STAT 361

HOMEWORK 5

KEY

$X_1 \sim N(\mu_1 = 10, \sigma_1^2)$ ⟶ we have a sample of size $n_1 = 10$ from this popn.

$X_2 \sim N(\mu_2 = 50, \sigma_2^2)$ ⟶ we have a sample of size $n_2 = 20$ from this popn.

We want to test the following hypothesis;

$$H_0: \sigma_1 = \sigma_2$$

$$H_1: \sigma_1 \neq \sigma_2$$

So our test statistic must be $\frac{s_1^2}{s_2^2} \sim F_{(n_1-1,\, n_2-1)}$

In order to test this hypothesis by Monte Carlo Simulation method, we should firstly find an estimate of σ $(\sigma_1 = \sigma_2 = \sigma)$. Under the assumption of equality of σ_1 & σ_2, we can estimate σ by combining s_1 & s_2.

$$\hat{\sigma} = S_{pooled} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1+n_2-2)}} = \sqrt{\frac{(10-1)\,36.8377 + (20-1)\,295.3674}{10+20-2}}$$

$$\hat{\sigma} = S_{pooled} = 14.5694$$

a) critical value approach:

① Generate a random sample of size $n_1 = 10$ from $N(\mu_1 = 10, \hat{\sigma}^2 = 14.5694^2)$

- Generate a random sample of size $n_2 = 20$ from $N(\mu_2 = 50, \hat{\sigma}^2 = 14.5694^2)$

② Calculate s_1^2 from the first sample and s_2^2 from the second sample and find $F = \frac{s_1^2}{s_2^2}$

③ Repeat steps 1 & 2 many times (s.a. $N = 1000$ times), and obtain a large sample of F statistics.

④ Calculate the critical values by using F sample.

$cv1 = (100 * (\alpha/2))$-th quantile of F sample.

$cv2 = (100 * (1 - \alpha/2))$-th quantile of F sample.

⑤ Calculate $F_{obs} = \frac{s_1^2}{s_2^2}$ from the original samples.

Check $cv1 < F_{obs} < cv2 \longrightarrow$ if satisfied then Fail to Rej. Ho.

o.w. Rej. Ho.

b) p-value approach:

① ② ③ } same with critical value approach.

④ Calculate $F_{obs} = \frac{s_1^2}{s_2^2}$ → from the original samples

If $s_1^2 > s_2^2$: $\text{p-value} = 2 * \frac{\# F \text{ stat.} > F_{obs}}{N}$

else $(s_1^2 < s_2^2)$: $\text{p-value} = 2 * \frac{\# F \text{ stat} < F_{obs}}{N}$

⑤ Compare p-value & α in order to conclude hypothesis test.

If p-value $< \alpha$ → Reject H_0

else (p-value $\geq \alpha$) → Fail to Reject H_0.

c) Checking type I error:

Type I error $= \alpha = P(\text{Reject } H_0 / H_0 \text{ is true})$

① ② ③ } same with critical value approach.

④ Use cv1 & cv2 values calculated in critical value approach.

$$\text{Type I error} = \frac{(\# F\text{stat} < cv1) + (\# F\text{stat} > cv2)}{N}$$

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d) <u>Type II error & power</u>:

Type II error = P(Fail to Reject H_0 / H_1 true)

power = P(Reject H_0 / H_1 true)

In order to find type II error or power we should check the test under alternative hypothesis ($\sigma_1 \neq \sigma_2$)

Define different alternative values for both σ_1 & σ_2 ($\sigma_1 \neq \sigma_2$).

① Generate a random sample of size $n_1 = 10$ from $N(\mu_1 = 10, \sigma_1^2)$

- Generate a random sample of size $n_2 = 20$ from $N(\mu_2 = 50, \sigma_2^2)$ <u>! (Be careful $\sigma_1 \neq \sigma_2$)!</u>

② Calculate s_1^2 from the first sample and s_2^2 from the second sample and find $F = \frac{s_1^2}{s_2^2}$

③ Repeat steps 1 & 2 many times (s.a. N=1000 times), and obtain a large sample of F statistics.

④ By using the cv1 & cv2 calculated in critical value approach calculate

$$\text{Type II error} = \frac{\#\text{ of obs.}(cv1 < F\text{ stat} < cv2)}{N}$$

$$\text{Power} = \frac{(\#\,F\text{stat} < cv1) + (\#\,F\text{stat} > cv2)}{N}$$