M	ath 219	Diff	erent	ial Equat	ions	II. Exam	17	7.04.2009	
Last Name : Name : Student No:				Dept./Sec Time Duration	.: :17: :100	40 minutes		Signature	
5 QUESTIONS ON 4 PAGES]	TOTAL 100	POINTS
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M E T U Northern Cyprus Campus

EACH PROBLEM - 20 POINTS.

Question 1. Consider the second order linear nonhomogeneous differential equation

$$y'' + y = \frac{1}{\sin(t)}, \quad 0 < t < \pi.$$

Using the Variation of Parameters Method, find the general solution of this equation. Do not use the exact formulas.

Question 2. Let f(t) be a piecewise continuous function defined as

$$f(t) = \begin{cases} 0 & \text{if } t < 1\\ 3 & \text{if } 1 \le t < 2\\ 4 & \text{if } 2 \le t < 3\\ t^2 - 8 & \text{if } 3 \le t \end{cases}$$

(a) Write down f(t) as a combination of step functions $u_1(t)$, $u_2(t)$ and $u_3(t)$.

(b) Based upon the result of (a) find the Laplace transform $\mathfrak{L}{f(t)}$ of the function f(t).

Question 3. Solve the initial value problem (IVP) $\begin{cases} y'' + y' + y = 1 + \delta(t-2) \\ y(0) = y'(0) = 0 \end{cases}$ using the Laplace transform. Do not use any convolution integral.

Question 4. Write down the general solution of the differential equation

$$y^{(5)} + 3y^{(4)} + y^{(3)} - y'' - 4 = 0.$$

Question 5. Prove the formula $\mathfrak{L}\left\{t^2\right\}(s) = \frac{2}{s^3}$ using the Convolution Integral technique. (*Hint: calculate the convolution* (1 * 1) * 1)