

MATH 262
Questions

1. Let A be a 3×3 matrix with real entries. Prove that if A is not similar over $\mathbb{R}$ to a triangular matrix then A is similar over $\mathbb{C}$ to a diagonal matrix.
 2. Let T be a linear operator on a finite dimensional vector space over an algebraically closed field $\mathbb{F}$. Let f be a polynomial over $\mathbb{F}$. Prove that c is a characteristic value of $f(T)$ if and only if $f(t) = c$ where t is a characteristic value of T .
 3. Let c be a characteristic value of T and let W be the space of characteristic vectors associated with the characteristic value c . What is the restriction operator $T|_W$.
 4. Every matrix A such that $A^2 = A$ is similar to a diagonal matrix.
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 6. Let T be a linear operator on V . If every subspace of V is invariant under T then it is a scalar multiple of the identity operator.
 7. Let V be the vector space of $n \times n$ matrices over $\mathbb{F}$. Let A be a fixed $n \times n$ matrix. Let T be a linear operator on V defined by $T(B) = AB$. Show that the minimal polynomial of T is the minimal polynomial of A .
 8. If E is a projection and f is a polynomial, then show that $f(E) = aI + bE$. What are a and b in terms of the coefficients of f ?
 9. Let V be a finite dimensional vector space and let W_1 be any subspace of V . Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.
 10. Let V be a real vector space and E be an idempotent linear operator on V , that is a projection. Prove that $I + E$ is invertible. Find $(I + E)^{-1}$.
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11. Let T be a linear operator on V which commutes with every projection operator on V . What can you say about T ?
12. Let V be the vector space of continuous real valued functions on the interval $[-1, 1]$ of the real line. Let W_e be the space of even functions, $f(-x) = f(x)$, and W_o be the space of odd functions, $f(-x) = -f(x)$.
 - a) Show that $V = W_e \oplus W_o$.
 - b) If T is the indefinite integral operator $(Tf)(x) = \int_0^x f(t)dt$, are W_e and W_o invariant under T ?
13. Let V be a finite dimensional vector space over the field $\mathbb{F}$, and let T be a linear operator on V , such that $\text{rank}(T) = 1$. Prove that either T is diagonalizable or T is nilpotent, but not both.
14. Let T be a linear operator on the finite dimensional vector space V . Suppose T has a cyclic vector. Prove that if U is any linear operator which commutes with T then U is a polynomial in T .
15. Give an example of a two 4×4 nilpotent matrices which have the same minimal polynomial but which are not similar.
16. Show that if N is a nilpotent linear operator on an n -dimensional vector space V , then the characteristic polynomial for N is x^n .
17. Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Prove that T has no cyclic vector. What is the T cyclic subspace generated by the vector $\beta = (1, -1, 3)$?

18. Find the minimal polynomial and rational form of the matrix

$$T = \begin{bmatrix} c & 0 & -1 \\ 0 & c & 1 \\ -1 & 1 & c \end{bmatrix}.$$