TUTORIAL PROBLEMS ON KINEMATICS

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PROBLEM 1

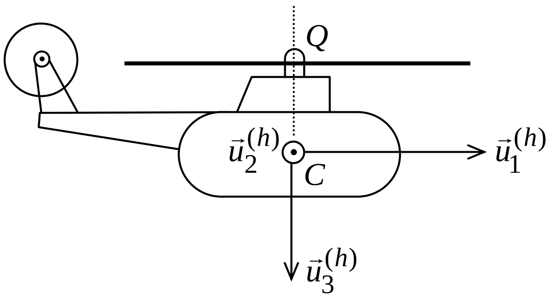


Figure 1

Figure 1 shows a helicopter moving with respect to an earth-fixed frame $\mathcal{F}_e(O)$. Let the frames attached to the helicopter and its rotor be $\mathcal{F}_h(C)$ and $\mathcal{F}_r(Q)$ respectively. The orientational relationships involving these frames are described as follows:

$$\mathcal{F}_e \xrightarrow[\psi]{\vec{u}_3^{(e)}} \mathcal{F}_m \xrightarrow[\theta]{\vec{u}_2^{(m)}} \mathcal{F}_n \xrightarrow[\phi]{\vec{u}_1^{(n)}} \mathcal{F}_h \xrightarrow[\gamma]{\vec{u}_3^{(h)}} \mathcal{F}_r$$

Let the dyadics $\breve{R}_{r/h}$ and $\breve{R}_{r/e}$ represent the rotation operators that orient the rotor with respect to the helicopter and the earth.

a) Obtain the transformation matrices $\hat{C}^{(e,h)}$, $\hat{C}^{(h,r)}$, and $\hat{C}^{(e,r)}$.

b) Obtain the matrix representations of $\breve{R}_{r/h}$ and $\breve{R}_{r/e}$ in $\mathcal{F}_h(C)$.

c) Obtain the matrix representations of $\breve{R}_{r/h}$ and $\breve{R}_{r/e}$ in $\mathcal{F}_e(O)$.

Express all the required matrices using exponential rotation matrices and their products.

d) The unit vector $\vec{w}$ (along one of the rotor blades) is obtained by rotating $\vec{w}_0 // \vec{u}_1^{(e)}$ (the first basis vector of $\mathcal{F}_e$) with $\breve{R}_{er} = \breve{R}_{r/e}$. Formulate this operation *separately* as observed in $\mathcal{F}_h$ and $\mathcal{F}_e$. Thus, express $\overline{w}^{(h)} = \{\vec{w}\}^{(h)}$ and $\overline{w}^{(e)} = \{\vec{w}\}^{(e)}$ using the results of the previous parts. Check your expressions by verifying the fact that $\overline{w}^{(e)} = \hat{C}^{(e,h)}\overline{w}^{(h)}$.

SOLUTION

a) $\hat{C}^{(e,h)} = \hat{C}^{(e,m)}\hat{C}^{(m,n)}\hat{C}^{(n,h)} = e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}$.

$\hat{C}^{(h,r)} = e^{\tilde{u}_3\gamma}$.

$\hat{C}^{(e,r)} = \hat{C}^{(e,h)}\hat{C}^{(h,r)} = e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}e^{\tilde{u}_3\gamma}$.

b) $\{\breve{R}_{r/h}\}^{(h)} = \hat{R}_{hr}^{(h)} = \hat{C}^{(h,r)} = e^{\tilde{u}_3\gamma}$.

$\{\breve{R}_{r/e}\}^{(h)} = \hat{R}_{er}^{(h)} = \hat{C}^{(h,e)}\hat{R}_{er}^{(e)}\hat{C}^{(e,h)} = \hat{C}^{(h,e)}\hat{C}^{(e,r)}\hat{C}^{(e,h)} = \hat{C}^{(h,r)}\hat{C}^{(e,h)}$

$= e^{\tilde{u}_3\gamma}e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi} = e^{\tilde{u}_3(\gamma+\psi)}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}$.

c) $\{\breve{R}_{r/h}\}^{(e)} = \hat{R}_{hr}^{(e)} = \hat{C}^{(e,h)}\hat{R}_{hr}^{(h)}\hat{C}^{(h,e)} = \hat{C}^{(e,h)}\hat{C}^{(h,r)}\hat{C}^{(h,e)}$

$= e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}e^{\tilde{u}_3\gamma}e^{-\tilde{u}_1\phi}e^{-\tilde{u}_2\theta}e^{-\tilde{u}_3\psi}$.

$\{\breve{R}_{r/e}\}^{(e)} = \hat{R}_{er}^{(e)} = \hat{C}^{(e,r)} = e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}e^{\tilde{u}_3\gamma}$.

d) $\vec{w} = \breve{R}_{er} \cdot \vec{u}_1^{(e)}$.

$\overline{w}^{(h)} = \hat{R}_{er}^{(h)}\overline{u}_1^{(e/h)} = \hat{R}_{er}^{(h)}\hat{C}^{(h,e)}\overline{u}_1^{(e/e)} = \hat{R}_{er}^{(h)}\hat{C}^{(h,e)}\overline{u}_1 = [\hat{C}^{(h,r)}\hat{C}^{(e,h)}]\hat{C}^{(h,e)}\overline{u}_1$

$= \hat{C}^{(h,r)}\overline{u}_1 = e^{\tilde{u}_3\gamma}\overline{u}_1; \quad \overline{w}^{(h)} = \overline{u}_1\cos\gamma + \overline{u}_2\sin\gamma$.

$\overline{w}^{(e)} = \hat{R}_{er}^{(e)}\overline{u}_1^{(e/e)} = \hat{R}_{er}^{(e)}\overline{u}_1 = \hat{C}^{(e,r)}\overline{u}_1$

$= e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}e^{\tilde{u}_3\gamma}\overline{u}_1$.

Verification:

$\overline{w}^{(e)} = \hat{C}^{(e,h)}\overline{w}^{(h)} = [e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}][e^{\tilde{u}_3\gamma}\overline{u}_1] = e^{\tilde{u}_3\psi}e^{\tilde{u}_2\theta}e^{\tilde{u}_1\phi}e^{\tilde{u}_3\gamma}\overline{u}_1$. (Checks!)

PROBLEM 2

Consider the same helicopter described in Problem 1.

a) Express $\overline{\omega}_{r/h}^{(h)} = \{\vec{\omega}_{r/h}\}^{(h)}$.

b) Express $\overline{\omega}_{r/e}^{(h)} = \{\vec{\omega}_{r/e}\}^{(h)}$.

c) Express $\overline{\alpha}_{r/e}^{(h)} = \{\vec{\alpha}_{r/e}\}^{(h)}$.

SOLUTION

a) $\vec{\omega}_{r/h} = \dot{\gamma}\vec{u}_3^{(h)} \Rightarrow \overline{\omega}_{r/h}^{(h)} = \dot{\gamma}\overline{u}_3$.

b) $\vec{\omega}_{r/e} = \vec{\omega}_{r/h} + \vec{\omega}_{h/n} + \vec{\omega}_{n/m} + \vec{\omega}_{m/e} = \dot{\gamma}\vec{u}_3^{(h)} + \dot{\phi}\vec{u}_1^{(h)} + \dot{\theta}\vec{u}_2^{(n)} + \dot{\psi}\vec{u}_3^{(m)}$;

$\overline{\omega}_{r/e}^{(h)} = \dot{\gamma}\overline{u}_3^{(h/h)} + \dot{\phi}\overline{u}_1^{(h/h)} + \dot{\theta}\overline{u}_2^{(n/h)} + \dot{\psi}\overline{u}_3^{(m/h)}$,

$\overline{\omega}_{r/e}^{(h)} = \dot{\gamma}\overline{u}_3 + \dot{\phi}\overline{u}_1 + \dot{\theta}\hat{C}^{(h,n)}\overline{u}_2 + \dot{\psi}\hat{C}^{(h,m)}\overline{u}_3$,

$\overline{\omega}_{r/e}^{(h)} = \dot{\gamma}\overline{u}_3 + \dot{\phi}\overline{u}_1 + \dot{\theta}e^{-\tilde{u}_1\phi}\overline{u}_2 + \dot{\psi}e^{-\tilde{u}_1\phi}e^{-\tilde{u}_2\theta}\overline{u}_3$,

$\overline{\omega}_{r/e}^{(h)} = \overline{u}_1(\dot{\phi} - \dot{\psi}s\theta) + \overline{u}_2(\dot{\theta}c\phi + \dot{\psi}s\phi c\theta) + \overline{u}_3(\dot{\gamma} - \dot{\theta}s\phi + \dot{\psi}c\phi c\theta)$.

c) $\vec{\alpha}_{r/e} = D_r\vec{\omega}_{r/e} = D_h\vec{\omega}_{r/e} + \vec{\omega}_{h/r} \times \vec{\omega}_{r/e} = D_h\vec{\omega}_{r/e} - \vec{\omega}_{r/h} \times \vec{\omega}_{r/e}$;

$$\bar{\alpha}_{r/e}^{(h)} = \dot{\bar{\omega}}_{r/e}^{(h)} - \dot{\gamma}\tilde{u}_3\bar{\omega}_{r/e}^{(h)},$$

$$\bar{\alpha}_{r/e}^{(h)} = \bar{u}_1(\ddot{\phi} - \ddot{\psi}s\theta + \cdots) + \bar{u}_2(\ddot{\theta}c\phi + \ddot{\psi}s\phi c\theta + \cdots) + \bar{u}_3(\ddot{\gamma} - \ddot{\theta}s\phi + \ddot{\psi}c\phi c\theta + \cdots)$$
$$+ \bar{u}_1\dot{\gamma}(\dot{\theta}c\phi + \dot{\psi}s\phi c\theta) - \bar{u}_2\dot{\gamma}(\dot{\phi} - \dot{\psi}s\theta).$$

PROBLEM 3

A reference frame $\mathcal{F}$, which is rotating about its fixed origin O, coincides consecutively with the reference frames $\mathcal{F}_0$, $\mathcal{F}_1$, and $\mathcal{F}_2$ according to the following sequence:

$$\mathcal{F}_0 \xrightarrow{\vec{u}_1^{(0)},\ \theta_1} \mathcal{F}_1 \xrightarrow{\vec{u}_2^{(1)},\ \theta_2} \mathcal{F}_2$$

Let P be a point with *constant* coordinates x_1, x_2, x_3 in the rotating frame $\mathcal{F}$.

a) Find the coordinates of P in $\mathcal{F}_1$ and $\mathcal{F}_0$ when $\mathcal{F}$ coincides with $\mathcal{F}_2$.

b) Find the coordinates of P in $\mathcal{F}_2$ while $\mathcal{F}$ is yet coincident with $\mathcal{F}_1$.

c) As $\mathcal{F}$ rotates all the way from $\mathcal{F}_0$ to $\mathcal{F}_2$, the observers in $\mathcal{F}_0$ and $\mathcal{F}_1$ use the rotation matrices $\hat{R}_{02}^{(0)}$ and $\hat{R}_{02}^{(1)}$ respectively in order to relate the initial and final coordinates of P that they observe. Express $\hat{R}_{02}^{(0)}$ and $\hat{R}_{02}^{(1)}$ using exponential rotation matrices.

SOLUTION

a)

$$\bar{r}_{P_2}^{(1)} = \hat{C}^{(1,2)}\bar{r}_{P_2}^{(2)}, \quad \bar{r}_{P_2}^{(0)} = \hat{C}^{(0,2)}\bar{r}_{P_2}^{(2)} = \hat{C}^{(0,1)}\bar{r}_{P_2}^{(1)}$$

$$\bar{r}_{P_2}^{(2)} = \bar{r}_{P_0}^{(0)} = \bar{u}_1x_1 + \bar{u}_2x_2 + \bar{u}_3x_3$$

$$\hat{C}^{(0,1)} = e^{\tilde{u}_1\theta_1}, \quad \hat{C}^{(1,2)} = e^{\tilde{u}_2\theta_2}, \quad \hat{C}^{(0,2)} = e^{\tilde{u}_1\theta_1}e^{\tilde{u}_2\theta_2}$$

$$\bar{r}_{P_2}^{(1)} = e^{\tilde{u}_2\theta_2}(\bar{u}_1x_1 + \bar{u}_2x_2 + \bar{u}_3x_3) = e^{\tilde{u}_2\theta_2}\bar{u}_1x_1 + \bar{u}_2x_2 + e^{\tilde{u}_2\theta_2}\bar{u}_3x_3$$

$$\bar{r}_{P_2}^{(1)} = (\bar{u}_1c\theta_2 - \bar{u}_3s\theta_2)x_1 + \bar{u}_2x_2 + (\bar{u}_3c\theta_2 + \bar{u}_1s\theta_2)x_3$$

$$\bar{r}_{P_2}^{(1)} = \bar{u}_1(x_1c\theta_2 + x_3s\theta_2) + \bar{u}_2x_2 + \bar{u}_3(x_3c\theta_2 - x_1s\theta_2)$$

$$\bar{r}_{P_2}^{(0)} = e^{\tilde{u}_1\theta_1}[\bar{u}_1(x_1 c\theta_2 + x_3 s\theta_2) + \bar{u}_2 x_2 + \bar{u}_3(x_3 c\theta_2 - x_1 s\theta_2)]$$

$$\bar{r}_{P_2}^{(0)} = \bar{u}_1(x_1 c\theta_2 + x_3 s\theta_2) + e^{\tilde{u}_1\theta_1}\bar{u}_2 x_2 + e^{\tilde{u}_1\theta_1}\bar{u}_3(x_3 c\theta_2 - x_1 s\theta_2)$$

$$\bar{r}_{P_2}^{(0)} = \bar{u}_1(x_1 c\theta_2 + x_3 s\theta_2) + (\bar{u}_2 c\theta_1 + \bar{u}_3 s\theta_1)x_2 + (\bar{u}_3 c\theta_1 - \bar{u}_2 s\theta_1)(x_3 c\theta_2 - x_1 s\theta_2)$$

$$\bar{r}_{P_2}^{(0)} = \bar{u}_1(x_1 c\theta_2 + x_3 s\theta_2) + \bar{u}_2(x_2 c\theta_1 + x_1 s\theta_1 s\theta_2 - x_3 s\theta_1 c\theta_2)$$
$$+ \bar{u}_3(x_2 s\theta_1 - x_1 c\theta_1 s\theta_2 + x_3 c\theta_1 c\theta_2)$$

b)

$$\bar{r}_{P_1}^{(2)} = \hat{C}^{(2,1)}\bar{r}_{P_1}^{(1)}$$

$$\bar{r}_{P_1}^{(1)} = \bar{r}_{P_0}^{(0)} = \bar{u}_1 x_1 + \bar{u}_2 x_2 + \bar{u}_3 x_3, \quad \hat{C}^{(2,1)} = e^{-\tilde{u}_2\theta_2}$$

$$\bar{r}_{P_1}^{(1)} = e^{-\tilde{u}_2\theta_2}(\bar{u}_1 x_1 + \bar{u}_2 x_2 + \bar{u}_3 x_3) = e^{-\tilde{u}_2\theta_2}\bar{u}_1 x_1 + \bar{u}_2 x_2 + e^{-\tilde{u}_2\theta_2}\bar{u}_3 x_3$$

$$\bar{r}_{P_1}^{(1)} = (\bar{u}_1 c\theta_2 + \bar{u}_3 s\theta_2)x_1 + \bar{u}_2 x_2 + (\bar{u}_3 c\theta_2 - \bar{u}_1 s\theta_2)x_3$$

$$\bar{r}_{P_1}^{(1)} = \bar{u}_1(x_1 c\theta_2 - x_3 s\theta_2) + \bar{u}_2 x_2 + \bar{u}_3(x_1 s\theta_2 + x_3 c\theta_2)$$

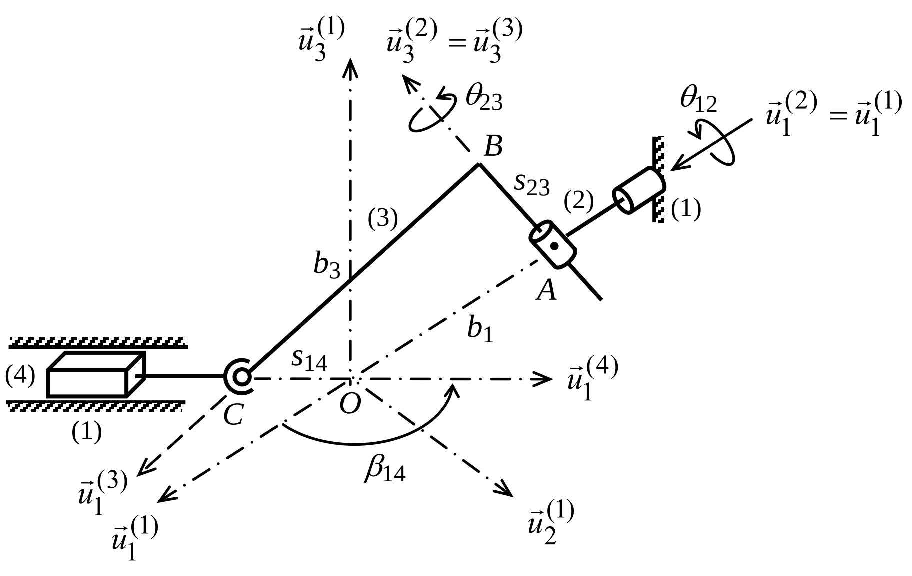
c)

$$\hat{R}_{02}^{(0)} = \hat{C}^{(0,2)} = e^{\tilde{u}_1\theta_1}e^{\tilde{u}_2\theta_2}$$

$$\hat{R}_{02}^{(1)} = \hat{C}^{(1,0)}\hat{R}_{02}^{(0)}\hat{C}^{(0,1)} = (e^{-\tilde{u}_1\theta_1})(e^{\tilde{u}_1\theta_1}e^{\tilde{u}_2\theta_2})(e^{\tilde{u}_1\theta_1})$$

$$\hat{R}_{02}^{(1)} = e^{\tilde{u}_2\theta_2}e^{\tilde{u}_1\theta_1}$$

PROBLEM 4



Consider the four-link *spatial* mechanism shown in the figure. Joint-12 is *revolute* with the joint variable θ_{12}, joint-23 is *cylindrical* with the joint variables θ_{23} and $s_{23} = AB$, joint-14 is *prismatic* with the joint variable $s_{14} = OC$, and joint-34 is *spherical*. At joint-34, the motion of link-4 relative to link-3 is described by three joint variables ϕ_{34}, θ_{34}, and ψ_{34}, which are the Euler angles of a suitable sequence. The link parameters of the mechanism are $b_1 = OA$, $b_3 = BC$, and β_{14} = angle from $\vec{u}_1^{(1)}$ to $\vec{u}_1^{(4)}$ about $\vec{u}_3^{(1)}$. Furthermore, angle(ABC) = 90° and AB is perpendicular to $\vec{u}_1^{(1)}$.

a) Using the *point-to-point loop closure equation*

$$\vec{r}_{OA} + \vec{r}_{AB} + \vec{r}_{BC} = \vec{r}_{OC}$$

written for the joint center locations, obtain formulas to find the joint variables θ_{23}, s_{23}, and s_{14} for a given value of the input joint variable θ_{12}. Indicate the *closure alternatives* clearly.

Suggestion: You may find it more convenient to formulate the solution as indicated below:

$$s_{14} = f_{14}(\theta_{12}, \sigma), \quad \sigma = \pm 1; \quad s_{23} = f_{23}(\theta_{12}, s_{14}), \quad \theta_{23} = f'_{23}(\theta_{12}, s_{14}).$$

b) Then, using the *orientational loop closure equation*

$$\hat{C}^{(1,2)}\hat{C}^{(2,3)}\hat{C}^{(3,4)} = \hat{C}^{(1,4)}$$

written for the link orientations, determine the remaining joint variables ϕ_{34}, θ_{34}, and ψ_{34} by choosing the most suitable sequence that gives them in the simplest possible way in terms of θ_{12}, β_{14}, and already determined θ_{23}.

c) In particular, find all the non-input joint variables *corresponding to each closure* of the mechanism for the following numerical values:

$$b_1 = 0.5 \text{ m}, \quad b_3 = 0.75 \text{ m}, \quad \beta_{14} = 120^\circ; \quad \theta_{12} = 30^\circ.$$

SOLUTION

a)

Point-to-Point Loop Closure Equation: $\vec{r}_{OA} + \vec{r}_{AB} + \vec{r}_{BC} = \vec{r}_{OC}$.

$$-b_1\vec{u}_1^{(1)} + s_{23}\vec{u}_3^{(2)} + b_3\vec{u}_1^{(3)} = -s_{14}\vec{u}_1^{(4)},$$

$$-b_1\bar{u}_1^{(1/1)} + s_{23}\bar{u}_3^{(2/1)} + b_3\bar{u}_1^{(3/1)} = -s_{14}\bar{u}_1^{(4/1)},$$

$$-b_1\bar{u}_1 + s_{23}\hat{C}^{(1,2)}\bar{u}_3 + b_3\hat{C}^{(1,3)}\bar{u}_1 = -s_{14}\hat{C}^{(1,4)}\bar{u}_1.$$

On the other hand,

$$\hat{C}^{(1,2)} = e^{\tilde{u}_1\theta_{12}}, \qquad \hat{C}^{(1,3)} = e^{\tilde{u}_1\theta_{12}}e^{\tilde{u}_3\theta_{23}}, \quad \hat{C}^{(1,4)} = e^{\tilde{u}_3\beta_{14}}.$$

Hence,

$$-b_1\bar{u}_1 + s_{23}e^{\tilde{u}_1\theta_{12}}\bar{u}_3 + b_3e^{\tilde{u}_1\theta_{12}}e^{\tilde{u}_3\theta_{23}}\bar{u}_1 = -s_{14}e^{\tilde{u}_3\beta_{14}}\bar{u}_1,$$

$$-b_1\bar{u}_1 + s_{23}\bar{u}_3 + b_3e^{\tilde{u}_3\theta_{23}}\bar{u}_1 = -s_{14}e^{-\tilde{u}_1\theta_{12}}(\bar{u}_1c\beta_{14} + \bar{u}_2s\beta_{14}),$$

$$-b_1\bar{u}_1 + s_{23}\bar{u}_3 + b_3(\bar{u}_1c\theta_{23} + \bar{u}_2s\theta_{23}) = -s_{14}(\bar{u}_1c\beta_{14} + \bar{u}_2s\beta_{14}c\theta_{12} - \bar{u}_3s\beta_{14}s\theta_{12}).$$

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The corresponding scalar equations are

$$b_3 c\theta_{23} = b_1 - s_{14} c\beta_{14} \quad , \qquad \text{Eq. } (i)$$

$$b_3 s\theta_{23} = -s_{14} s\beta_{14} c\theta_{12} \,, \qquad \text{Eq. } (ii)$$

$$s_{23} = s_{14} s\beta_{14} s\theta_{12} \,. \qquad \text{Eq. } (iii)$$

By adding squares of Eqs. (i) and (ii), we get

$$(c^2\beta_{14} + s^2\beta_{14} c^2\theta_{12}) s_{14}^2 - 2(b_1 c\beta_{14}) s_{14} - (b_3^2 - b_1^2) = 0 \,.$$

Two possible solutions to this equation are

$$s_{14} = \frac{b_1 c\beta_{14} + \sigma\sqrt{b_3^2 c^2\beta_{14} + (b_3^2 - b_1^2) s^2\beta_{14} c^2\theta_{12}}}{c^2\beta_{14} + s^2\beta_{14} c^2\theta_{12}} \,; \qquad \sigma = \pm 1 \,.$$

With this solution, Eq. (iii) gives s_{23} readily as

$$s_{23} = s_{14} s\beta_{14} s\theta_{12} \,.$$

Finally, from the ratio of Eqs. (i) and (ii), we get

$$\theta_{23} = \operatorname{atan}_2(-s_{14} s\beta_{14} c\theta_{12} : \, b_1 - s_{14} c\beta_{14}) \,.$$

Here, σ is the *closure indicator*. Once, its value (+1 or −1) is selected, the closure will be defined and the corresponding variables (s_{14}, s_{23}, and θ_{23}) will be uniquely determined by the preceding expressions.

b)

Orientational Loop Closure Equation: $\hat{C}^{(1,2)} \hat{C}^{(2,3)} \hat{C}^{(3,4)} = \hat{C}^{(1,4)}$

$$\hat{C}^{(1,2)} = e^{\tilde{u}_1 \theta_{12}} \,, \quad \hat{C}^{(2,3)} = e^{\tilde{u}_3 \theta_{23}} \,, \quad \hat{C}^{(3,4)} = ? \,, \quad \hat{C}^{(1,4)} = e^{\tilde{u}_3 \beta_{14}} \,.$$

The orientational equation leads to

$$\hat{C}^{(3,4)} = e^{-\tilde{u}_3 \theta_{23}} e^{-\tilde{u}_1 \theta_{12}} e^{\tilde{u}_3 \beta_{14}} \,.$$

This equation implies that the most suitable sequence for the spherical joint (Joint-34) is 3-1-3. That is,

$$\hat{C}^{(3,4)} = e^{\tilde{u}_3 \phi_{34}} e^{\tilde{u}_1 \theta_{34}} e^{\tilde{u}_3 \psi_{34}} = e^{-\tilde{u}_3 \theta_{23}} e^{-\tilde{u}_1 \theta_{12}} e^{\tilde{u}_3 \beta_{14}} \,.$$

Hence, by direct comparison, the associated joint variables are determined simply as

$$\phi_{34} = -\theta_{23} \,, \quad \theta_{34} = -\theta_{12} \,, \quad \psi_{34} = \beta_{14} \,.$$

c)

With the given numerical values, we get the following solutions for each closure:

First closure with $\sigma = +1$;

$$s_{14} = 0.3846 \text{ m}, \quad s_{23} = 0.1665 \text{ m}, \quad \theta_{23} = -22.62^{\circ};$$

$$\phi_{34} = 22.62^{\circ}, \quad \theta_{34} = -30^{\circ}, \quad \psi_{34} = 120^{\circ}.$$

Second closure with $\sigma = -1$;

$$s_{14} = -1.000 \text{ m}, \quad s_{23} = -0.433 \text{ m}, \quad \theta_{23} = 90^{\circ};$$

$$\phi_{34} = -90^{\circ}, \quad \theta_{34} = -30^{\circ}, \quad \psi_{34} = 120^{\circ}.$$