METU
Mathematics Department

Room No.	ACCELERATED CALCULUS I Mini Midterm	List No.

Code : *Math 155* Academic Year : *1994–1995* Semester : *Fall* Instructor : *ZN, HTK, YO* Date : *7.10.1994* Time : *17:40* Duration : *45 minutes*	Last Name : Department : Name : α Student No. : α Signature : 4 QUESTIONS OF 5 POINTS EACH ON 4 PAGES TOTAL: 20 POINTS

1	2	3	4	M. U. Muhiddin	

YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT.
WRITE YOUR LIST NUMBER AND THE ROOM NUMBER ABOVE.

Question 1. (a) Evaluate $\lim_{x\to0^+}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)$ and $\lim_{x\to0^-}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)$

$$\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}+2}\right)=\frac{1}{x^2}-\left(\frac{1}{x^2}+2\right)=-2$$

$$\lim_{x\to0^+}\frac{-2}{\frac{1}{x}+\sqrt{\frac{1+2x^2}{x^2}}}=\lim_{x\to0^+}\frac{-2}{\frac{1}{x}+\frac{1}{|x|}\sqrt{1+2x^2}}=\lim_{x\to0^+}\frac{-2}{\frac{1}{x}\left(1+\sqrt{1+2x^2}\right)}=\lim_{x\to0^+}\frac{-2x}{1+\sqrt{1+2x^2}}=0$$

but $\lim_{x\to0^-}\frac{-2}{\frac{1}{x}-\frac{1}{x}\sqrt{1+2x^2}}\to-\infty$

(b) Does $\lim_{x\to0}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)$ exist? Explain.

since $\lim_{x\to0^+}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)\neq\lim_{x\to0^-}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)$

$\lim_{x\to0}\left(\frac{1}{x}-\sqrt{\frac{1}{x^2}+2}\right)$ doesn't exist

Question 2. Consider the function given by $f(x) = \begin{cases} 0, & \text{if } x = 0; \\ x^2 \sin \frac{1}{x}, & \text{if } x \neq 0. \end{cases}$ Is $f(x)$ continuous at $x = 0$? Explain.

$f(0) = 0$

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$-1 \leq \sin x \leq 1$

$-1 \leq \sin \frac{1}{x} \leq 1$

$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$

$\underbrace{\lim_{x \to 0} (-x^2)}_{\to 0} \leq \lim_{x \to 0} x^2 \sin \frac{1}{x} \leq \underbrace{\lim_{x \to 0} x^2}_{\to 0}$

So by sequezing theorem

$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$ and $f(0) = 0$

so $f(x)$ is cont at $x = 0$

Question 3. For any real number x, $[\![x]\!]$ is the greatest integer not larger than x. So $[\![3]\!] = 3$, $[\![3.9]\!]=3$, and $[\![-3.5]\!] = -4$. Consider the function $g(x) = [\![2x]\!]$.

(a) Is g continuous at $x = \frac{1}{2}$? Explain.

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$g(x) = [\![2x]\!]$ $\quad$ $g(\frac{1}{2}) = [\![2 \cdot \frac{1}{2}]\!] = 1$

$\lim_{x\to\frac{1}{2}^+} g(x) = \lim_{x\to\frac{1}{2}^+} [\![2x]\!] = 1$

$\lim_{x\to\frac{1}{2}^-} g(x) = \lim_{x\to\frac{1}{2}^-} [\![2x]\!] = 0$

$\lim_{x\to\frac{1}{2}} g(x)$ doesn't exist.

$\lim_{x\to\frac{1}{2}} g(x)$ doesn't exist so g is not cont. at $x=\frac{1}{2}$

(b) Is g continuous at $x = \frac{3}{4}$? Explain.

$\lim_{x\to\frac{3}{4}^-} [\![2x]\!] =$ ✓ $\quad$ $g(\frac{3}{4}) = [\![2 \cdot \frac{3}{4}]\!] = [\![\frac{6}{4}]\!] = 1$ ✓

$\lim_{x\to\frac{3}{4}^+} [\![2x]\!] = 1$ ✓

so $g(x)$ is cont at $x = \frac{3}{4}$

Question 4. (a) If $\lim_{x\to 2}(h(x))^2 = 4$, is it true that $\lim_{x\to 2} h(x) = 2$? Either prove or give a counterexample.

$\lim_{x\to 2} (h(x))^2 = 4$ let $h(x) = -2$

$\lim_{x\to 2} (h(x))^2 = \lim_{x\to 2} (-2)^2 = 4$

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but

$\lim_{x\to 2} h(x) = -2 \neq 2$

(b) Show that (a) is true if $h(x) \geq 0$ for all x.

$\lim_{x\to 2} (h(x))^2 = 4$ $\forall \varepsilon > 0$, $\exists\, \delta > 0$ such that

$|(h(x))^2 - 4| < \varepsilon$ whenever $|x-2| < \delta$

$|h(x) - 2|\,|h(x)+2| < \varepsilon$ " $x-2|$

$|h(x)-2| < \dfrac{\varepsilon}{|h(x)+2|} \leq \dfrac{\varepsilon}{2}$ since $h(x) \geqslant 0\ \forall x$

so choose $\varepsilon_1 = \frac{\varepsilon}{2}$

then $|h(x)-2| < \varepsilon_1$ $|x-2| < \delta$

$\delta > 0$

so $\boxed{\lim_{x\to 2} h(x) = 2}$