# MATH 371: Differential Geometry <br> Course Syllabus 

ELEMENTARY
EAR*ETT O~HEHL
DIFFERENTIAL stcons sourion
GEOMETRY


## Course information

| Instructor: | Yıldıray Ozan |
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| Classes: | Monday 08:40-10:30, Wednesday 10:40-12:30 |
| Office-Hours: | Tu 15:40-17:30 and also by appoiontment |
| Teaching Assistant: Özgür Karabayır, Room: Z-42 |  |
| Textbook | Barrett O'neill "Elementary Differential Geometry" 2nd Ed. |

## Course Content

Curves in 3 space: Local Theory of curves. Frenet formulas and Fundamental Theorem.Regular surfaces, definition and examples. Inverse image of regular values. Change of parameters, differentiable functions on surfaces The tangent plane; The differential of a map, vector fields, the first fundamental form. Gauss map, second fundamental form, normal curvature,principal curvature and principal directions, asymptotic directions. Gauss map in local coordinates. Covariant derivative, geodesics, Some Global Theorems including Gauss-Bonnet Theorem.

## Grading

| Midterm 1 | $30 \%$ | April 20th, Wednesday at 17:40 |
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| Midterm 2 | $30 \%$ | May 25th, Wednesday at 17:40 |
| Final | $40 \%$ |  |

## Schedule

| Weeks | Topics | Problems |
| :---: | :--- | :--- |
| 1 | 1.1 Euclidean Space | 1.2 Tangent Vectors |
|  | 1.3 Directional Derivatives | $1.1: 3,4$ |
|  | 1.4 Curves in R^3 | $1.2: 3(\mathrm{~d}, \mathrm{e}), 5(\mathrm{~b})$ |
| 2 | 1.5 1-Forms | $1.3 .1(\mathrm{a}), 3(\mathrm{c}, \mathrm{f}) 4,5$ |
|  | 1.6 Differential Forms | $1.5: 1(\mathrm{c}), 3,4(\mathrm{~b}, \mathrm{c}), 6(\mathrm{c}), 7,10$ |
|  | 1.7 Mappings | $1.6: 1-9$ |


|  | 2.1 Dot Product |  |
| :---: | :---: | :---: |
| 3 | 2.2 Curves <br> 2.3 The Frenet Formulas <br> 2.4 Arbitrary Speed Curves | 1.4: $4,6,7,9$ <br> 2.1: $5,11,12$ <br> 2.2: $3,5,6,8,10,11$ <br> 2.3: $1,2,6,7,10,11$ |
| 4 | Planar Curves (4.7 of E. Bloch) <br> 2.5 Covariant Derivatives <br> 2.6 Frame Fields | $\begin{aligned} & \text { 2.4: } 1,2,3,5,7,12,16,17,18 \\ & \text { 2.5: 1(b),2(c,d,e),3,5 } \\ & \text { 2.6: } 1,2(\mathrm{c}) \end{aligned}$ |
| 5 | 3.1 Isometries of $\mathrm{R}^{\wedge} 3$ <br> 3.2 The Tangent Map of an Isometry <br> 3.3 Orientation <br> 3.4 Euclidean Geometry | $\begin{aligned} & \text { 3.1: } 4,6,9 \\ & \text { 3.2: } 3,4 \\ & \text { 3.3: } 3,4,5 \\ & \text { 3.4: } 1(\mathrm{~b}), 2,4,5 \end{aligned}$ |
| 6 | 3.5 Conguence of Curves + Fundamental Theorem of Curves (4.6 of E. Bloch) | 3.5: 1,3,6,7 |
| 7 | 4.1 Surfaces in $\mathrm{R}^{\wedge} 3$ <br> 4.2 Patch Computations | $\begin{aligned} & \text { 4.1: } 1,4,5,8,9,10,11 \\ & 4.2: \\ & 1,2,3,5,7,8,9(\mathrm{a}, \mathrm{~b}), 10(\mathrm{~b}), 11(\mathrm{c}) \end{aligned}$ |
| 8 | 4.3 Differentiable Functions and Tangent Vectors <br> 5.1 The Shape Operator of $M$ in $R^{\wedge} 3$ + Gauss map | $\begin{aligned} & \text { 4.3: 1(b),2,3,4,5,6(b),7,12 } \\ & 5.1: 3(\mathrm{c}, \mathrm{~d}), 4,5,7,9 \end{aligned}$ |
| 9 | 5.2 Normal Curvature <br> 5.3 Gaussian Curvature <br> 5.4 Computational Techniques | $\begin{aligned} & \text { 5.2: } 1 \\ & 5.3: 1-4,7 \\ & 5.4: 1-3,5,7-15 \end{aligned}$ |
| 10 | 5.5 The Implicit Case |  |
| 11 | Isometries and Theorema Egregium | 6.4:1,8,9,14 |
| 12 | 5.6 Special Curves in Surface: Geodesics | 5.6:3,17a,19 |
| 13 | 6.3 Some Global Theorems | 6.3: 1,3 |
| 14 | Gauss-Bonnet Theorem* |  |

## Additional Sources

A First Course in Geometric Topology and Differential Geometry, Ethan D. Bloch

