

Calculus and Analytical Geometry

II. Midterm

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|-------------------------------|---|---|-------------------------------|---|-------------|---|
| Code : <i>Math 119</i> | | | Last Name: | | | |
| Acad. Year : <i>2008-2009</i> | | | Name : | | Student No: | |
| Semester : <i>Fall</i> | | | Department: | | Section: | |
| Date : <i>1.12.2008</i> | | | Signature: | | | |
| Time : <i>17:40</i> | | | 7 QUESTIONS ON 6 PAGES | | | |
| Duration : <i>120 minutes</i> | | | TOTAL 100 POINTS | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |

1.(15 points) Find the absolute maximum and absolute minimum values of f on the given interval

$$f(x) = x + 2x^{\frac{2}{3}}, \quad [-1, 1]$$

2.(10 points) Use the Mean value theorem to prove the inequality

$$\frac{|\sin(\cos(a)) - \sin(\cos(b))|}{|a - b|} \leq 1$$

for all a and b such that $a \neq b$.

3.(15 points) A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is

- (a) a maximum
- (b) a minimum

4.(20 points) Use the guidelines for sketching a curve to graph the function $f(x) = \frac{x^2}{3x+6}$

5.(12 points) Evaluate

$$\frac{d}{dx} \left(\int_x^{x^2} \tan t \, dt + \int_1^{\sin \pi} \sqrt{1-t^3} \, dt \right)$$

6.(18 points) Evaluate the integrals

(a) $\int \sqrt{x^3 + 1} x^8 dx$

(b) $\int_{-2}^2 \frac{x^2 \sin x}{x^4 - 9} dx$

(c) $\int_{-2}^5 |4x - x^2| dx$

7. (10 points) Prove that

$$\int_0^2 \sqrt{1+x^3} dx \leq 4$$

Hint: Show that $\sqrt{1+x^3}$ is concave upward on $[0, 2]$