

M E T U
Northern Cyprus Campus

Math 219 Differential Equations I. Exam 20.03.2009								
Last Name : Name : Student No:					Dept./Sec. : Time : 17: 40 Duration : 110 <i>minutes</i>		Signature	
5 QUESTIONS ON 5 PAGES						TOTAL 100 POINTS		
1	2	3	4	5				

EACH PROBLEM - 20 POINTS.

Question 1. Consider the second order linear nonhomogeneous differential equation

$$y'' + y = \frac{1}{\sin(t)}, \quad 0 < t < \pi.$$

Using the Variation of Parameters Method find the general solution of this equation.

Question 2. Write down the general solution of the differential equation

$$y^{(5)} + 3y^{(4)} + y^{(3)} - y'' - 4 = 0.$$

Question 3. Let $f(t)$ be a piecewise continuous function defined as

$$f(t) = \begin{cases} 0 & t < 1 \\ 3 & 1 \leq t < 2 \\ 4 & 2 \leq t < 3 \\ t^2 & 3 \leq t \end{cases}$$

(a) Write down $f(t)$ as a combination of step functions $u_1(t)$, $u_2(t)$ and $u_3(t)$?

(b) Based upon the result of the item (a) find the Laplace transform $\mathcal{L}\{f(t)\}$ of the function $f(t)$.

Question 4. Solve the initial value problem (IVP) $\begin{cases} y'' + y' + y = 1 + \delta(t - 2) \\ y(0) = y'(0) = 0 \end{cases}$ using the Laplace transform.

Question 5. Prove the formula $\mathfrak{L}\{t^2\}(s) = \frac{2}{s^3}$ using the Convolution Integral technique.
(*Hint: calculate the convolution $(1 * 1) * 1$*)