	5.0	1.7-1.9	20	20
$\Sigma_4^{\Xi^*\to\Xi}$	5.0	1.2-1.4	20	20

Table 1: The continuum thresholds, s_0, the Borel mass, M^2, regions and the contributions of the continuum and the highest dimensional operator to the various structures for various processes

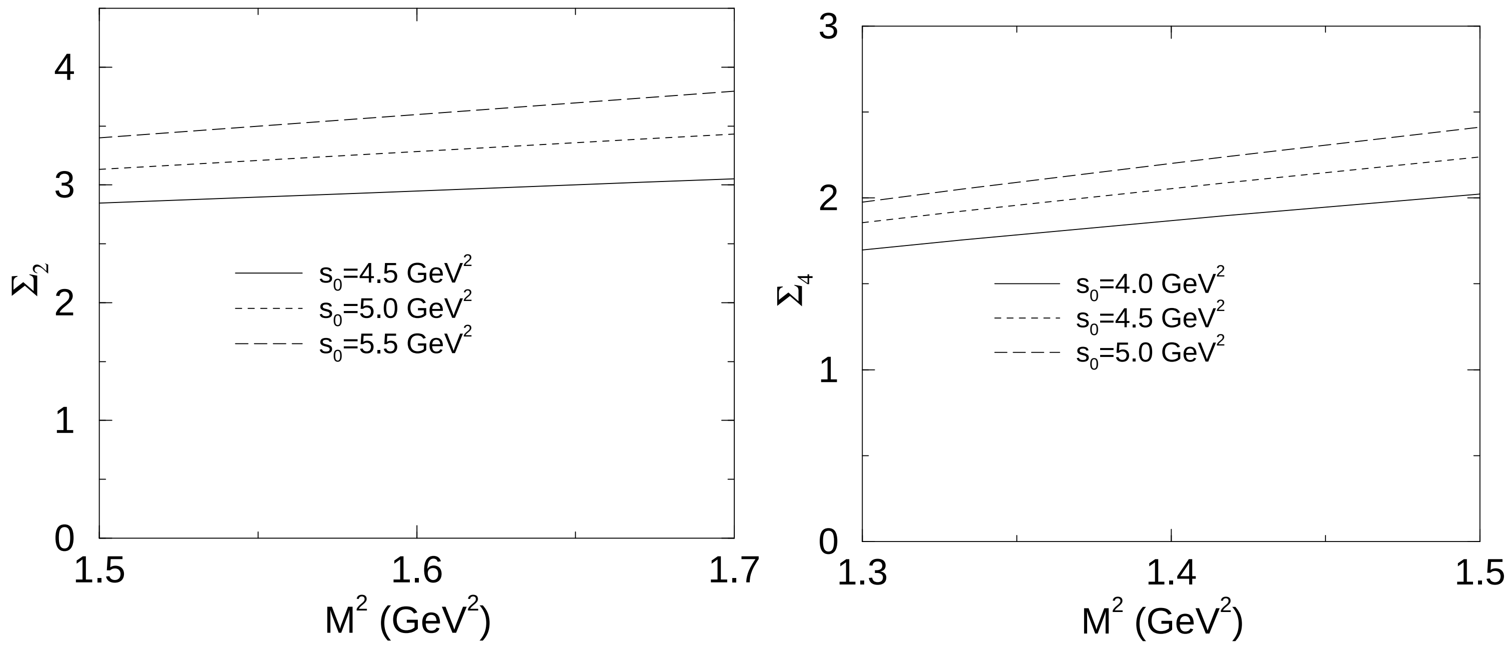

Figure 2: The M^2 dependence of $\Sigma_i^{\Delta^+ \to p}$

Process	Σ_2	Σ_4
$\Delta^+ \to p$	3.3 ± 0.5	2.0 ± 0.4
$\Delta^0 \to n$	-3.3 ± 0.5	-2.0 ± 0.4
$\Sigma^{*+} \to \Sigma^+$	2.6 ± 0.3	1.3 ± 0.2
$\Sigma^{*0} \to \Sigma^0$	1.2 ± 0.2	0.6 ± 0.1
$\Sigma^{*0} \to \Lambda$	-1.3 ± 0.2	-0.5 ± 0.1
$\Sigma^{*-} \to \Sigma^-$	-0.28 ± 0.01	-0.05 ± 0.02
$\Xi^{*0} \to \Xi^0$	-2.1 ± 0.3	-0.7 ± 0.2
$\Xi^{*-} \to \Xi^-$	0.26 ± 0.02	-0.003 ± 0.017

Table 2: The predictions of Σ_2 and Σ_4 for various decays.

Process	G_1	G_2	G_E	G_M	$G_E/G_M(\%)$
$\Delta^+ \to p$	1.5 ± 0.2	-1.1 ± 0.7	0.015 ± 0.057	1.7 ± 0.3	1 ± 4
$\Delta^0 \to n$	-1.5 ± 0.2	1.1 ± 0.7	-0.015 ± 0.057	-1.7 ± 0.3	1 ± 4
$\Sigma^{*+} \to \Sigma^+$	1.2 ± 0.1	-0.9 ± 0.4	-0.005 ± 0.021	1.8 ± 0.2	-0.2 ± 1.2
$\Sigma^{*0} \to \Sigma^0$	0.56 ± 0.09	-0.4 ± 0.2	0 ± 0.01	0.8 ± 0.1	-0.1 ± 1.5
$\Sigma^{*0} \to \Lambda$	-0.6 ± 0.1	0.6 ± 0.2	0.016 ± 0.016	-0.75 ± 0.12	-2 ± 2
$\Sigma^{*-} \to \Sigma^-$	-0.12 ± 0.01	0.17 ± 0.02	0.006 ± 0.002	-0.18 ± 0.01	-3.2 ± 0.8
$\Xi^{*0} \to \Xi^0$	-0.95 ± 0.14	0.9 ± 0.3	0.028 ± 0.024	-1.5 ± 0.2	-1.7 ± 1.4
$\Xi^{*-} \to \Xi^-$	0.11 ± 0.01	-0.17 ± 0.02	-0.009 ± 0.002	0.17 ± 0.02	- 5.5 ± 0.8

Table 3: The predictions on the moments for various decays. The magnetic moments are given in terms of natural magnetons

Process	G_E	$G_E^{[6]}$	G_M	$G_M^{[6]}$	$\mathcal{R}_{EM}(\%)$	$\mathcal{R}_{EM}^{[6]}(\%)$
$\Delta^+ \to p$	0.015 ± 0.057	-0.05(13)	1.7 ± 0.3	2.46(41)	-1 ± 4	3(8)
$\Delta^0 \to n$	-0.015 ± 0.057	0.05(13)	-1.7 ± 0.3	-2.46(41)	-1 ± 4	3(8)
$\Sigma^{*+} \to \Sigma^+$	-0.005 ± 0.021	-0.08(10)	1.8 ± 0.2	2.61(20)	0.2 ± 1.2	5(6)
$\Sigma^{*0} \to \Sigma^0$	0 ± 0.01	-0.03(5)	0.8 ± 0.1	1.07(8)	0.1 ± 1.5	4(6)
$\Sigma^{*-} \to \Sigma^-$	0.006 ± 0.002	0.024(12)	-0.18 ± 0.01	-0.47(5)	3.2 ± 0.8	8(4)
$\Xi^{*0} \to \Xi^0$	0.028 ± 0.024	0.04(5)	-1.5 $\pm$ 0.2	-2.77(17)	1.7 ± 1.4	2.4(27)
$\Xi^{*-} \to \Xi^-$	-0.009 $\pm$ 0.002	-0.022(9)	0.17 ± 0.02	0.47(4)	5.5 ± 0.8	7.4(30)

Table 4: Our results together with the results from lattice [6]

	Particle Data Group	Our Work
$A_{1/2}(\times 10^{-3} GeV^{-1/2})$	-135 ± 6	-107 ± 18
$A_{3/2}(\times 10^{-3} GeV^{-1/2})$	-255 ± 8	-192 ± 32
$\mathcal{R}_{EM}(\%)$	-2.5 ± 0.5	-1 ± 4

Table 5: Comparison of our results with the experimental results for the decay $\Delta^+ \to p\gamma$

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