Valuation and Clean Surplus Accounting for Operating and Financial Activities*

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Abstract. This paper models the relation between a firm's market value and accounting data concerning operating and financial activities. Book value equals market value for financial activities, but they can differ for operating activities. Market value is assumed to equal the net present value of expected future dividends, and is shown, under *clean surplus accounting*, to also equal book value plus the net present value of expected future *abnormal earnings* (which equals accounting earnings minus an interest charge on opening book value).

A linear model specifies the dynamics of an information set that includes book value and abnormal earnings for operating activities. Model parameters represent *persistence* of abnormal earnings, *growth*, and *accounting conservatism*. The model is sufficiently simple to permit derivation of closed form expressions relating market value to accounting data and other information.

Three kinds of analyses develop from the model. The first set deals with value as it relates to anticipated realizations of accounting data. The second set examines in precise terms how value depends on contemporaneous realizations of accounting data. The third set examines asymptotic relations comparing market value to earnings and book values, and how earnings relate to beginning of period book values.

The paper demonstrates that in all three sets of analyses the conclusions hinge on the extent to which the accounting is conservative as opposed to unbiased. Further, the absence/presence of growth in operating activities is relevant if, and only if, the accounting is conservative.

Résumé. Les auteurs présentent sous forme de modèle la relation entre la valeur marchande d'une entreprise et les données comptables relatives à ses activités d'exploitation et ses activités financières. La valeur comptable est égale à la valeur marchande lorsqu'il s'agit d'activités financières, mais elle peut être différente dans le cas des activités d'exploitation. Les auteurs supposent que la valeur marchande est égale à la valeur actualisée nette des dividendes futurs prévus et démontrent que, lorsqu'on applique la *méthode du résultat global*, la valeur marchande est aussi égale à la valeur comptable additionnée de la valeur actualisée nette des *bénéfices extraordinaires* futurs prévus (qui

sont égaux aux bénéfices comptables diminués de frais d'intérêt implicites sur la valeur comptable nette).

Un modèle linéaire précise la dynamique d'un ensemble de données, incluant la valeur comptable et les bénéfices extraordinaires, relatives aux activités d'exploitation. Les paramètres du modèle traduisent la persistance des bénéfices extraordinaires, la *croissance* et le *principe de prudence*. Le modèle est suffisamment simple pour permettre de dériver des expressions fermées qui mettent en relation la valeur marchande et les données comptables et autres.

Du modèle se dégagent trois formes d'analyses. La première porte sur la valeur, dans sa relation avec la matérialisation anticipée des données comptables. La deuxième porte sur l'examen précis du lien entre la valeur et la matérialisation actuelle des données comptables. Enfin, la troisième porte sur l'examen des relations asymptotiques à travers lesquelles se comparent la valeur marchande, d'une part, et les bénéfices et la valeur comptable, d'autre part, ainsi que sur la façon dont les bénéfices se rattachent aux valeurs comptables du début de l'exercice.

Les auteurs établissent que dans les trois formes d'analyses, les conclusions s'orientent vers la mesure dans laquelle, dans le domaine comptable, l'accent est mis sur la prudence par opposition à l'impartialité. En outre, l'absence ou la présence de croissance dans les activités d'exploitation n'est pertinente que si et seulement si le principe de prudence est appliqué à la comptabilité.

This paper models how a firm's market value relates to accounting data that discloses results from both operating and financial activities. Each of the two activities raises distinct accounting measurement issues, which, in turn, influence the analysis of a firm's market value as a function of the financial statements' components. Financial activities involve assets and liabilities for which there are relatively perfect markets. Hence, one can plausibly conceptualize accounting measurements such that book values and market values coincide for these assets and liabilities. Accrual accounting for financial activities can be viewed as either redundant or straightforward (e.g., the accounting for interest accruals). In contrast, the accounting for operating assets (receivables, inventory, etc.) precipitates more intricate concerns because these assets are typically not individually traded in perfect markets. Thus, measurements of operating accounting earnings focus on cash flows adjusted for accruals, and the use of accounting conventions for accruals generally leads to differences between a firm's market and book values. The existence of the latter discrepancy, referred to as (unrecorded) goodwill, institutes the problem of how to determine the factors and information that bear on its sign and magnitude. Hence, in broad terms, this paper analyzes how accrual accounting relates to the valuation of a firm's equity and goodwill.

The model starts from the assumption that the value of the firm's equity equals the net present value of the expected dividends that will be distributed to equity holders. The accounting system records the creation and distribution of wealth. Links between the creation of wealth, as recorded by the accounting system, and the dividends paid to equity holders provide the basis for alternative expressions for the value of the firm's equity.

Three basic statements supply accounting data: income statement, balance sheet, and statement of changes in owners' equity. We postulate a "going concern" dynamic environment in which the statements are disclosed at regular dates (e.g., end of fiscal years). In each period the firm realizes cash flows from operations, and the difference between cash flows and operating earnings reconcile with the balance sheet accruals. Thus the model admits four "flow" variables: operating earnings, (net) interest revenues (expenses), cash flows, and dividends. The "stock" variables consist of three balance sheet items: (net) operating assets, (net) financial assets (i.e., marketable securities minus debt), and book value (which is the sum of the operating and financial assets, thus representing owners' equity).

The first set of analyses explores the relation between value and expectations about future accounting numbers. Three concepts, which impose structure on the accounting variables, play a central role in the derivation of accounting-based expressions of value.

First, the income statements and balance sheets reconcile via the clean surplus relation. From this powerful restriction on the financial reporting model one infers that a firm's goodwill equals the present value of anticipated future "abnormal earnings," where abnormal earnings are defined to equal reported earnings minus the risk-free interest rate times the book value of the firm's equity.[1] As a consequence, the analysis of a firm's value and goodwill as a function of accounting data, and their attributes, depends on how these affect the prediction of the future abnormal earnings sequence.

Second, the analysis incorporates Modigliani and Miller's (1958, 1961) (MM) basic concept regarding debt. The firm's borrowing (and lending) activities, whether incremental or on average, yield zero net present value. Financing activities, including the dividend policy, separate from the operating activities, to ensure that a firm's equity value equals the value of the operating activities plus the value of the financial assets (which consist of marketable securities minus debt). Moreover, the value of the financial assets is assumed to equal their book value; that is, the model assumes that "perfect" accounting applies for financial assets. This feature of financing activities implies that a firm's goodwill is attributable solely to its operating activities, and that goodwill equals the present value of a firm's expected abnormal operating earnings. Analogous to the definition of abnormal earnings, operating earnings minus an interest charge for the use of operating assets defines abnormal operating earnings.

Third, the cash flow concept evolves naturally if one appreciates that the difference between cash (operating) flows and operating earnings is due to accruals, that is, cash flows equal operating earnings minus the change in (net) operating assets. Consistent with standard concepts of

value, one infers from this framework that a firm's market value equals the present value of expected cash flows plus the value of financial assets.

The second set of analyses explore the relation between value and current accounting numbers. These analyses are based on a model that relates current accounting data to the prediction of future realizations of accounting data. The model specifies a set of information dynamics in which the information set is assumed to consist of current abnormal operating earnings, operating assets, financial assets, and some primitive variables representing "other" prediction-relevant information. The information dynamics are assumed to be linear and they are specified so that one obtains a parsimonious model in which there is a precise parametric representation of three key characteristics of the dynamics: the *persistence* in abnormal operating earnings, the *growth* in operating assets (and operating earnings), and the *conservatism* in reporting operating assets.

The dichotomy between unbiased versus conservative accounting is defined in terms of how the market value differs, on average, from the book value. Unbiased (conservative) accounting obtains if, on average, the market value equals (exceeds) the book value. The analysis establishes that unbiased accounting implies a valuation function such that the market value is a weighted average of a "stock" model (based on the firm's book value) and a "flow" model (based on the firm's earnings), plus a zero mean variable that adjusts for other information. This result is consistent with Ohlson's (1995) earlier work, and the weight on the "flow" model increases with the persistence in abnormal earnings. The valuation function under conservative accounting is similar, but it requires additionally an adjustment for the understatement of operating assets. Hence, the analysis shows that when the accounting is conservative, it is important to separate the reporting of financial and operating assets. However, the financial and operating components of earnings aggregate without any loss of information. This aggregation result is surprising because the two components differ significantly in their stochastic behavior (i.e., persistence and growth).

The third set of analyses examine expectations with respect to the asymptotic relations of market value and changes in market value to contemporaneous earnings, and the relation of book value to subsequent earnings. The use of asymptotic relations permits us to abstract from the idiosyncratic (i.e., realization specific) effects of information, thereby identifying the *average* relation. The results for unbiased accounting are straightforward. On average, the price/earnings relation is identical to the certainty case with "properly" measured earnings, accounting earnings equal the change in market value, and accounting rate of return equals the risk-free rate of return. The results for conservative accounting are more complex. The analysis shows that, on average, both the market value and the change in market value are large relative to earnings if, and only if, in

addition to conservative accounting, the operating assets are expected to grow. That is, growth and conservatism have "synergistic" effects in these relations.

The impact of conservative accounting on the book rate of return is even more subtle. To examine this relation we assume a "full payout" dividend policy (i.e., future dividends equal future earnings), which results in a constant book value. The analysis demonstrates that earnings (or, equivalently, the book rate of return) increase to a finite bound if there is conservative accounting and no growth, whereas it increases without bound if there is conservative accounting and growth.

The fourth set of analyses examine how conservative accounting influences the response of value to increments in various components of earnings and assets, subject to debits equal credits. It is shown that an incremental dollar of cash earnings is worth less than an incremental dollar of non-cash earnings if, and only if, the accounting is conservative. Thus, cash earnings are of "lower quality" than accrual earnings given conservative accounting measurements. A parallel result applies with respect to next-period expected earnings, i.e., an incremental dollar of non-cash earnings has a more favorable effect on expected next-period earnings as compared to an incremental dollar of cash earnings.

Conservatism results in unrecorded goodwill and fundamentally affects the relations examined in our analysis. Goodwill can reflect either the understatement of the value of existing assets or the anticipation of future positive net present value investments. The final analysis in the paper demonstrates that the results in the paper hold even if the firm undertakes only zero net present value projects (and, hence, the firm initially has zero unrecorded goodwill). In this case, unbiased accounting results in full capitalization of the initial investment in operating assets. Conservative accounting, on the other hand, results in capitalization of only a fraction of that investment and expensing of the remainder. Consequently, conservative accounting results, on average, in low earnings in the early periods and offsetting large earnings in later periods.

Relations between value and expectations about future accounting numbers

The analyses in this paper are based on a model of a firm in a multiple-date, neo-classical setting. At each date t ($t = 0, 1, ...$), the firm discloses accounting data pertaining to its operating and financial activities. The data, which are random prior to their disclosure, bear upon the firm's value. The following variables represent these data:

bv_t = book value of the firm's equity, date t

x_t = earnings for period (t-1,t)

d_t = dividends, net of capital contributions, date t

fa_t = financial assets, net of financial obligations, date t

i_t = interest revenues, net of interest expenses, for period $(t\text{-}1,t)$
oa_t = operating assets, net of operating liabilities, date t
ox_t = operating earnings for period $(t\text{-}1,t)$
c_t = cash flows realized from operating activities,
   net of investments in those activities, date t
P_t = market value of the firm's equity, date t.

The following analysis first specifies the assumed relations among the accounting variables, and then states how the market value depends on the anticipated sequence of dividends. These relations are then integrated to derive three fundamental relations between expected accounting data realizations and market value.

Accounting relations

The model segregates the firm's activities into financial and operating activities. The book value (of the firm's equity) at date t is $bv_t = fa_t + oa_t$, and its period $(t\text{-}1,t)$ earnings are $x_t = i_t + ox_t$.

Consistent with Ohlson (1995), we assume that the accounting measurements satisfy the *clean surplus relation*, i.e., all changes in book value are reported as either income or dividends:

$$bv_t = bv_{t\text{-}1} + x_t - d_t. \qquad \text{(CSR)}$$

Dividends are declared and paid at the end of the period. They directly reduce the book value of the assets retained in the firm, $\partial bv_t/\partial d_t = -1$, but do not influence the income earned during the period, $\partial x_t/\partial d_t = 0$.

The model permits only cash dividends (and cash capital contributions), and the marginal effect of dividends on book value is due to a reduction in financial assets or an increase in debt. We refer to the difference between financial assets ("marketable securities") and debt ("bonds payable") as simply financial assets, fa_t. The correct language for $fa_t < 0$ is debt net of financial assets, but our reference to fa_t as financial assets should not be a source of confusion. (The convention is analogous to referring to d_t as dividends, regardless of its sign.) The interest rate is assumed to be the same for financial assets and liabilities and, hence, the interest rate is independent of the sign of fa_t. The following *net interest relation* is assumed for positive and negative fa_t:[2]

$$i_t = (R_F - 1)fa_{t\text{-}1}, \qquad \text{(NIR)}$$

where R_F denotes one plus the risk-free interest rate. NIR expresses the certain zero net present value economic return on the net financial position, and the relation imposes a flat, non-stochastic, term-structure on interest rates. Further, NIR also determines the accounting for financial assets so that their book and market values coincide to equal fa_t for all t. This modelling of the accounting for the (net) financial assets makes sense if one thinks of risk-free financial assets and liabilities as, virtually by definition, trading in perfect markets.[3]

Financial activities begin period $(t\text{-}1,t)$ with a stock of financial assets fa_{t-1}. Interest i_t is earned on fa_{t-1} during the period, dividends d_t are paid at the end of the period, and cash from operating activities c_t are received at the end of the period. The net result is an ending stock of financial assets fa_t. The *financial assets relation* among these accounting variables is:

$$fa_t = fa_{t-1} + i_t - [d_t - c_t]. \tag{FAR}$$

The dividends minus cash flows from operations $(d_t - c_t)$ directly reduce the ending financial asset balance, but do not influence the interest earned during the period. The investment in financial assets changes only because the firm does not equate dividends to the cash flows plus net interest earned. Of course, no interest is earned or incurred if the firm always equates dividends to cash flows. That is, $fa_0 = 0$ and $d_t = c_t$, all t, imply $fa_t = 0$, all t, and, conversely, $fa_t = 0$, all t, implies $d_t = c_t$.

Operating assets oa_t consist of *all* asset (liability) accounts that do *not* generate earnings as proscribed by NIR (e.g., cash held for operating purposes, accounts receivable, inventory, prepaid expenses, property, plant and equipment net of depreciation, and operating liabilities, such as accounts payable, and accrued wages). Similarly, operating earnings consist of all non-interest items (e.g., sales, cost of goods sold, selling and administration expenses, and gains and losses on the disposal of operating assets).

Since the firm's activities are either financial or operating, CSR and FAR imply the following *operating asset relation*:[4]

$$oa_t = oa_{t-1} + ox_t - c_t. \tag{OAR}$$

This relation closely parallels the clean surplus relation (CSR). Operating activities begin period $(t\text{-}1,t)$ with operating assets oa_{t-1}, generate operating income ox_t during the period, transfer cash flows c_t to the financial assets at the end of the period ($c_t < 0$ represents net capital expenditures in operating assets), and end the period with operating assets oa_t. The cash flows from operations represent the "dividends paid" by the operating activities, but these cash flows can be put into financial assets and need not be immediately distributed to the equity holders.

Since OAR and FAR comprehensively describe the firm's two activities, the "transfer" of assets (cash flows) from the operating account to the financial account does not yield any gain or loss. This claim holds regardless of how operating assets are valued per the books. Moreover, due to FAR and NIR, the asset (cash flow) transfer must be recorded at market value. Thus the cash flow concept is independent of the accounting rules for operating assets, and one can view cash flows as "objectively" measured.

The cash flow concept specified by OAR and FAR generally conforms with the "free cash flow" concept used in finance. The same can be said for the "enterprise cash flow" concept discussed in CON-6.[6] On the other hand, c_t differs from the SFAS-95 concept of "cash flows from operations". Roughly, the SFAS-95 "cash flows from operations" minus capital expenditures and minus (net) interest revenue corresponds to our c_t.

Basic market value relation

The firm's market value, P_t, is assumed to equal the present value of expected dividends discounted at the risk-free interest rate R_F (the *present value relation*):

$$P_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{d}_{t+\tau}],$$ (PVR)

where $E_t[.]$ denotes the expected value operator conditioned on the information available at date t. Implicit in the present value relation is the assumption that investors are risk neutral with respect to the risks associated with this firm and, hence, the PVR formula does not adjust risk in the expectation (or the discount rate).[7]

The equivalence of the risk-free interest rate in NIR and PVR is central to our analysis because Modigliani/Miller (MM) concepts will apply. The model structure with NIR, PVR, and FAR ensures that the valuation of operating activities does not depend on the extent to which the firm distributes financial assets as dividends.[8] This aspect of the model is exploited throughout the analysis.

Relation of value to future accounting data and operating cash flows

PVR emanates from the concept that the expected transfer of wealth from the firm to investors, $E_t[\tilde{d}_{t+\tau}]$, $\tau \geq 1$, suffices to determine the firm's equity value. Since this distribution of wealth ultimately must articulate with the creation of wealth, one may consider how the current value depends on accounting measures of the wealth creation process. This section develops three additional value representations that are equivalent to PVR; each representation focuses on expected realizations of accounting data, including cash flows.

We first consider the significance of expected future cash flows. FAR shows that (operating) cash flows increase financial assets — the creation of wealth — whereas dividends reduce financial assets — the distribution of wealth. Further, via NIR, interest on undistributed cash flows add to financial assets. Combining NIR and FAR one thus reconciles the difference between wealth distributed and wealth created:

$$d_t = c_t + R_F fa_{t-1} - fa_t.$$ (1)

For any realized sequence of cash flows and financial assets,

$\{c_{t+\tau}, fa_{t+\tau-1}\}_{\tau\geq1}$, one next infers the realized sequence of dividends, $\{d_{t+\tau}\}_{\tau\geq1}$. Using (1), it follows immediately that

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{d}_{t+\tau}] = fa_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{c}_{t+\tau}], \tag{2}$$

provided $R_F^{-\tau} E_t[\tilde{fa}_{t+\tau}] \to 0$ as $\tau \to \infty$.[9] That is, the NIR and FAR assumptions suffice for the present value of expected dividends to equal the book value of financial assets plus the present value of the expected cash flows from operations.

Expression (2) shows how the value of a firm's equity depends on the firm's two separate activities: (i) the value of firm's financial activities, which equals its book value due to NIR and FAR; and (ii) the value the firm's operating activities as determined by the present value of expected (operating) cash flows. In the absence of operating activities, $P_t = fa_t$, because for this case $\tilde{c}_{t+\tau} \equiv 0$ ($\tau > 1$), and the accounting is "perfect". Operating activities, on the other hand, are evaluated through their perceived cash flow consequences, $\sum_{\tau} R_F^{-\tau} E_t[\tilde{c}_{t+\tau}]$. Expression (2) is thus independent of OAR, CSR, and any accounting principles that determine the book value of operating assets (because one derives (2) from PVR, FAR, and NIR alone). The valuation concept remains valid even if, for example, one uses "cash accounting" principles (which put $ox_t = c_t$ and $oa_t = 0$).

Although the model does not specify the principles that determine the book value of operating assets, CSR *by itself* ensures that the difference between book and market values reconciles via a measure of future expected profitability. To develop this relation, define *abnormal earnings* as

$$x_t^a \equiv x_t - (R_F - 1)bv_{t-1}.$$

The terminology is motivated by the idea that $(R_F-1)bv_{t-1}$ is a measure of "normal" earnings for period $(t-1,t)$. Since CSR implies

$$d_t = x_t^a + R_F bv_{t-1} - bv_t, \tag{3}$$

one infers the realized sequence of dividends, $\{d_{t+\tau}\}_{\tau\geq1}$, from the realized sequence of abnormal earnings and book values, $\{x_{t+\tau}^a, bv_{t+\tau-1}\}_{\tau\geq1}$. Using (3), it follows immediately that

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{d}_{t+\tau}] = bv_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{x}_{t+\tau}^a], \tag{4}$$

provided $R_F^{-\tau} E_t[\tilde{bv}_{t+\tau}] \to 0$ as $\tau \to \infty$.[11] That is, CSR and the definition of x_t^a suffice for the present value of expected dividends to equal the book value of the firm's assets plus the present value of expected abnormal earnings.

Now consider the distinction between financial and operating activities. Let ox_t^a denote the *abnormal operating earnings*, where

$$ox_t^a \equiv ox_t - (R_F - 1)oa_{t-1}.$$

Since OAR implies

$$c_t = ox_t^a + R_F oa_{t-1} - oa_t, \tag{5}$$

each realized sequence of abnormal operating earnings and operating assets, $\{ox_{t+\tau}^a, oa_{t+\tau-1}\}_{\tau \geq 1}$, determines a realized sequence of cash flows, $\{c_{t+\tau}\}_{\tau \geq 1}$. Similar to the way (3) leads to (4), from (5) it follows that

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{c}_{t+\tau}] = oa_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[o\tilde{x}_{t+\tau}^a], \tag{6}$$

provided $R_F^{-\tau} E_t[o\tilde{a}_{t+\tau}] \to 0$ as $\tau \to \infty$.[12] That is, OAR and the definition of ox_t^a suffice for the present value of cash flows to equal the book value of operating assets plus the present value of expected abnormal operating earnings.

Adding fa_t to both sides of (6), using $bv_t = fa_t + oa_t$, and substituting into (2) results in

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{d}_{t+\tau}] = bv_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[o\tilde{x}_{t+\tau}^a]. \tag{7}$$

Alternatively, one can derive (7) from (4) because $ox_t^a = x_t^a$; the last equivalence is immediate from NIR, $x_t = i_t + ox_t$, and $bv_t = fa_t + oa_t$.

As a summary of expressions (2), (4), and (7) in conjunction with PVR, one obtains the following proposition.

Proposition 1:[13] Assume accounting relations CSR, NIR, FAR and OAR, and valuation relation PVR. Then the firm's equity value, P_t, can be represented equivalently as:

(a) $P_t = fa_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{c}_{t+\tau}],$

(b) $P_t = bv_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{x}_{t+\tau}^a]],$

(c) $P_t = bv_t + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[o\tilde{x}_{t+\tau}^a]].$

We interpret the above proposition as follows. Expression (a) provides the usual "finance" approach to valuation and is independent of the accounting measures for operating activities. As noted, it follows directly from PVR and (1), which depends only on NIR and FAR. The key is that the cash flows represent the economic value of resources obtained from operations, and it makes no difference to the equity holders whether the firm pays out the cash flows immediately as dividends or retains them in

the firm by investing in zero net present value projects (i.e., in financial assets).

Expression (b) follows directly from PVR and (4), which depends only on CSR.[14] The distinction between financial and operating assets is irrelevant, as are NIR and any cash flow concept. This approach to value can be recast in terms of (unrecorded) goodwill, defined and denoted by

$$g_t \equiv P_t - bv_t,$$

$$g_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{x}_{t+\tau}^a]. \tag{8}$$

The amount of goodwill at any date obviously depends on the accounting principles employed. However, as Preinreich (1938) and, more recently, Peasnell (1982) emphasize, the analysis that leads to expression (8) (and (b)) remains valid for *all* accounting principles satisfying CSR. The result is surprisingly useful; it permits the introduction of an accounting framework in valuation without specifying accounting principles.

Expression (c) derives from NIR, FAR, and CSR, in addition to the starting point PVR. Since the approach demands the partitioning of the income statement and balance sheet into operating and financial activities, (c) depends on a more elaborate accounting structure than (b). With regard to (a) versus (c), (a) obtains as a special case of (c). Recall that valuation expression (c) (and (b)) works for *any* accounting measurement rules pertaining to the firm's operating assets. As a possibility, consider cash accounting, in which case one puts $oa_t \equiv 0$, all t, even though such assets may exist on the basis of (conventional) accrual accounting. It follows that $bv_t = fa_t$, and $ox_t^a = ox_t = c_t$; further, with these restrictions, (c) reduces to (a).

One concludes that the discounting of expected cash flows can be viewed as a special application of the *more general* CSR based valuation expression (b), because (a) derives from (b) if one uses "cash accounting." Accrual accounting and discounting of expected future abnormal earnings therefore provides a broader framework than cash flow discounting, and one need not worry that accruals will "distort" the analysis. In sharp contrast to cash flow discounting, the role of profitability in valuation becomes apparent. Formula (c), in particular, emphasizes that with appropriate constructs one can discount future expected abnormal operating earnings to derive a firm's value.

Unbiased versus conservative accounting for operating assets

Valuation expression (c) in Proposition 1 subsumes a well-known MM concept. The market value implications of financing activities separate additively from operating activities:

Value of Equity = Value of Financing Activities + Value of Operating

Activities $= fa_t + [oa_t + g_t]$.

Goodwill is entirely attributable to the accounting for operating assets. The claim is immediate because the financial activities have zero abnormal earnings due to NIR (i.e., NIR implies $i_t - (R_F-1)fa_{t-1} = 0$). Given this powerful property of MM separation in a setting with NIR and FAR, we naturally next consider valuation issues bearing on operating activities.

The valuation of operating activities,

$$P_t - fa_t = oa_t + g_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{c}_{t+\tau}], \tag{9}$$

introduces at least two complications, neither of which arises in the simple case of financial activities. First, goodwill may differ from zero (i.e., oa_t is not likely to equal the market value of the operating activities). Second, goodwill may be biased systematically in the sense that $g_t = P_t - bv_t$ deviates, *on average*, from zero. In other words, not only is $P_t - bv_t \neq 0$ generally, but the (long run) expected value of $P_t - bv_t$ may also differ from zero. The latter possibility points toward the theoretical (and practical) importance of *conservative* versus *unbiased* accounting for (net) operating assets. We use the following definitions to analyze the (valuation) implications of these alternative attributes of accounting:[15]

Definitions: *Unbiased accounting* obtains if

$E_t[\tilde{g}_{t+\tau}] \to 0$ as $\tau \to \infty$

regardless of the dividend policy and the date t information.

Conservative accounting obtains if

$E_t[\tilde{g}_{t+\tau}] > 0$ as $\tau \to \infty$

regardless of the dividend policy and the date t information.

The following characterizations of unbiased versus conservative accounting immediately follow from their respective definitions and Proposition 1.

Proposition 2: Given accounting and value relations CSR, NIR, FAR, OAR, and PVR, unbiased accounting obtains if, and only if,

$$E_t[o\tilde{a}_{t+T}] = E_t\left[\sum_{\tau=1}^{\infty} R_F^{-\tau} E_{t+T}[\tilde{c}_{t+T+\tau}] \right] \text{ as } T \to \infty$$

or, equivalently,

$$E_t\left[\sum_{\tau=1}^{\infty} R_F^{-\tau} E_{t+T}[o\tilde{x}^a_{t+T+\tau}]\right] \to 0 \text{ as } T \to \infty.$$

For conservative accounting one replaces '=' with '<' and '→ 0' with '> 0'.

In words, unbiased accounting occurs if, on the average, oa_t equals the present value of future cash flows, or if, on average, the present value of anticipated abnormal operating earnings equal zero.

Proposition 2 suggests that conservative accounting reduces the book value of operating assets but increases future expected abnormal operating earnings. Expression (6) brings out this idea more explicitly. Due to the objectivity of the cash flow measure, $\sum_\tau R_F^{-\tau} E_t[\tilde{c}_{t+\tau}]$ (the LHS of (6)) is independent of the accounting measurements employed. A conservative assessment of date t operating assets must accordingly be offset by an optimistic assessment of future expected abnormal operating earnings. That is, given (6), a decrease in oa_t is exactly offset by an increase in $\sum_\tau R_F^{-\tau} E_t[o\tilde{x}^a_{t+\tau}]$, if one keeps $\sum_\tau R_F^{-\tau} E_t[\tilde{c}_{t+\tau}]$ fixed.

Relation between value and current accounting numbers

The preceding analysis identifies general relations between the market value of a firm's equity and *future* accounting numbers. These relations hold even if the accounting numbers are not part of the information used by investors. However, investors receive accounting reports and accounting variables can be used in representing investor information. We now introduce an explicit model of the dynamics of the investors' information and assume that the values at date t of the previously introduced accounting variables form part of the sufficient statistic representing the investors' information at date t.[16] This permits us to relate value to *current* accounting numbers.

A dynamic linear information model

We continue to distinguish between financial and operating activities. Since we assume perfect accounting for the financial activities, the financial activities are given only limited attention. The model focuses on the operating activities and assumes that abnormal operating earnings and the book value of operating assets form part of the sufficient statistic representing investor information. This seems reasonable in the light of Proposition 1(c), which establishes that the value of operating assets can be expressed as the book value of those assets plus the discounted future expected abnormal earnings. The model provides explicit representation of three key characteristics of the dynamics associated with abnormal operating earnings and operating assets: *persistence* in abnormal operating earnings, *growth* in operating assets (and, hence, growth in operating

earnings), and *conservatism* in the accounting for operating assets. The rates of persistence and growth are influenced by both the economics of the firm and the accounting procedures that are employed. We do not explicitly model these two components. Our objective is to develop a simple model that captures key characteristics that are likely to influence the observed contemporaneous relation between a firm's market value and its accounting numbers.

Proposition 1 established that value is related to investor beliefs about future abnormal operating earnings. Hence, we naturally develop a model in which *current* abnormal operating earnings, and other accounting and nonaccounting data, provide the basis for predicting *future* abnormal operating earnings (and, by inference, future cash flows). This information maps into the value of the firm's operating activities and, via MM separation, into the value of the firm's equity. Thus, we assume the prediction of future abnormal operating earnings, $\{o\tilde{x}^a_{t+\tau}\}^\infty_{\tau=1}$, depends on (i) current abnormal operating earnings ox^a_t, (ii) current operating assets oa_t, and (iii) other information v_t. The latter feature takes on prominence by ruling out extreme—and unrealistic—settings in which ox^a_t and oa_t suffice to determine $P_t - fa_t$.

To keep the model analytically tractable, we restrict the other information to two random numbers, $v_t = (v_{1t}, v_{2t})$, and assume the evolution of all information follows a linear, Markovian structure. Specifically, our *linear information model* (LIM) is based on the following four linear recursive equations:

$$o\tilde{x}^a_{t+1} = \omega_{11}ox^a_t + \omega_{12}oa_t + v_{1t} + \tilde{\varepsilon}_{1t+1} \tag{10a}$$

$$o\tilde{a}^a_{t+1} = \qquad\quad + \omega_{22}oa_t + v_{2t} + \tilde{\varepsilon}_{2t+1} \tag{10b}$$

$$\tilde{v}_{1t+1} = \qquad\qquad\quad \gamma_1 v_{2t} + \tilde{\varepsilon}_{3t+1} \tag{10c}$$

$$\tilde{v}_{2t+1} = \qquad\qquad\quad \gamma_2 v_{2t} + \tilde{\varepsilon}_{4t+1} \tag{10d}$$

The random terms, $\tilde{\varepsilon}_{jt+\tau}$, satisfy the nonpredictability, mean zero, condition $E_t[\tilde{\varepsilon}_{jt+\tau}] = 0, j=1,...,4$, all t and $\tau > 0$. These terms constitute the only source of uncertainty, and a realization of $(\varepsilon_{1t+1},...,\varepsilon_{4t+1})$ updates the information vector from $(ox^a_t, oa_t, v_{1t}, v_{2t})$ to $(ox^a_{t+1}, oa_{t+1}, v_{1t+1}, v_{2t+1})$ via the four equations in (10). The innovations $(\varepsilon_{1t+1},...,\varepsilon_{4t+1})$ can correlate across equations for each t, and their variances/covariances can depend on the date t information.

It is important to appreciate that LIM embeds the process that determines the evolution of cash flows. Since $c_{t+1} = ox^a_{t+1} + R_F oa_t - oa_{t+1}$, expressions (10a) and (10b) imply the recursive equation[17]

$$\tilde{c}_{t+1} = \omega_{11}ox^a_t + [(R_F - \omega_{22}) + \omega_{12}]oa_t + [v_{1t} - v_{2t}] + [\tilde{\varepsilon}_{1t+1} - \tilde{\varepsilon}_{2t+1}] \tag{11}$$

By putting $\varepsilon_{1t+1} = \varepsilon_{2t+1} = 0$ in (11) one identifies the prediction equation for $E_t[\tilde{c}_{t+1}]$. Similarly, one can also derive the more general expressions for $E_t[\tilde{c}_{t+\tau}]$, $\tau=1,2,...$ via the equations in LIM.

The appendix derives the expected asymptotic behavior of the variables governed by LIM. To make sure that the convergence/divergence of these variables is appropriate, and for other reasons discussed below, we impose the following *a priori* restrictions on the parameters in LIM:
(i) $|\gamma_h| < 1$, $h = 1, 2$; (ii) $0 \le \omega_{11} < 1$; (iii) $1 \le \omega_{22} < R_F$; and (iv) $\omega_{12} \ge 0$.

Condition (i) ensures that the random events influencing other information have no long run effect on future other information, i.e., as $E_t[\tilde{v}_{ht+\tau}] \to 0$ as $\tau \to \infty$, $h=1,2$. The other information acts as serially correlated, but convergent, noise in the prediction of abnormal earnings and operating assets.

Condition (ii) restricts the (marginal) persistence in abnormal earnings. The lower bound, $\omega_{11} \ge 0$, eliminates implausible oscillating persistence. The upper bound, $\omega_{11} < 1$, permits positive (or zero) persistence, but implies the (marginal) effect decays (geometrically) with time. A corresponding decaying persistence effect applies to future cash flows.[18] Hence,

$$\frac{\partial E_t[o\tilde{x}^a_{t+\tau}]}{\partial ox^a_t} = \frac{\partial E_t[\tilde{c}_{t+\tau}]}{\partial ox^a_t} \to 0 \text{ as } \tau \to \infty.$$

Condition (iii) restricts the (long run) growth in operating assets. The lower bound, $\omega_{22} \ge 1$, is necessary and sufficient to rule out

$$E_t[o\tilde{a}^a_{t+\tau}] = E_t[o\tilde{x}_{t+\tau}] = E_t[\tilde{c}_{t+\tau}] = 0 \text{ as } \tau \to \infty.$$

This outcome implies asymptotic liquidation of the firm's operations, which is of no interest given our focus on conservative versus unbiased accounting within a "going operating concern" context. The upper bound, $\omega_{22} < R_F$, eliminates growth paradoxes, i.e., the requirement is necessary for absolute convergence in the present value calculations of expected abnormal operating earnings and expected cash flows.

Finally, condition (iv) represents the dichotomous possibilities of unbiased ($\omega_{12} = 0$) versus conservative ($\omega_{12} > 0$) accounting. Proposition 4 establishes that this characterization is consistent with the definitions of unbiased and conservative accounting provided in the preceding section. The lower bound, $\omega_{12} \ge 0$, eliminates the opposite of conservative accounting. We rule out "aggressive" accounting to keep matters simple and (presumably) more consistent with real world accounting.

The asymptotic behavior of $E_t[o\tilde{x}_{t+\tau}]$ and $E_t[o\tilde{a}^a_{t+\tau}]$ describe important aspects of the model dynamics. The appendix (see expressions (A.1) and (A.3)) derives the asymptotic solution. With unbiased accounting ($\omega_{12} = 0$), $E_t[o\tilde{x}^a_{t+\tau}] \to 0$ as $\tau \to \infty$, irrespective of the values for ω_{11} and

ω_{22}. If $\omega_{22} = 1$, there is no asymptotic growth; both $E_t[o\widetilde{x}_{t+\tau}^a]$ and $E_t[o\widetilde{a}_{t+\tau}]$ converge to finite values as $\tau \to \infty$. If $\omega_{22} > 1$ and $\omega_{12} > 0$, one obtains asymptotic growth in both variables; that is, $E_t[o\widetilde{x}_{t+\tau}^a] \to \infty$ and $E_t[o\widetilde{a}_{t+\tau}] \to \infty$, as $\tau \to \infty$. On the other hand, if $\omega_{22} > 1$ and $\omega_{12} = 0$, only $E_t[o\widetilde{a}_{t+\tau}] \to \infty$. These asymptotic results establish that a model with $\omega_{22} > 1$ is appropriately termed a growth setting and a model with $\omega_{22} = 1$ is appropriately termed a no growth setting.

Of course, a firm can be expected to exhibit "short" to "intermediate" growth (positive or negative) regardless of the value of ω_{22} because υ_{2t} influences $E_t[o\widetilde{a}_{t+\tau+1} - o\widetilde{a}_{t+\tau}]$. For example, if $\omega_{22} = 1$ (no growth), the difference equals $\gamma_2^\tau \upsilon_{2t}$, and the direction of the expected change in operating assets depends uniquely on the sign of υ_{2t}. Although negative values of υ_{2t} may result in temporary negative growth, we assume throughout the analysis that such declines are limited and current and expected operating assets are always positive.

Linear Valuation Functions

Combined with the accounting structure and the concepts set forth in previous sections, LIM leads to a closed form, linear, valuation solution. Specifically, one can derive the market value by calculating the present value of expected abnormal operating earnings (i.e., P_t obtains via an application of Proposition 1(c)). This approach has the advantage of bypassing the need for modelling the firm's dividend policy and the behavior of financial assets.[19]

The market value of the firm's equity, P_t, equals its book value, $bv_t = fa_t + oa_t$, plus a linear function of the date t LIM variables $(ox_t^a, oa_t, \upsilon_{1t}, \upsilon_{2t})$. The four coefficients for these variables can be expressed as functions of the parameters $(\omega_{11}, \omega_{12}, \omega_{22}, \gamma_1, \gamma_2)$ and R_F (see Proposition 3 below). Subsequent analyses exploit this result to develop a succession of insights concerning attributes of accounting data—conservative versus unbiased accounting in particular—and their relation to value. Since all conclusions depend on an elaborate set of assumptions, a summary may be useful before proceeding. The assumptions fall into three categories:

    (i)   the basic accounting relations consisting of the clean surplus relation (CSR), the net interest relation (NIR), the financial assets relation (FAR), and the operating assets relation (OAR);

    (ii) the requirement that the market value of the firm's equity (P_t) equals the present value of expected (net) dividends (PVR); and

    (iii) the linear information model (LIM) for $(ox_t^a, oa_t, \upsilon_t)$, consisting of (10) and the restrictions on the parameters $(\omega_{11}, \omega_{12}, \omega_{22}, \gamma_1, \gamma_2)$.

To avoid cumbersome repetition, the formal propositions (and related discussion) that follow do not explicate the above assumptions. We emphasize, however, that the derivations rely on all three sets.

The next proposition provides the simple closed form solution showing how the date t accounting data and other information relate to the firm's date t market value, P_t.

Proposition 3:[20] The valuation function can be expressed as[21]

$$P_t = bv_t + \alpha_1 ox_t^a + \alpha_2 oa_t + \beta \cdot v_t$$

where

$$\alpha_1 = \frac{\omega_{11}}{R_F - \omega_{11}}$$

$$\alpha_2 = \frac{\omega_{12} R_F}{(R_F - \omega_{22})(R_F - \omega_{11})}$$

and

$$\beta = (\beta_1, \beta_2) = \left[\frac{R_F}{(R_F - \omega_{11})(R_F - \gamma_1)} , \frac{\alpha_2}{(R_F - \gamma_2)} \right]$$

The valuation function coefficients for operating assets and earnings, α_1 and α_2, play an important role in subsequent analysis. The coefficients for the other information, β_1 and β_2, are less significant.

From the proposition one infers that goodwill equals

$$g_t = P_t + bv_t = \alpha_1 ox_t^a + \alpha_2 oa_t + \beta \cdot v_t.$$

Therefore, the present value of future expected abnormal operating earnings can be expressed as a function of the current operating profitability as measured by current abnormal operating earnings, ox_t^a, the current book value of operating assets, oa_t, and other information relevant to the prediction of future abnormal operating earnings, v_t. Given the restrictions on the LIM parameters, one further infers from the valuation function that goodwill is an increasing function of current abnormal earnings unless there is no persistence (i.e., $\alpha_1 > 0$ if $\omega_{11} > 0$ and $\alpha_1 = 0$ if $\omega_{11} = 0$). The result makes intuitive sense. Valuation coefficient α_2 is likewise non-negative, and goodwill relates non-negatively to operating assets (assuming fixed ox_t^a and v_t). As the reader may suspect, the possibility of $\alpha_2 > 0$ as opposed to $\alpha_2 = 0$ occurs because of the correction for understated book values associated with conservative accounting.[22]

Before proceeding with an analysis of the critical role of the coefficient α_2, we note that the valuation solution reflects the previously discussed dividend policy irrelevancy. An incremental dollar of dividends simply displaces a dollar of market value in the sense that $\partial P_t / \partial d_t = -1$; the conclusion follows because ox_t^a, oa_t, and v_t do not vary with the dividends, but $\partial fa_t / \partial d_t = \partial bv_t / \partial d_t = -1$.

Impact of conservative accounting on the structure of the valuation function

We now turn to a core issue: How does the structure of the valuation function depend on conservative versus unbiased accounting?

Proposition 4: Unbiased accounting is equivalent to $\alpha_2 = \omega_{12} = 0$; conservative accounting is equivalent to $\alpha_2, \omega_{12} > 0$.

The above result is somewhat technical in nature. However, it brings out the basic concept that conservative accounting, on average, understates book values in that the valuation function requires an additional term $\alpha_2 oa_t$ if, and only if, $\omega_{12} > 0$. One also sees that conservative accounting leads to an upward adjustment in the prediction of future profitability, consistent with the discussion in the previous section.

One can instructively compare conservative and unbiased accounting by restating P_t as a linear function of $(fa_t, oa_t, ox_t, c_t, v_t)$. This transformation is readily accomplished by substituting $ox_t^a = ox_t - (R_F{-}1)(oa_t + c_t - ox_t)$ into valuation function (12). Similarly, one can restate P_t as a linear function of (bv_t, x_t, d_t, v_t) by substituting $ox_t^a = x_t^a = x_t - (R_F{-}1)(bv_t + d_t - x_t)$ into (12). The result for unbiased accounting is as follows.

Corollary 1: Unbiased accounting obtains if, and only if,

$$P_t = k(\emptyset x_t - d_t) + (1 - k)bv_t + \beta_1 v_{1t} \tag{13a}$$

or $\quad P_t = fa_t + k(\emptyset ox_t - c_t) + (1 - k)oa_t + \beta_1 v_{1t}, \tag{13b}$

where $\quad \emptyset \equiv \dfrac{R_F}{R_F - 1}$

and $\quad k = \omega_{11} \dfrac{R_F - 1}{R_F - \omega_{11}} \in [0,1).$

The valuation solution does not depend on ω_{22} or γ_2.

Expression (13a) shows that unbiased accounting reduces to the model developed in Ohlson (1995). The market value, P_t, equals a weighted average of a pure "flow" model based on (x_t, d_t) and a pure "stock" model based on bv_t, plus other information. To expand on this weighted average concept, observe that if $\omega_{11} = 1,$[23] then $k = 1$ and

$$P_t = k(\emptyset x_t - d_t) + \beta_1' v_{1t}, \quad \text{where } \beta_1' \equiv \dfrac{R_F}{(R_F - 1)(R_F - \gamma_1)}.$$

On the other hand, if $\omega_{11} = 0$, then $k = 0$ and

$$P_t = bv_t + \beta_1'' \upsilon_{1t}, \quad \text{where } \beta_1'' \equiv \frac{1}{R_F - \gamma_1}.$$

In the more general case, $\omega_{11} \in (0,1)$, $k \in (0,1)$, and one obtains P_t as a weighted average of the two extreme models (with $\beta_1 = k\beta_1' + (1 - k)\beta_1''$). Since ω_{11} acts as a "persistence" parameter for abnormal operating earnings, this factor logically determines the weight one places on the "flow" model relative to the "stock" model.[24]

Conservative accounting leads to a different conclusion concerning the weights on "flows" and "stocks".

Corollary 2: For conservative accounting the valuation function equals

$$P_t = k(\phi x_t - d_t) + (1 - k)bv_t + \alpha_2 oa_t + \beta \cdot \upsilon_t, \tag{14a}$$

$$\text{or } P_t = fa_t + k_1(\phi ox_t - c_t) + k_2 oa_t + \beta \cdot \upsilon_t, \tag{14b}$$

where $k_1 = k \geq 0$, $k_2 = 1 - k + \alpha_2 > 0$, and $k_1 + k_2 > 1$.

With unbiased accounting (13a) is a special case of (14a). The only difference is that (14a) adjusts for the understatement of operating assets if they are conservatively reported. Similarly, (13b) is a special case of (14b). Expression (14b) applies for all $\omega_{12} \geq 0$, $k_1 = k$ regardless of ω_{12}, and $k_2 = 1 - k_1 + \alpha_2$. One concludes immediately that *the sum of k_1 and k_2 exceeds one if, and only if, the accounting is conservative*. Since k equals k_1 independently of ω_{12}, the coefficient adjustment necessary for conservative accounting focuses singularly on how one "interprets" the book value of operating assets. These results make intuitive sense because conservative accounting concerns the valuation of operating assets relative to the present value of expected cash flows.

The LIM requirement $\omega_{11} < 1$ implies $k_2 > 0$ for any $\omega_{12} \geq 0$ and, hence, the valuation functions (13 & 14) always attach a strictly positive weight to the operating "stock" measure, oa_t. However, the operating "flow" measure, $\phi ox_t - c_t$, vanishes in expressions (13b) and (14b) if there is no earnings persistence. In this case, $\omega_{11} = k_1 = 0$, so that (14b) reduces to

$$P_t = fa_t + k_2 oa_t + \beta \cdot \upsilon_t, \tag{15}$$

where $k_2 = 1 + \alpha_2 \geq 1$; $k_2 > 1$ if, and only if, $\omega_{12} > 0$. Since $\beta \cdot \upsilon_t$ varies around zero, (15) illustrates how conservative accounting requires the valuation coefficient associated with oa_t to reflect the on average understatement of the operating assets' value. Of course, the phrase 'on average' is important. At any given date t the information vector (oa_t, υ_t) may result in $\alpha_2 oa_t + \beta \cdot \upsilon_t < 0$, and the state of other information, υ_t, may induce a market value of operating assets less than its book value. Conservative accounting permits a *realization* $P_t < bv_t$ even though, by definition, P_t is expected to exceed bv_t.[25]

Corollaries 1 and 2 bear on two other intertwined issues: the possibility of aggregation and the relevance of cash flows. The income statement and the balance sheet both satisfy a simple informational aggregation property for unbiased accounting. One can add the income statement items (i_t, ox_t), and the balance sheet items (fa_t, oa_t) without losing any information when one infers the market value. The claim follows immediately because of expression (13a). From the possibility of aggregation one also sees that separating earnings into cash earnings $(i_t + c_t)$ and non-cash earnings (Δoa_t) plays no informational role. Nor is any information content attached to cash flows. To be precise, cash flows are informationally redundant (or irrelevant) in the sense that one infers P_t from (x_t, bv_t, d_t, v_t) and yet the latter vector does not reveal c_t.

In contrast, the above aggregation property is violated if conservative accounting is used for operating assets, while unbiased reporting is used for financial assets. In that case, inference of P_t requires separate information concerning the valuation of operating and financial activities. To illustrate this concept, assume $\omega_{12} > 0$ and consider the extreme no persistence case $\omega_{11} = 0$, which leads to valuation expression (15) with $k_2 > 1$. The value P_t cannot be inferred unless one knows the components of book value, (fa_t, oa_t). The necessity of using disaggregated data when one infers value applies no less when $\omega_{11} > 0$. Provided only that $\omega_{12} > 0$, one readily shows that there exist two realizations $(fa_t', oa_t', ox_t', i_t', d_t, v_t)$ and $(fa_t'', oa_t'', ox_t'', i_t'', d_t, v_t)$ satisfying $fa_t' + oa_t' = fa_t'' + oa_t''$ and $ox_t' + i_t' = ox_t'' + i_t''$, but $P_t' \neq P_t''$. (The two vectors are valuation sufficient because one infers c_t from fa_t, i_t and d_t: $c_t = fa_t - i_t + d_t - fa_{t-1}$ and $fa_{t-1} = i_t/(R_F - 1)$.) However, this conclusion is unavailable if one adds the restriction $(fa_t', oa_t') = (fa_t'', oa_t'')$, even when $k_1 > 0$. With conservative accounting, the Proposition 3 valuation model requires balance sheet disaggregation, but one need not distinguish between the financial and operating components of x_t. Expression (14a) makes the point obvious.

The flow components of valuation models (13) and (14) involve the multiplication of either aggregate earnings x_t or operating earnings ox_t by $\emptyset = R_F/(R_F - 1)$ and then deducting either the dividends paid d_t or the operating cash flows c_t. In both cases, these deductions reflect the fact that multiplying earnings by $\emptyset$ provides a flow model of the value of the *total* aggregate or *total* operating assets generated at the end of the period. An adjustment must then be made for either the assets distributed to the equity holders (d_t) or the operating assets transferred to the financial assets (c_t).

If there is no persistence in the abnormal operating earnings $(\omega_{11} = 0)$, then the flow model is given zero weight and there is no need to know either the dividends paid or the cash flow from operations. On the other hand, if there is persistence in the abnormal operating earnings $(\omega_{11} > 0)$, then the flow model is given positive weight and an adjustment is made

for *either* the dividends paid (see (13a) and (14a)) *or* the operating cash flows (see (13b) and (14b)). These observations are independent of whether the accounting is unbiased or conservative.

The informational redundancy of current cash flows for unbiased accounting extends to the dynamics predicting cash flows and operating assets. The point is readily appreciated by noting that the valuation coefficients (α_1, α_2, β_1, β_2) do not depend on the LIM parameters ω_{22} and γ_2 if, and only if, $\omega_{12} = 0$ (see Proposition 3). The irrelevant parameters ω_{22} and γ_2 specify the operating assets' dynamics, equations (10b) and (10d). Hence, given $\omega_{12} = 0$, valuation function (12) derives from (10a) and (10c) alone. Since the latter equations cannot predict future cash flows without predictions of future (changes) in operating assets (i.e., accruals), unbiased accounting implies that one can derive the value P_t regardless of the anticipated sequence of cash flows. Of course, the converse applies for conservative accounting. The operating assets' dynamic equations (10b) and (10d) take on relevance, and these specify the expected cash flow sequence when combined with the always relevant abnormal earnings' dynamics, (10a) and (10c). One concludes that valuation analysis does not depend, in any substantive sense, on current and future cash flows (or future operating accruals) if, and only if, the accounting is unbiased.

Asymptotic relations among value, value changes, and contemporaneous accounting numbers

At any given date t, the relation between value (or changes in value) and current accounting numbers are influenced by idiosyncratic events that influence the accounting numbers and other information that is used in determining value. To remove these idiosyncratic effects and thereby focus on *average* relations, we explore the relation of the asymptotic expectations for the variables of interest.

Price/earnings relation

Accounting textbooks frequently discuss how conservative accounting influences the behavior of earnings. The prototypical analysis concerns the effects of straightline versus accelerated depreciation methods on earnings for alternative acquisition scenarios. Simple numerical examples demonstrate that accelerated depreciation brings lower earnings than straightline depreciation for all periods, provided that acquisitions grow over time. On the other hand, in a steady state of constant acquisitions both methods result in identical earnings.[26] These straightforward conclusions depend only on the clean surplus relation, and, of course, they generalize to other assets and accounting issues. As an immediate implication one obtains the hypothesis that, in a world of conservative accounting, growth firms tend to have larger P/E ratios than no growth firms, and no

growth firms tend to have the same ratios as firms using unbiased accounting.

The analysis of how conservative accounting affects a firm's value relative to its earnings poses no problems in simple certainty settings. One derives conclusions by comparing the accounting earnings with the "true" economic earnings based on the present value of cash flows. Since, by assumption, the evolution of these variables are certain, neither serves as information in estimating future cash flows. Uncertainty settings require more complex analysis because one must now consider how accounting information affects the firm's value. Earnings may not be value relevant information in extreme cases. As previously noted, if there is no persistence in operating abnormal earnings ($\omega_{11} = 0$), then P_t depends only on fa_t, oa_t and υ_t (see expression (15)). The current income items are therefore irrelevant, except for their updating effect on fa_t and oa_t. Furthermore, the prediction of future operating earnings does not depend on the current income items. Given such apparent value irrelevance of current operating earnings, it may seem unlikely that aggregate earnings relate systematically, on average, to price. Interestingly, the average relation between earnings and price does not depend particularly on whether earnings provide value relevant information. As Proposition 5 below demonstrates, conservative accounting ($\omega_{12} > 0$) and growth ($\omega_{22} > 1$) are the key factors, rather than persistence (ω_{11}).

Under certainty and unbiased accounting, the predividend value of equity is a multiple of current earnings, $P_t + d_t = \phi x_t$. Using this relation as a point of reference, the following proposition identifies conditions such that the asymptotic expectation of the predividend value equals ϕ times the expectation of contemporaneous earnings.

Proposition 5:[27] Conservative accounting ($\omega_{12} > 0$) *and* growth ($\omega_{22} > 1$) imply

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] > 0 \text{ as } \tau \to \infty.$$

Unbiased accounting *or* no growth imply[28]

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] \to 0 \text{ as } \tau \to \infty. \tag{16}$$

The case in which earnings are not value relevant information ($\omega_{11} = 0$) illustrates how the LIM parameters determine the conclusion. Under this restriction, for large τ:[29]

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] \approx \left[\frac{1}{R_F - \omega_{22}} - \frac{1}{(R_F - 1)\omega_{22}} \right] \omega_{12} \, \omega_{22}^{\tau}$$

$$\times \left[oa_t + \frac{\upsilon_{2t}}{\omega_{22} - \gamma_2} \right]$$

This expression is positive if, and only if, accounting is conservative ($\omega_{12} > 0$), there is growth ($\omega_{22} > 1$), and expected operating assets are positive ($oa_t + v_{2t}(\omega_{22}-\gamma_2)^{-1} > 0$). Furthermore, the expected difference between predividend value ($P_{t+\tau} + d_{t+\tau}$) and earnings scaled by the multiple ($\emptyset x_{t+\tau}$) increases as the degree of conservativeness or rate of growth increases. With growth, both the firm's (expected) value and earnings increase over time, but only for conservative accounting does value grow at a rate faster than earnings. This conclusion remains valid even if abnormal earnings exhibit positive persistence ($\omega_{11} > 0$). In fact, the difference in growth rates for value and earnings increases as ω_{11} increases (given $\omega_{12} > 0$ and $\omega_{22} > 1$). Accounting conservatism and growth accordingly lead to more pronounced effects on the price/earnings relation when there is greater persistence in abnormal earnings.

Relation between change in value and accounting earnings
Proposition 5 conceptualizes accounting earnings x_{t+1} as a potential indicator of the *stock* of value, $P_{t+1} + d_{t+1}$. An alternative perspective recasts the role of accounting earnings as potentially indicating the *change* in value, $P_{t+1} + d_{t+1} - P_t$. The idealized certainty case with unbiased accounting lends precision to this concept because these two conditions imply $P_{t+1} + d_{t+1} - P_t = x_{t+1}$. Using this relation as a reference point, the following proposition identifies conditions such that the asymptotic expectation of the change in value equals the expectation of contemporaneous earnings.

Proposition 6: Conservative accounting ($\omega_{12} > 0$) *and* growth ($\omega_{22} > 1$) imply

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}) - \tilde{x}_{t+\tau}] > 0 \text{ as } \tau \to \infty.$$

Unbiased accounting *or* no growth imply

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}) - \tilde{x}_{t+\tau}] \to 0 \text{ as } \tau \to \infty.$$

As the appendix shows, the proof of the proposition is immediate from proposition 5 and the fact that PVR, by itself, implies that, for every $\tau > 0$,

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \emptyset\tilde{x}_{t+\tau}] = \emptyset E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}) - \tilde{x}_{t+\tau}].$$

Hence, independently of LIM, accounting earnings indicate the change in the firm's value if, and only if, accounting earnings also indicate the firm's (stock of) value via the multiplier $\emptyset$. Since this statement is valid for every horizon $\tau > 0$, it also applies on average or as $\tau \to \infty$.

The essence of condition (17), and (16), revolves around the behavior of the expected rate of change in goodwill. An alternative proof of Proposition 6 brings out this simple but instructive concept. Applying

CSR, the difference between the value change and accounting earnings equals

$$(P_{t+\tau} + d_{t+\tau} - P_{t+\tau-1}) - (bv_{t+\tau} + d_{t+\tau} - bv_{t+\tau-1}) = g_{t+\tau} - g_{t+\tau-1} \equiv \Delta g_{t+\tau}$$

It follows that (17) is satisfied if, and only if, $E_t[\Delta \tilde{g}_{t+\tau}] \to 0$ as $\tau \to \infty$. Unbiased accounting therefore suffices for (17) because, by definition, it implies $E_t[\tilde{g}_{t+\tau}] \to 0$. More generally, if one thinks of $g_{t+\tau}$ as an "error" in book value at date τ, then (17) can be satisfied because the (expected) errors in book values at two adjacent dates "cancel" each other. Conservative accounting allows for this possibility precisely because the "errors" will cancel in the absence of expected growth in operating assets. But these two settings are the only ones leading to $E_t[\Delta \tilde{g}_{t+\tau}] \to 0$; without difficulty one shows that growth and conservative accounting imply $E_t[\Delta \tilde{g}_{t+\tau}] > 0$ as $\tau \to \infty$. In this case one therefore concludes that accounting earnings are downward biased relative to economic earnings because the expected value deficiency in book value (i.e., an "understatement error") increases with time.

Relation between book value and accounting earnings

Propositions 3 through 6 combine LIM with the modelling of financial activities to show how the stochastic evolution of accounting data (fa_t, oa_t, i_t, ox_t, c_t, and d_t) relates to value and value changes. We now examine how accounting affects the relation between the book value of the firm's equity (financial plus operating assets) and subsequent aggregate accounting earnings (financial plus operating earnings). To eliminate the idiosyncratic effects of current abnormal earnings and other information, we again focus on asymptotic expectations; that is, we consider the relation between $E_t[\tilde{x}_{t+\tau+1}]$ and $E_t[\tilde{bv}_{t+\tau}]$.

The inclusion of the financial activities opens the possibility that the relations depend on the dividend policy. Recall that dividends directly reduce the firm's book value ($\partial bv_t/\partial d_t = -1$). The impact of dividends on future earnings is slightly less straightforward to derive, but it is equally straightforward to interpret. A reduction in the asset base resulting from the payment of dividends reduces the firm's capacity to generate earnings. In particular, financial assets are reduced, and this reduces the interest earned on those assets (or increases the interest paid on the debt).

More formally, the expected value of (aggregate) earnings for $t+1$, given the date t book value, abnormal earnings, and other information, is readily derived from NIR and (10a):

$$E_t[\tilde{x}_{t+1}] = (R_F - 1)bv_t + \omega_{11}x_t^a + \omega_{12}oa_t + v_{1t}.$$

Since $\partial bv_t/\partial d_t = -1$ and $\partial x_t^a/\partial d_t = \partial oa_t/\partial d_t = \partial v_{1t}/\partial d_t = 0$, one concludes that $\partial E_t[\tilde{x}_{t+1}]/\partial d_t = -(R_F - 1)$.[30] This relation shows that the *marginal* effect of an incremental dollar of dividends on expected earnings is

accounted for "properly," in the sense that one derives the same result in a certainty model with unbiased accounting. The possibilities of conservative accounting or persistence in earnings introduce no distortions because future interest revenues capture the marginal effect of a change in dividends.

To avoid prejudicing the behavior of expected earnings due to dividends/capital contributions, we invoke a "full payout" dividend policy, $d_{t+\tau} = x_{t+\tau}$, all τ.[31] This simplifies the analysis by holding bv_t constant and, hence, we can investigate how $E_t[\tilde{x}_{t+\tau+1}]$ relates to bv_t, or to the base projection $(R_F - 1)bv_t$. The full payout policy loses no generality because $E_t[\tilde{x}_{t+\tau}] - (R_F - 1)E_t[b\tilde{v}_{t+\tau-1}]$ is independent of the dividend policy. At any rate, the full payout policy is only hypothetical, and it may deviate from a firm's "actual" policy.

We first consider the behavior of asymptotic (full payout) earnings in settings with unbiased accounting. Given the full payout policy, a firm's expected earnings equal, on average, the risk-free interest rate times the current book value, i.e., one obtains[32]

$$E_t[\tilde{x}_{t+\tau}] \to (R_F - 1)bv_t \text{ as } \tau \to \infty, \tag{18}$$

if $\omega_{12} = 0$ and $d_{t+\tau} = x_{t+\tau}$, $\tau > 0$. Conclusion (18) depends neither on the growth parameter, nor on the change in the mix of expected operating and financial assets.

The sufficiency of current aggregate book value for predicting long run earnings is unsurprising if one keeps in mind that unbiased accounting combined with $d_{t+\tau} = x_{t+\tau}$ $(bv_{t+\tau} = bv_t)$ imply $E_t[\tilde{P}_{t+\tau}] \to bv_t$, and $E_t[\tilde{P}_{t+\tau}] \to (R_F - 1)^{-1}E_t[\tilde{x}_{t+\tau}]$ due to proposition 5. The last two expressions imply (18). Further note that because $P_t \neq bv_t$ generally, the long run, full payout, expected earnings do not determine the market value.[33]

We next consider the behavior of asymptotic (full payout) earnings in settings with conservative accounting. One should now expect asymptotic expected earnings to exceed $(R_F - 1)bv_t$ because, as noted earlier, conservative accounting makes a firm "look" profitable, i.e., $E_t[\tilde{x}_{t+\tau}/bv_{t+\tau-1}]$ $= E_t[\tilde{x}_{t+\tau}/bv_t] > R_F - 1$, where the equality follows from $d_{t+\tau} = x_{t+\tau}$. This type of bias in the book rate of return also seems reasonable in that conservative accounting allows a firm to understate values which can be expected to surface as additional future earnings. As shown below, it follows indeed that $E_t[\tilde{x}_{t+\tau}] - (R_F - 1)bv_t > 0$, $\tau \to \infty$, if, and only if, $\omega_{12} > 0$. Nevertheless, one distinguishes usefully between what occurs for no growth ($\omega_{22}=1$) as opposed to growth ($\omega_{22}>1$) in expected operating assets.[34]

First, for $\omega_{12} > 0$ and $\omega_{22} = 1$, one obtains

$$E_t[\tilde{x}_{t+\tau}] \to (R_F - 1)bv_t + K_t \text{ as } \tau \to \infty,$$

where $K_t \equiv \omega_{12}(1 - \omega_{11})^{-1}[oa_t + v_{2t}(1 - \gamma_2)^{-1}] > 0$.

That is, with *no growth*, conservatism results in positive expected abnormal earnings that converge to a finite bound, that is, $E_t[\tilde{x}^a_{t+\tau}] \rightarrow K_t$ (implied by $x^a_t = ox^a_t$ and the appendix).

Second, using similar analysis for $\omega_{12} > 0$ but $\omega_{22} > 1$ yields

$$E_t[\tilde{x}_{t+\tau}] \rightarrow \infty \text{ as } \tau \rightarrow \infty.$$

The result is immediate from the LIM specification because the parametric restrictions imply $E_t[\tilde{x}^a_{t+\tau}] = E_t[ox^a_{t+\tau}] \rightarrow \infty$. Hence, with growth and conservative accounting, expected abnormal earnings grow without bound.

The last setting leads to an infinite expected book rate of return, $E_t[\tilde{x}_{t+\tau}/bv_{t+\tau-1}] \rightarrow \infty$, and this outcome may appear too strong. However, one has to keep in mind that the conditions $\omega_{12} > 0$ and $\omega_{22} > 1$ imply growth in expected net *operating* assets, and the same applies to the expected goodwill, $E_t[\tilde{g}_{t+\tau}] \rightarrow \infty$. Hence, $E_t[\tilde{P}_{t+\tau}] \rightarrow \infty$ even though the full payout policy results in $bv_{t+\tau}$ remaining constant at bv_t. The exceptional nature of the unbounded expected growth in the book rate of return mirrors the unbounded expected growth in the market to book ratio. The full payout policy is, in effect, sufficiently conservative to allow for growth in dividends and earnings.

The preceding analysis has identified three outcomes for asymptotic full payout expected earnings, based on the parameters ω_{12} and ω_{22}. This partitioning of outcomes can be used to highlight the nature of Propositions 4 and 5 (or 6). That is, we identify and link the joint behavior of $E_t[\tilde{P}_{t+\tau} - b\tilde{v}_{t+\tau}]$ and $E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}]$ to the asymptotic properties of full payout earnings.

Proposition 7: Assume $d_{t+\tau} = \tilde{x}_{t+\tau}, \tau > 0$. Then, as $\tau \rightarrow \infty$:

(a) $E_t[\tilde{x}_{t+\tau}] \rightarrow (R_F - 1)bv_t$ implies, and is implied by,

$$E_t[\tilde{P}_{t+\tau} - b\tilde{v}_{t+\tau}] \rightarrow 0 \text{ and } E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] \rightarrow 0;$$

(b) $E_t[\tilde{x}_{t+\tau}] \rightarrow (R_F - 1)bv_t + K_t, K_t \in (0,\infty)$, implies, and is implied by,

$$E_t[\tilde{P}_{t+\tau} - b\tilde{v}_{t+\tau}] > 0 \text{ and } E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] \rightarrow 0;$$

(c) $E_t[\tilde{x}_{t+\tau}] \rightarrow \infty$ implies, and is implied by,

$$E_t[\tilde{P}_{t+\tau} - b\tilde{v}_{t+\tau}] > 0 \text{ and } E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] > 0.$$

It should be stressed that the expressions concerning price relative to book value, or earnings, do not depend on the dividend policy. That is, although the statements regarding the asymptotic limits of $E_t[\tilde{x}_{t+\tau}]$

assume $d_{t+\tau} = x_{t+\tau}$, the asymptotic limits of $E_t[\tilde{P}_{t+\tau} - b\tilde{v}_{t+\tau}]$ and $E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}]$ do *not* depend on that assumption.

Part (a) provides the benchmark relating price in an unbiased fashion to both book value and earnings. Part (b) shows a bias in price relative to book value, but not in price relative to earnings. This outcome occurs because the (expected) goodwill is positive but bounded due to no growth, and thus earnings are measured, on average, as if the earnings were unbiased. Part (c) shows biases in both price relative to book value and price relative to earnings. In this case the unrecorded goodwill grows exponentially, and this leads to an understated change in book value, i.e. in understated earnings.[35]

We conclude more generally from this section that the core issues surrounding earnings behavior and value must ultimately come to grips with implications of unbiased versus conservative accounting, and, in the case of conservative accounting, the absence or presence of growth in assets accounted for conservatively. The persistence in abnormal earnings—i.e., the LIM parameter ω_{11} – is, in sharp contrast, of at most subordinated interest.

Comparative dynamics: cash earnings versus accrued earnings

The preceding section examines the average (asymptotic) relation of aggregate earnings to the firm's market value and to its book value. This section examines the effect on market value of *incremental changes* in the accounting numbers. In particular, we examine how an incremental dollar of cash operating earnings versus an incremental dollar of accrued operating earnings affects price. This analysis requires some care because a change in one accounting variable realization, such as earnings, must be offset by a change in some other accounting variable realization. One would otherwise violate the basic accounting relation, debits equal credits, within the CSR framework. This duality in accounting also implies that any comparative dynamic analysis leads to multiple possibilities when one evaluates the incremental effect of a dollar for some specific variable. As shown below, an incremental dollar of cash earnings, for example, is consistent with numerous alternative scenarios concerning changes in the remaining accounting variables. This aspect is critical in the analysis.

To illustrate the significance of the debits-equals-credits requirement, consider valuation function (14b) in Corollary 2. On the surface, one may (perhaps surprisingly) conclude that

$$\partial P_t/\partial c_t = -k_1 \leq 0 \ !$$

However, as indicated, the partial derivative operation makes no sense because FAR and OAR require a (small) change in c_t to impart an offsetting (small) change in at least one other accounting variable.

The definitions below specify changes in accounting variables that satisfy the debits-equals-credits relations inherent in FAR, OAR, and CSR. Subsequent analysis relies on these definitions to evaluate the impact of changes in earnings, and their composition, on market value.

Definitions:

A. An *incremental dollar of cash (operating) earnings* (δcash earnings) results in an increase in the cash flows, but not in the operating accruals:

$$\delta ox_t = 1, \delta c_t = 1, \delta oa_t = 0.$$

B. An *incremental dollar of accrued earnings* (δaccrued earnings) results in an increase in the operating accruals, but not in the cash flows:

$$\delta ox_t = 1, \delta c_t = 1, \delta oa_t = 1.$$

C. An *incremental dollar of investment in operating assets* (δinvestment) decreases cash flows and increases operating accruals, but has no immediate impact on earnings:

$$\delta ox_t = 0, \delta c_t = -1, \delta oa_t = 1.$$

The operating accounting relation OAR, $oa_t = oa_{t-1} + ox_t - c_t$, with oa_{t-1} fixed, provides the basic relation to define the three change specifications. However, changes in the OAR variables also potentially influence the financial accounting relation, FAR, and the clean surplus relation, CSR. The effects on these two relations must be identified for a proper understanding of how changes in operating variables impact on the market value.

An incremental dollar of cash earnings (A) increases aggregate earnings, book value, and financial assets; this effect occurs because operating assets remain unchanged:

$$\delta x_t = \delta bv_t = \delta fa_t = 1.$$

An incremental dollar of accrued earnings (B) also increases aggregate earnings and book value, but it does not affect financial assets because the increase is offset by an increase in operating assets:

$$\delta x_t = \delta bv_t = 1, \delta fa_t = 0.$$

An incremental dollar of investment in operating assets (C) does not change earnings or book value, but the financing of the investment reduces financial assets:

$$\delta x_t = \delta bv_t = 0, \delta fa_t = -1.$$

Further observe that an incremental dollar of accrued earnings is equivalent to an incremental dollar of cash earnings plus the reinvestment of those cash earnings in operating assets: δaccrued earnings = δcash earnings + δinvestment.

The impact of the three types of changes on value and future expected earnings depends on whether the accounting is unbiased or conservative. The following proposition characterizes these differences.

Proposition 8: The following five statements are equivalent:

(a) the accounting is unbiased;

(b) $\partial P_t/\partial$accrued earnings$_t$ = $\partial P_t/\partial$cash earnings$_t$;

(c) $\partial P_t/\partial$investment$_t$ = 0;

(d) $\partial E_t[\tilde{x}_{t+1}]/\partial$accrued earnings = $\partial E_t[\tilde{x}_{t+1}]/\partial$cash earnings$_t$;

(e) $\partial E_t[\tilde{x}_{t+1}]/\partial$investment$_t$ = 0.

One replaces the '=' signs in statements (b) through (e) with '>' signs if the accounting is conservative.

The proof of the proposition (see the Appendix) derives the changes in price from valuation function (12), and the changes in expected earnings from expression (10a) combined with $i_{t+1} = (R_F - 1)fa_t$. Observe the close parallel between the price responses in (b) and (c) and the expected earnings responses in (d) and (e). In cases (b) and (d) the signs are, of course, positive, and (d) reflects the notion that additional current earnings, on average, beget additional earnings in the future. Unbiased accounting renders the distinction between cash and accrued earnings unnecessary. For conservative accounting one obtains differential effects in (b) and (d) because accrued earnings, as opposed to cash earnings, defer some of the value recognition to future periods. Thus one can view accrued earnings, on the margin, as being of higher "quality" than cash earnings.

Statement (b) permits straightforward analysis for the two accounting regimes. First consider the no persistence in abnormal earnings case (i.e., $\omega_{11} = 0$). From expression (15) one infers that $\partial P_t/\partial$cash earnings = 1, because each incremental dollar of cash earnings increases fa_t but leaves oa_t (and υ_t) unchanged. A dollar of cash earnings therefore equals a dollar of value. This same conclusion applies for unbiased accounting and an (incremental) dollar of accrued earnings; fa_t now remains unchanged, oa_t increases by a dollar, and $\alpha_2 = 0$. However, for conservative accounting the increase in economic value always exceeds accrued earnings in that $\partial P_t/\partial$accrued earnings = $1 + \alpha_2 = 1 + \omega_{12}/(R_F - \omega_{22}) > 1$. The impact of cash earnings on cash flows is immediate, whereas accrued earnings result from an increase in expected future cash flows. Under conservative accounting, the present value of the increase in expected future cash flows is greater than the recorded increase in current accruals.

If there is persistence in abnormal earnings (i.e., $\omega_{11} > 0$), then both types of earnings are less than the increase in economic value. In particular, under conservative accounting

$$\partial P_t/\partial\text{accrued earnings} = 1 + \alpha_1 + \alpha_2 = \frac{R_F}{(R_F - \omega_{11})}\left[1 + \frac{\omega_{12}}{R_F - \omega_{22}}\right]$$

$$> \partial P_t/\partial \text{cash earnings} = 1 + \alpha_1 = \frac{R_F}{R_F - \omega_{11}} > 1.$$

Hence, the more persistent the abnormal earnings (ω_{11}), the greater is the multiple associated with both accrued and cash earnings. The more conservative the accounting (ω_{12}) the greater is the multiple associated with accrued earnings.

Statements (c) and (e), which refer to "δinvestment," require guarded interpretation. Investment in operating assets is not an object of choice in our analysis. The model postulates a set of information dynamics that reflect some underlying, preset, sequence of operating activities, which include the decision rules determining the investment in operating assets. The examination of the impact of δinvestment bears on how price and expected earnings respond to an increase in investment given that the increase results from events imbedded in the LIM information dynamics.

To interpret properly the "δ" in "δinvestment" one must keep firmly in mind that the increment in operating assets is neutral as to types of operating assets or their vintages. Likewise, the "δ" is neutral as to any causation of the outcome. The idea behind the "δ" in results (c) and (e) is much simpler. This comparative analysis concerns the effect on price and next period expected earnings resulting from two sets of financial reports that differ *only* with respect to their mixes of financial and operating assets. Unbiased accounting renders the mix of financial and operating assets irrelevant; for conservative accounting, a given level of book value induces a higher price, and expected earnings, if it is comprised of a higher proportion of operating accruals. The value of an additional dollar of oa_t exceeds the loss of a dollar of fa_t because, on average, oa_t is less than the present value of its related anticipated cash flows. The linear dynamics ensures that what happens on average works on the margin as well.

Conservative accounting and zero net present value investments

When we first discussed unbiased versus conservative accounting, we noted that conservative accounting makes a firm look profitable, on average, by inducing relatively large earnings compared to the investment base generating the earnings. In particular, the second part of Proposition 2 shows that conservative accounting is equivalent to a positive present value of expected abnormal earnings, on average. Intuitively, there are two major ways in which conservative accounting produces this result. First, conservative accounting can result in book values that are less than the present value of the cash flows that will be generated by current or prior investments. This occurs if investments are expensed immediately (e.g., investments in R&D) or are amortized more quickly than their drop in value (e.g., the use of declining balance depreciation methods or short estimated lives with straightline depreciation). Second, virtually all

accounting procedures are conservative in that they do not recognize today the expected *net* present value of *future* investment projects.

A question arises as to whether our results are due entirely to the second form of conservatism. To demonstrate this is not the case, we consider a setting in which the firm undertakes only zero net present value investments. Under these conditions, the firm has zero value prior to any investment in the firm. Let $-d_0 = fa_0 = bv_0 > 0$ represent the initial investment by the equity holders. After that investment, the firm's equity has a market value of $P_0 = -d_0$, which implies the goodwill g_0 equals zero. From (8) we obtain

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_0[o\tilde{x}_\tau^a] = 0.$$

If the firm subsequently invests in operating assets, that is, $c_1 < 0$, then Proposition 2 combined with the above zero present value expression implies that, for conservative accounting, there exists some $T \in (0,\infty)$ such that

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_0[o\tilde{x}_\tau^a] < 0. \tag{19}$$

Hence, given $g_0 = 0$ and conservative accounting, the firm can expect negative abnormal earnings for at least some of the early years; only in later years are abnormal earnings expected to be positive.

Expression (19) permits us to make another point. Even if a firm plans to undertake only zero net present value projects, the number of years with negative expected abnormal earnings may be small, but the number of years with positive expected abnormal earnings could be infinite. The notion that conservative accounting can lead to apparent but not "real" profitability, on average, makes sense as long as "on average" refers to the arithmetic rather than the discounted-weighted average.

The preceding comments are independent of the information dynamics. Additional insights are obtained by considering this setting within LIM. Assume there are no existing operations at $t=0$, that is, $ox_0 = oa_0 = 0$, and an initial capital contribution of $-d_0 > 0$ is invested in financial assets, so that $-d_0 = fa_0 = bv_0$. The date zero valuation function is

$$P_0 = fa_0 + \beta_1 \upsilon_{10} + \beta_2 \upsilon_{20},$$

or $\quad P_0 + d_0 = g_0 = \beta_1 \upsilon_{10} + \beta_2 \upsilon_{20}.$

The initial goodwill, g_0, is positive if, and only if, the firm is expected to have an opportunity, in the future, to invest in positive net present value projects. Given LIM and $ox_0 = oa_0 = 0$, the absence or presence of initial goodwill hinges on the nonaccounting data υ_{10} and υ_{20}, and their related valuation coefficients β_1 and β_2.

As demonstrated earlier, ω_{12} determines whether the accounting is unbiased ($\omega_{12} = 0$) or conservative ($\omega_{12} > 0$). However, knowledge of the

sign of ω_{12} does not suffice to determine the sign of g_0, even though ω_{12} alone determines the sign of $E_0[\tilde{g}_\tau]$, as $\tau \to \infty$ (see Proposition 4).[36] Thus it makes no sense to think of ω_{12} as an indicator of the absence (or presence) of zero (positive) expected net present value investments.

Given expression (19) one should expect, generally, that conservative accounting implies $E_0[o\tilde{x}_1^q] < 0$.[37] Indeed, in the context of LIM one obtains the following result.

Proposition 9: Consider the zero *ex ante* net present value initialization $P_0 = -d_0 = fa_0$ with $ox_0 = oa_0 = 0$, but $E_0[o\tilde{a}_1] > 0$.

Then, for conservative accounting,

$$E_0[o\tilde{x}_1^q] = E_0[o\tilde{x}_1] < 0.$$

For unbiased accounting the last inequality is replaced by an equality.

With $ox_0^a = oa_0 = 0$, (10) and (11) imply $E_0[o\tilde{x}_1^q] = E_0[o\tilde{x}_1] = \upsilon_{10}$, $E_0[o\tilde{a}_1] = \upsilon_{20}$, and $E_0[\tilde{c}_1] = \upsilon_{10} - \upsilon_{20}$. Under the assumptions of the proposition and unbiased accounting, $\upsilon_{10} = 0$ and $\upsilon_{20} > 0$. The total cash expected to be invested in operations at date t=1 is υ_{20} and the entire amount is capitalized on the balance sheet. Under conservative accounting, on the other hand, $\upsilon_{10} < 0$ and $\upsilon_{20} > 0$. The total cash expected to be invested in operations is $-\upsilon_{10} + \upsilon_{20}$. This investment has an expected net present value of zero, but only the proportion $\upsilon_{20}/(-\upsilon_{10}+\upsilon_{20}) < 1$ is capitalized on the balance sheet, while the remainder is expensed (resulting in negative earnings in the first period).[38]

In summary, this section demonstrates how the model discriminates between investment activities with zero as opposed to positive expected net present values. One resolves the issue by focusing on the initialization of the dynamic system and the state of other information at date zero. Given the zero net present value condition, conservative accounting leads to negative expected abnormal earnings for a finite number of years. Nonetheless, in the long run, on average, the expected abnormal earnings will be positive (regardless of the present value condition). Put simply in analytical terms, the sign of the present value condition $P_0 + d_0 = g_0$ is independent of the sign of $E_0[\tilde{g}_\tau]$, $\tau \to \infty$, i.e., the conservative accounting condition.

Concluding remarks

While the explicit modelling in this paper relies on the LIM dynamics to yield clear-cut solutions, we emphasize that this specification serves a useful purpose only because the accounting structure satisfies the broader properties delineated in our initial analysis of the relation between value and future accounting numbers. The central driving force in the

analysis emanates from the separation of accounting for financial and operating activities within a clean surplus context. This framework, combined with the "perfect" accounting for financial activities, ensures that wealth creation aligns with wealth distribution, as is apparent from Proposition 1, and classical MM concepts apply. One values the financial and operating activities separately – though the activities interact because of cash flows – and the analysis remains independent of the dividend policy. This structure is subtle because it never requires a firm to actually engage in the financial activity. The key assumptions revolve around the notion that there *exists* a financial asset (liability) defined by the NIR relation, and the accounting for this asset equates book and market values. This financial asset therefore functions as a numeraire in the accounting model. That is, dividends (capital contributions) reduce (increase) the financial assets held by the firm, and this transaction does not influence a firm's predividend value or provoke accounting measurement issues. A firm without an "inventory" of financial assets satisfying NIR equates dividends to cash flows by definition, and the accounting for this transaction is unproblematic. Thus one sees how closely MM concepts relate to the existence of an unambiguous numeraire asset/liability in a firm's economic environment.

Our modelling regards the operating activity as a "residual" activity, i.e., it reflects the outcome of all activities except for those that relate to pure borrowing and lending as characterized by NIR (and FAR). The observation makes it clear that nothing prevents us from introducing non–NIR financial activities which potentially are accounted for conservatively. Thus one can extend the model to include several independent (or "nonsynergistic") LIM activities, one (or more) of which could represent non-NIR financial assets or liabilities. All of the propositions permit this generalization because one readily establishes that the general, "aggregate", concept of unbiased accounting obtains if, and only if, each individual activity is accounted for without bias.

The possibility of generalizations beyond LIM raises the issue of necessary conditions for many of our results. Examination of Propositions 5 through 9 clearly suggest robustness with respect to the underlying information dynamics, i.e., the conclusions depend only on conservative/unbiased accounting and the absence/presence of growth. Aside from the use of multiple, independent LIMs, the analysis can expand on the dimensionality of the nonaccounting information (v_t). Other kinds of generalizations are less obvious. However, to focus on the set of all feasible linear models and related parametric restrictions would not by itself appear to be sufficiently substantive. The more fundamental issue concerns the modelling of accounting data as the outcome of two separate elements: accounting measures and primitive transactions (or economic events). Only by maintaining a distinction between measurements and transac-

tions can one evaluate how alternative accounting measurement principles influence mappings from accounting data to market value.[39] We believe the success of future financial accounting theory research will critically hinge on the development of analyses in such a direction.

Appendix

LIM Asymptotic Properties

Let $\quad y_t \begin{bmatrix} ox_\tau^a \\ oa_t \\ \upsilon_{1t} \\ \upsilon_{2t} \end{bmatrix}$ and H $= \begin{bmatrix} \omega_{11} & \omega_{12} & 1 & 0 \\ 0 & \omega_{22} & 0 & 1 \\ 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & \gamma_2 \end{bmatrix}$

and observe that LIM implies

$$E_t[\tilde{y}_{t+\tau}] = H^\tau y_t \tag{A.1}$$

One can show that

$$H^\tau = \begin{bmatrix} \omega_{11}^\tau & \omega_{12}\dfrac{\omega_{22}^\tau - \omega_{11}^\tau}{\omega_{22} - \omega_{11}} & \dfrac{\omega_{11}^\tau - \gamma_1^\tau}{\omega_{11} - \gamma_1} & H_{14}^\tau \\ 0 & \omega_{22}^\tau & 0 & \dfrac{\omega_{22}^\tau - \gamma_2^\tau}{\omega_{22} - \gamma_2} \\ 0 & 0 & \gamma_1^\tau & 0 \\ 0 & 0 & 0 & \gamma_2^\tau \end{bmatrix}$$

where $H_{14}^\tau = \omega_{12}\left[\dfrac{\omega_{11}^\tau}{(\omega_{11} - \omega_{22})(\omega_{11} - \gamma_2)} + \dfrac{\omega_{22}^\tau}{(\omega_{22} - \omega_{11})(\omega_{22} - \gamma_2)} \right.$

$$\left. + \dfrac{\gamma_2^\tau}{(\gamma_2 - \omega_{11})(\gamma_2 - \omega_{22})} \right]$$

Under LIM conditions (i) and (ii), ω_{11}^τ, γ_1^τ, $\gamma_2^\tau \to 0$ as $\tau \to \infty$. Hence, for large τ,

$$H^\tau \approx \begin{bmatrix} 0 & \dfrac{\omega_{12}\omega_{22}^\tau}{\omega_{22} - \omega_{11}} & 0 & \dfrac{\omega_{12}}{\omega_{22} - \gamma_2} \quad \dfrac{\omega_{22}^\tau}{\omega_{22} - \omega_{11}} \\ 0 & \omega_{22}^\tau & 0 & \dfrac{\omega_{22}^\tau}{\omega_{22} - \gamma_2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.2}$$

If $\omega_{22} = 1$, then the model converges as τ goes to ∞; in particular,

$$H^\tau \rightarrow \begin{bmatrix} 0 & \dfrac{\omega_{12}}{1 - \omega_{11}} & 0 & \dfrac{\omega_{12}}{1 - \gamma_2} & \dfrac{1}{1 - \omega_{11}} \\ 0 & 1 & 0 & \dfrac{1}{1 - \gamma_2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A.3)}$$

Given the parametric restriction on LIM, the maximum characteristic root of H equals ω_{22}. Since $\omega_{22} < R_F$, LIM ensurses convergence in the Proposition 1 present value calculations.

Proof of Proposition 3:

Recall that $g_t \equiv P_t - bv_t$. Multiplying both sides of (8) by R_F leads directly to

$$R_F g_t = E_t[\tilde{g}_{t+1} + o\tilde{x}^a_{t+1}] \quad \text{(A.4)}$$

(A similar relation is obtained by multiplying both sides of PVR by R_F to obtain $P_t R_F = E_t[\tilde{P}_{t+1} + \tilde{d}_{t+1}]$; see Samuelson [1965] and Ohlson [1991] for a discussion of this relation.)

Next, conjecture a linear solution for (A.4):

$$g_t = \alpha_1 ox^a_t + \alpha_2 oa_t + \beta \cdot \upsilon_t$$

Substituting the above into (A.4) and substituting for $E_t[o\tilde{x}^a_{t+1}]$, $E_t[o\tilde{a}_{t+1}]$, and $E_t[\tilde{\upsilon}_{t+1}]$ based on LIM, implies that the following equation must be satisfied with probability one:

$$R_F \alpha_1 ox^a_t + R_F \alpha_2 oa_t + R_F \beta \cdot \upsilon_t$$

$$= (\alpha_1 + 1)(\omega_{11} ox^a_t + \omega_{12} oa_t + \upsilon_{1t}) + \alpha_2(\omega_{22} oa_t + \upsilon_{2t})$$

$$+ \beta_1 \gamma_1 \upsilon_{1t} + \beta_2 \gamma_2 \upsilon_{2t}$$

Since the above equation must hold for all values of ox^a_t, oa_t, and υ_t, one obtains:

$$R_F \alpha_1 = (\alpha_1 + 1)\omega_{11}$$

$$R_F \alpha_2 = (\alpha_1 + 1)\omega_{12} + \alpha_2 \omega_{22}$$

$$R_F \beta_1 = (\alpha_1 + 1) + \beta_1 \gamma_1$$

$$R_F \beta_2 = \alpha_2 + \beta_2 \gamma_2$$

Solving the above system of four equations and four unknowns — α_1, α_2, β_1, and β_2 — yields the coefficient values stated in the proposition.

Proof of Proposition 4:

Given Proposition 3, α_2 is either zero or positive, depending on whether ω_{12} is zero or positive. Hence, there are only two relevant cases.
Case (i): $\omega_{12} = \alpha_2 = 0$. In this case, valuation function (12) and $g_t = P_t - bv_t$, imply

$$g_t = \alpha_1 ox_t^a + \beta_1 \upsilon_{1t}.$$

(A.1) and (A.2) imply $E_t[o\tilde{x}_{t+\tau}^a]$, $E_t[\tilde{\upsilon}_{t+\tau}] \to 0$ as $\tau \to \infty$. Thus, $E_t[\tilde{g}_{t+\tau}] \to 0$ as $\tau \to \infty$.
Case (ii): $\omega_{12}, \alpha_2 > 0$. In this case, valuation function (12) and $g_t = P_t - bv_t$ imply

$$g_t = \alpha_1 ox_t^a + \alpha_2 oa_t + \beta \upsilon_t,$$

where $\alpha_1 \geq 0$ and $\alpha_2 > 0$. The conclusion $E_t[\tilde{g}_{t+\tau}] > 0$ as $\tau \to \infty$, follows immediately since (A.1) and (A.2) imply $E_t[\tilde{\upsilon}_{kt+\tau}] \to 0$ while $E_t[o\tilde{x}_{t+\tau}^a]$, $E_t[o\tilde{a}_{t+\tau}] > 0$ as $\tau \to \infty$.

Proof of Corollary 1:

Given unbiased accounting, $\omega_{12} = \alpha_2 = \beta_2 = 0$, and valuation function (12) simplifies to

$$P_t = bv_t + \alpha_1 ox_t^a + \beta_1 \upsilon_{1t}.$$

Further, since $ox_t^a = x_t^a = x_t - (R_F - 1)bv_{t-1}$,

$$P_t = bv_t + \alpha_1[x_t - (R_F - 1)bv_{t-1}] + \beta_1 \upsilon_{1t}.$$

Expression (13) is obtained by substituting $bv_{t-1} = bv_t + d_t - x_t$ into the last expression, and simplifying.
Similarly, expression (14) is obtained by substituting $bv_t = fa_t + oa_t$ and

$$ox_t^a = ox_t - (R_F - 1)oa_{t-1} = ox_t - (R_F - 1)(oa_t + c_t - ox_t)$$

into valuation function (12) and simplifying.

Proof of Corollary 2:

The proof follows the same principles as the proof of the previous corollary. That is, in valuation function (12), one replaces bv_t and ox_t^a with $bv_t = fa_t + oa_t$ and

$$ox_t^a = ox_t - (R_F - 1)(oa_t + c_t - ox_t),$$

and simplifies. Thereafter one readily shows that

$$k_1 = \omega_{11} \frac{R_F - 1}{R_F - \omega_{11}} \geq 0$$
$$k_2 = 1 - k_1 + \alpha_2 > 0$$

and $k_1 + k_2 > 1$ since $\alpha_2 > 0$ when the accounting is conservative.

Proof of Proposition 5:

Substituting $ox_t^a = x_t^a = x_t - (R_F - 1)bv_t$ into valuation function (12) yields

$$P_t = bv_t + \alpha_1 x_t + \alpha_1(R_F - 1)bv_{t-1} + \alpha_2 oa_t + \beta \bullet \upsilon_t.$$

Using the same substitution in (11b), adding $i_{t+1} = (R_F - 1)fa_t$ to obtain x_{t+1}, and taking the expectation yields

$$E_t[\tilde{x}_{t+1}] = (R_F - 1)bv_t + \omega_{11}x_t - \omega_{11}(R_F - 1)bv_{t-1} + \omega_{12}oa_t + \upsilon_{1t}.$$

Hence,

$\partial P_t/\partial$accrued earnings$_t = 1 + \alpha_1 + \alpha_2$,

$\partial P_t/\partial$cash earnings$_t = 1 + \alpha_1$,

$\partial E_t[\tilde{x}_{t+1}]/\partial$accrued earnings$_t = (R_F - 1) + \omega_{11} + \omega_{12}$,

$\partial E_t[\tilde{x}_{t+1}]/\partial$cash earnings$_t = (R_F - 1) + \omega_{11}$.

Since either $\omega_{12} = \alpha_2 = 0$ or $\omega_{12}, \alpha_2 > 0$, Proposition 4 implies the equivalence of statements (a), (b), and (d). Finally, since

∂accrued earnings $- \partial$cash earnings $= \partial$investment

one proves the equivalence of (a), (c), and (e) as well.

Proof of Proposition 6:

Adding d_t to both sides of valuation function (12) provides

$$P_t + d_t = bv_t + d_t + \alpha_1 ox_t^a + \alpha_2 oa_t + \beta \bullet \upsilon_t.$$

Note that

$$ox_t^a = x_t^a = x_t - (R_F - 1)bv_{t-1} = x_t - (R_F - 1)[bv_t + d_t - x_t]$$
$$= (R_F - 1)[\emptyset x_t - (bv_t + d_t)],$$

which implies

$$bv_t + d_t + \alpha_1 ox_t^a = \emptyset x_t + \left[\alpha_1 - \frac{1}{R_F - 1}\right] ox_t^a.$$

Substituting into (A.6), and rearranging terms, establishes that

$$(P_t + d_t) - \emptyset x_t = \alpha_1 ox_t^a + \alpha_2 oa_t + \beta \bullet \upsilon_t.$$

where

$$\hat{\alpha}_1 = \alpha_1 - \frac{1}{R_F - 1} = -\frac{R_F(1 - \omega_{11})}{(R_F - 1)(R_F - \omega_{11})}$$

(A.1), (A.2) and (A.7) imply, for large τ,

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi\tilde{x}_{t+\tau}] \approx \left[\hat{\alpha}_1 \frac{\omega_{12}}{\omega_{22} - \omega_{11}} + \alpha_2\right]\omega_{22}^\tau\left[oa_t + \frac{\upsilon_{2t}}{\omega_{22} - \gamma_2}\right]$$

$$= \left[\frac{1}{R_F - \omega_{22}} - \frac{1 - \omega_{11}}{(R_F - 1)(\omega_{22} - \omega_{11})}\right]\frac{\omega_{12}\omega_{22}^\tau R_F}{R_F - \omega_{11}}$$

$$\times \left[oa_t + \frac{\upsilon_{2t}}{\omega_{22} - \gamma_2}\right]$$

The latter quantity is readily shown to be positive if $\omega_{22} \in (1, R_F)$, $\omega_{12} > 0$ and $oa_t + \upsilon_{2t}/(\omega_{22} - \gamma_2) > 0$, and to equal zero if either $\omega_{22} = 1$ or $\omega_{12} = 0$.

Proof of Proposition 7:

For any $\tau \geq 1$, PVR implies

$$E_t[\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}] = E_t\left[E_{t+\tau-1}[\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}]\right] = E_t[R_F\tilde{P}_{t+\tau-1}] = R_F E_t[\tilde{P}_{t+\tau-1}]$$

and similarly,

$$E_t[\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}] = (R_F - 1)E_t[\tilde{P}_{t+\tau-1}].$$

Hence, one obtains

$$E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \phi x_{t+\tau}] = \phi E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}) - \tilde{x}_{t+\tau}]$$

regardless of how $\tilde{x}_{t+\tau}$ is determined. The proof now follows directly from Proposition 6.

Proof of Proposition 8:

$P_0 = -d_0$, $oa_0 = ox_a^0 = 0$, and (12) imply $\beta_1\upsilon_{10} + \beta_2\upsilon_{20} = 0$.

$E_0[o\tilde{a}_1] > 0$, $oa_0 = 0$, and (10a) imply $\upsilon_{20} > 0$.

$ox_a^0 = oa_0 = 0$ and (10b) imply $E_0[o\tilde{x}_1^a] = \upsilon_{10}$.

Hence,

$$E_0[o\tilde{x}_1^a] = \frac{\beta_2}{\beta_1}\upsilon_{20} = -\frac{\omega_{12}(R_F - \gamma_1)}{(R_F - \gamma_2)(R_F - \omega_{22})}\upsilon_{20} < (=) \ 0 \ \text{if} \ \omega_{12} > (=) \ 0$$

Endnotes

1   One can view "abnormal earnings" as a contraction of "above normal earnings," where normal earnings equal the risk-free interest rate times the book value of firm's equity. Some, such as Canning (1929) and Preinreich (1937) refer to this as "excess earnings," Edey (1957) uses the term "super-profits," and the management accounting literature typically refers to it as "residual income."
     Discounting future abnormal earnings to determine the difference between market value and the cost or book value of an investment or firm is found in Preinreich (1937), Edey (1957), Edwards and Bell (1961), and Peasnell (1981, 1982).

2   To simplify the exposition, we assume that the risk-free interest rate is time-independent and the return on the financial assets is certain. The results pertaining to the relation between value and *future* accounting numbers can be extended to encompass uncertainty and systematic risk, with time and information dependent risk-free interest rates. In that setting, NIR is replaced with $E_{t-1}^*[i_t] = (R_{t-1}-1)fa_{t-1}$, where $E_{t-1}^*[i_t]$ is the risk-adjusted expected return on the financial assets during period $(t-1,t)$ and R_{t-1} is the risk-free return for period $(t-1,t)$, where both are conditioned on investor information at date $t-1$. See Feltham and Ohlson (1994b).

3   One can view fa_t as a numeraire asset that is measured without error. For the importance of an asset measured without error, see Morgenstern (1963). If some financial assets satisfy NIR, whereas others do not, then the latter can be included with the operating assets. Alternatively, the model can be extended to explicitly recognize those financial assets that do not satisfy NIR; they would be handled in the same way as the operating assets.

4   Observe that any two of the relations CSR, FAR, and OAR implies the third.

5   Any notion that oa_t, oa_{t-1}, and ox_t determine c_t because $c_t = ox_t - [oa_t - oa_{t-1}]$ is as false as the notion that bv_t, bv_{t-1}, and x_t determine d_t because $d_t = x_t - [bv_t - bv_{t-1}]$.

6   The following example illustrates the cash flow concept inherent in OAR. Suppose there are three operating assets (and no operating liabilities): accounts receivable (ar), inventory (inv), and property, plant and equipment, net of depreciation (ppe). The cash flow is

c_t = cash from customers$_t$ – cash paid to suppliers$_t$ – capital invested in operating assets$_t$

= [sales$_t$ - Δar$_t$] - [cost of goods sold$_t$ + Δinv$_t$] - [depreciation expense$_t$ + Δppe$_t$]

= operating earnings$_t$ - [Δar$_t$ + Δinv$_t$ + Δppe$_t$].

The example shows that one can think of operating earnings as cash flows adjusted for changes in accruals in operating assets and liabilities. In this sense the cash flow concept corresponds to what CON-6 refers to as "enterprise cash flow". Hence,

c_t = "enterprise cash flow" = "free cash flow."

7   The analysis relating value to *future* accounting numbers can be extended in a straightforward manner to encompass risk averse investors and systematic risk. In that case, $E_t[.]$ is replaced with the risk-adjusted expectation $E_t^*[\bullet]$, and $R_F^{-\tau}$ is replaced by the risk-free discount factor between date t and date $t+\tau$ based on investor information at date t. See Feltham and Ohlson (1994b).

8   We assume there are no information asymmetries that result in the use of dividends and financial structure as means of communicating information.

9   This encompasses the special case in which the firm has a finite life T, which results in $fa_t = 0$ for all $t \geq T$ and $d_t = c_t = 0$ for all $t > T$.

10   If the model is extended to recognize investor risk aversion and systematic risk, then the appropriate definition of abnormal earnings is $x_t^a \equiv x_t - (R_{t-1}-1)bv_{t-1}$, where $R_{t-1}-1$ is the risk-free interest rate for period $(t-1,t)$ given the information available at date $t-1$. See Feltham and Ohlson (1994b).

11   If the firm has finite life T, then $bv_t = 0$ for all $t \geq T$ and $d_t = ox_t^a = 0$ for all $t > T$.

12   If the firm has finite life T, then $oa_t = 0$ for all $t \geq T$ and $c_t = ox_t^a = 0$ for all $t > T$.

13   The proof follows directly from PVR and equations (2), (4) and (7). If investor risk aversion and systematic risk are recognized, then, in each case, $E_t[.]$ is a risk-adjusted expectation and $R_F^{-\tau}$ is replaced by the risk-free discount factor between date t and $t+\tau$, given investor information at date t.

14   This valuation formula can be traced to Preinreich (1938), who derives it in a con-
tinuous time, certainty setting. Interestingly, he also claims (p.240) that "In prac-
tice, it is the well-known formula for appraising the capital value of a business by
past experience" and "bookkeeping accordingly has great influence upon capital
value after all." See also Preinreich (1936, 1937).
      Edwards and Bell (1961) also basically apply formula (b); see their Appendix B,
particularly equations (2), (3) and (4). Our market price is similar to their "subjec-
tive value" of the firm, our earnings is similar to their "realizable profit," our abnor-
mal earnings is similar to their "excess realizable profit," and our book value is
similar to their market value of assets.
      Peasnell (1981,1982) uses the clean surplus relation to develop a T-period version
of expression (b). This requires explicit recognition of the terminal value of the
firm at date T, and either bv_T equals the terminal value or an adjustment must be
made for that difference. Of course, if the firm liquidates and disperses all assets at
date T, then CSR implies that no date T adjustment is necessary.
      Edey (1957) refers to abnormal profits as "super-profits" and examines valuation
expression: $P = bv + (x - Rbv)/j$, where "j is the appropriate rate for capitalisation
of the super-profit ..." The concept that goodwill equals the capitalization of future
abnormal profits is also recognized in ARS #10 by Catlett and Olson (1968).

15   In the US, the equity market value tends to exceed the related book value much
more frequently than the other way around. Examination of compustat data for the
past thirty years shows that the market value exceeds the book value in more than
2/3 of all cases.
      The median market to book ratio, P_t/bv_t, is better approximated by 2 rather than 1.

16   A statistic (which is often a vector) is sufficient if investor beliefs (at a specified
date) can be expressed as a function of the statistic, which is a function of the
available information. In any setting, there are many possible representations of a
sufficient statistic (including all of the available information). Accounting variables
can be part of a sufficient statistic either because the investors use accounting num-
bers in forming their beliefs or because the accounting numbers are functions of
the information used by investors. Changes in accounting procedures may affect
the information available to investors or may merely affect the functional relation
between the underlying information and the accounting numbers. It is unlikely that
accounting variables alone constitute a sufficient statistic. Other information must
be included as well. Differences in accounting procedures can affect the "other
information" required to obtain a sufficient statistic.

17   While LIM parsimoniously represents the information evolution, it is not unique.
Any scheme that allows us to infer the four components in the OAR equation
works as well. For example, LIM can be transformed using c_t and ox_t as informa-
tion variables instead of ox_t^a. The LIM expressions for oa_{t+1} and v_{t+1} are
unchanged, but (10a) is replaced by:

$$\tilde{c}_{t+1} = -\omega_{11}(R_F - 1)c_t + \omega_{11}R_F ox_t + [(R_F - 1)(1 - \omega_{11}) - (\omega_{22} - 1)]$$
$$+ \omega_{12}]oa_t + (v_{1t} - v_{2t}) + (\tilde{\varepsilon}_{1t} - \tilde{\varepsilon}_{2t})$$

$$o\tilde{x}_{t+1} = -\omega_{11}(R_F - 1)c_t + \omega_{11}R_F ox_t + [(R_F - 1)(1 - \omega_{11}) + \omega_{12}]oa_t$$
$$+ v_{1t} + \tilde{\varepsilon}_{1t}.$$

18   We have assumed that current abnormal earnings have no marginal effect on next
period's operating assets (i.e., $\omega_{21} = 0$). A non zero effect would significantly com-
plicate the explication of key concepts and results.

19   The financial assets are increased by the operating cash flows (see (11)) and
reduced by the dividends paid. The dividends paid in any given period depend on

the dividend policy. For example, assume that the dividends paid depend on abnormal earnings, operating assets, and financial assets for the preceding period, i.e.,

$$\tilde{d}_{t+1} = \pi_1 ox_t^a + \pi_2 oa_t + \pi_3 fa_t + \tilde{\varepsilon}_{dt+1},$$

where π_1, π_2 and π_3 are policy parameters. Under these conditions, the dynamics for financial assets are represented by

$$f\tilde{a}_{t+1} = (R_F - \pi_3)fa_t + (\omega_{11} - \pi_1)ox_t^a + (R_F - \omega_{22} + \omega_{12} - \pi_2)oa_t$$
$$+ (\upsilon_{1t} - \upsilon_{2t}) + (\tilde{\varepsilon}_{1t+1} - \tilde{\varepsilon}_{2t+1} - \tilde{\varepsilon}_{dt+1}).$$

The dividend policy parameters will influence the magnitude of fa_t, but those parameters have no impact on the valuation of either the financial or operating assets. Under LIM, one can view $(d_t, fa_t, ox_t^a, oa_t, \upsilon_{1t}, \upsilon_{2t})$ as the full specification of the firm's value relevant information.

20   The appendix provides proofs for all propositions and corollaries, other than those for which the text provides the proof.

21   From valuation function (12) and LIM, one can derive the expression for returns, $R_{t+1} \equiv (P_{t+1} + d_{t+1})/P_t$, as a function of the innovations $(\varepsilon_{1t+1},...,\varepsilon_{4t+1})$:

$$R_{t+1} = R_F + [(1 + \alpha_1)\varepsilon_{1t+1} + \alpha_2\varepsilon_{2t+1} + \beta_1\varepsilon_{3t+1} + \beta_2\varepsilon_{4t+1}]/P_t.$$

22   For an illustration of how conservative depreciation policies can result in this type of adjustment see Feltham and Ohlson (1994a).

23   $\omega_{11} = 1$ technically violates condition (ii) of LIM, but it is instructive to consider the case in which ω_{11} equals its least upper bound.

24   In this regard, while $k \neq \omega_{11}$ when $\omega_{11} \in (0,1)$, k is an increasing function of ω_{11}.

25   To be sure, one can specify variances in the LIM innovations such that $Pr_t\{ P_{t+\tau} < bv_{t+\tau}\} > 0$ as $\tau \to \infty$ for any realization of date t information, even though $\omega_{12} > 0$.

26   See, for example, Stickney, Weil, and Davidson (1991).

27   As stated earlier, we assume sufficient regularity for current and expected operating accruals to be positive. The latter holds as $\tau \to \infty$ if $oa_t + \upsilon_{2t}(\omega_{22} - \gamma_2)^{-1} > 0$, i.e., the other information υ_{2t} cannot be "too negative" relative to oa_t.

28   Condition (16) is equivalent to $E_t[\phi^{-1}(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau}) - \tilde{x}_{t+\tau}] \to 0$, as $\tau \to \infty$. By defining $\phi^{-1}(P_t + d_t)$ as the "economic earnings" for period $(t-1,t)$, one can interpret (16) as a comparison of "economic earnings" and accounting earnings. A setting satisfying (16) can then be interpreted as one in which accounting earnings, on average, equal "economic earnings." This concept of economic earnings is due to Ryan (1986). Intuitively, it is the earnings which when "capitalized" at the risk-free rate results in the observed pre-dividend value, $P_t + d_t$.

29   The Proposition 5 proof in the Appendix provides the more general statement of this expression, permitting $\omega_{11} \geq 0$.

30   This analysis applies more generally. Consider the earnings accruing to an investor over T periods: $\Sigma_{\tau=1}^{T} x_{t+\tau} + \Sigma_{\tau=1}^{T} (R_F^{T-\tau} - 1)d_{t+\tau}$. The second term is necessary since any dividends paid at date $t+\tau$ produce earnings on private account for the ensuing $T - \tau$ periods. One can show that $\partial E_t[\Sigma_{\tau=1}^{T} \tilde{x}_{t+\tau} + \Sigma_{\tau=1}^{T}(R_F^{T-\tau} - 1)\tilde{d}_{t+\tau}]/\partial d_t = -(R_F^T - 1)$. Again, the accounting for dividends is unbiased at the margin. The result highlights that for this analysis one can add earnings across all sources — operating activities, financial activities, and earnings on "private account" — and across periods. An unweighted aggregation scheme is of course desirable since it conforms with the basic structure of accounting. FAR and OAR aggregate into CSR without weights, and CSR aggregates without weights over periods so that $bv_{t+\hat{t}} = bv_t + \Sigma_{\tau=1}^{T} x_{t+\tau} - \Sigma_{\tau=1}^{T} d_{t+\tau}$. Accounting measurement problems do not arise with respect to the OAR variables in this marginal analysis because they are unaffected by dividends, and for the FAR variables economic and accounting values coincide.

31 A full payout policy permits growth in operating assets via borrowing, which reduces financial assets.

32 This observation is due to Ram Ramakrishnan.

33 Even though $d_t = x_t$ permits a conversion of PVR to $P_t = \Sigma_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{x}_{t+\tau}]$, none of the valuation concepts $P_t = (R_F - 1)^{-1} x_t$, $P_t = (R_F - 1)^{-1} E_t[\tilde{x}_{t+1}]$, or $P_t = (R_F - 1)^{-\tau} E_t[\tilde{x}_{t+\tau}]$, $\tau \to \infty$, make much sense in an uncertainty setting. The literature frequently suggests otherwise.

34 The latter case is feasible even with an underlying fixed book value since the growth in the operating assets can be offset by growth in "bonds payable," $-E_t[f\tilde{a}_{t+\tau}] = E_t[o\tilde{a}_{t+\tau}] - bv_t$

35 From case (c) in Proposition 7 one also infers the following somewhat surprising observation. Given a full payout policy, an asymptotic infinite expected return is necessary and sufficient for an expected *downward* bias in accounting earnings relative to economic earnings. That is, given $x_{t+\tau} = d_{t+\tau}$, $E_t[\tilde{x}_{t+\tau}/bv_{t+\tau-1}] \to \infty$ if, and only if, $E_t[(\tilde{P}_{t+\tau} + \tilde{d}_{t+\tau} - \tilde{P}_{t+\tau-1}) - \tilde{x}_{t+\tau}] > 0$; in this context, recall the essential equivalence of Propositions 5 and 6.

36 If $\omega_{12} = 0$, then $\beta_2 = 0$ and the sign of g_0 depends uniquely on the sign of υ_{10}. On the other hand, if $\omega_{12} > 0$, then $\beta_2 > 0$ and the sign of g_0 depends on both υ_{10} and υ_{20}.

37 The possibility of $E_0[o\tilde{x}_1^a] > 0$ but $E_0[o\tilde{x}_{\tau*}^a] < 0$, some $\tau^* > 1$, implies an exceptional case of oscillations since there also exists some $\tau > \tau^*$ such that $E_0[o\tilde{x}_\tau^a] > 0$ (Proposition 2).

38 This analysis depends on $g_0 = 0$. If $g_0 > 0$ and $\omega_{12} = 0$, then $\upsilon_{10} > 0$ and the expected earnings in the first period are positive. However, if $g_0 > 0$ and $\omega_{12} > 0$, then the sign of υ_{10} (and hence the sign of $E_0[o\tilde{x}_1]$) can be positive or negative. More specifically, if $g_0 > 0$ and $\omega_{12} > 0$, then $\upsilon_{10} = [g_0 - \beta_2\upsilon_{20}]/\beta_1$ and its sign is the same as the sign of $[g_0 - \beta_2\upsilon_{20}]$.

39 See Feltham and Ohlson (1994a) for an illustration of this approach.

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