MATH 332 Theoretical Aspects of Stochastic Processes

Credit: (3-0) 3

Catalog description: Introduction to stochastic processes. Emergence and applications of stochastic processes in various areas of mathematics such as geometry and group theory. Finite and countable Markov chains. Classification of states with proofs. Continuous-time Markov chains; Poisson process. Conditional expectation. Martingales. Brownian motion. Fractal nature of zero sets of Brownian motion.

Prerequisites: Math 301 or consent of the instructor.

Course Instructor: Yıldıray Ozan (Office: 217, Mathematics Department.
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Course Website: https://users.metu.edu.tr/ozan/

Textbooks:

- "Introduction to Stochastic Processes" by Lawler, G. F., Chapman & Hall /CRC, second edition.

Additional Resources:

- "Stochastic Processes" by Ross, S. M., John Wiley & Sons, 1996.

Exams and Grading: The grading will be based on two midterm examinations, one final examination and class participation.

- Midterm 1: 30% 25^{th} of March, 2026, Wednesday, 17:40
- Midterm 2: 30% 29^{th} of April, 2026, Wednesday, 17:40
- Final: 40% (date to be announced)

NA Policy: A student who misses all exams will receive a grade of NA for the course.

Make-up Policy: In order to be eligible to enter a make-up examination for a missed examination, a student should have a documented or verifiable, and officially acceptable excuse. A student cannot get make-up examinations for two missed exams. The make-up examination for all exams will be after the final exam and will include all topics.

Lectures:

Section, Instructor	Lecture Time
S1. Yildiray Ozan	Tue 15:40-17:30 (M-102) Thu 12:40-13:30 (M-102)

Office Hours: Thursdays, 10:40-12:30

Important Dates:

- **February 16:** Classes begin
- **February 23-27:** Add-drop and advisor approvals
- **March 30-April 1:** Religious holiday
- **April 23:** National Sovereignty and Children's Day (Thursday)
- **April 27-May 3:** Course withdrawal applications
- **May 1:** Labor and Solidarity Day (Friday)
- **May 19:** Commemoration of Atatürk & Youth and Sports Festival (Tuesday)
- **May 27-30:** Religious holiday
- **June 5:** Last day of classes
- **June 8-19:** Final exams period
- **June 29:** The announcement of letter grades

Course Schedule:

The table below is a rough guideline for the content of course lectures. The lectures may be reordered or reorganized if necessary.

Week 1: Feb.17-21	Finite Markov chains: Definition and examples (§ 1.1)
Week 2: Feb.24-28	Large time behavior and invariant probability (§ 1.2)
	Classification of states: Reducibility, periodicity (§ 1.3)
	Irreducible aperiodic chains, Reducible or periodic chains
Week 3: Mar.3-7	Return times. Transient states (§ 1.4, 1.5)
Week 4: Mar.10-14	Examples of finite Markov chains (§ 1.6)
	Countable Markov chains: Intro. (§ 2.1)
Week 5: Mar.17-21	Recurrence and transience (§ 2.2)
	Positive recurrence and null recurrence (§ 2.3)
Week 6: Mar.24-28	Branching process (§ 2.4)
	Continuous time Markov chains: Poisson process (§ 3.1)
Week 7: Mar.31-Apr.4	Finite state space. Birth and death process (§ 3.2, 3.3)
Week 8: Apr.7-11	Martingales: Conditional expectation (§ 5.1)
	Definition and examples (§ 5.2)
Week 9: Apr.14-18	Optional Sample Theorem. (§ 5.3)
Week 10: Apr.21-25	Uniform integrability (§ 5.4)
	Martingale Convergence Theorem (§ 5.5)
Week 11: Apr.28-May.2	Maximal inequalities (§ 5.6)
	Brownian motion: Introduction (§ 8.1)
Week 12: May.5-9	Markov property (§ 8.2)
	Zero set of Brownian motion (§ 8.3)
Week 13: May.12-16	Brownian motion in several dimensions (§ 8.4)
	Recurrence and transience (§ 8.5)
Week 14: May.19-23	Fractal nature of Brownian motion (§ 8.6)
	Hausdorff dimension. Scaling rules (§ 8.7)
Week 15: May.26-30	Brownian motion with drift (§ 8.8)