	Calculus and Analytical Geometry I. Midterm	

Code : *Math 119* Acad.Year: *2008-2009* Semester : *Fall* Date : *4.11.2008* Time : *17:40* Duration : *120 minutes*	Last Name: Name : Student No: Department: Section: Signature:
	7 QUESTIONS ON 6 PAGES TOTAL 100 POINTS

1	2	3	4	5	6	7		

1. (6+6+6=18 points) Evaluate the following limits, if they exist. Show your work. Do not use L'Hospital's rule.

(a) $\lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{x - 4}$

(b) $\lim_{x \to 0} x^3 \sin(\pi/x)$

(c) $\lim_{x \to 0} \frac{\tan 2x}{\sin 3x}$

2. (10 points) Prove that $\lim_{x \to 2} \sqrt{4x+1} = 3$ using the precise definition of a limit.

3. (15 points) Find the values of a and b which make the following function differentiable at each $x \in \mathbb{R}$:

$$f(x) = \begin{cases} \cos(\pi x), & x \le \frac{1}{2} \\ x^2 + ax + b, & x > \frac{1}{2} \end{cases}$$

4. (6+6+6=18 points) Find the indicated derivatives. Do not simplify your answers in parts (a) and (b).

(a) $f(x) = x\cos(\sin(x^2+1))$. Find $f'(x)$.

(b) $f(x) = \dfrac{x\tan x}{2x+1}$. Find $f'(x)$.

(c) $f(x) = \dfrac{1}{3x+1}$. Find $f^{(n)}(x)$ for each positive integer n.

5. (9 points) Suppose that f is a function that satisfies the following equation

$$f(x+h) = f(x) + f(h) + x^2h + xh^2$$

for all $x, h \in \mathbb{R}$. Suppose that $\lim_{h \to 0} \frac{f(h)}{h} = 1$.

(a) Find $f(0)$.

(c) Find $f'(x)$.

6. (15 points) Suppose that $ABCD$ is a trapezoid with the sides AB and CD parallel. Suppose that $|AB|$ is increasing at a rate of $1cm/min$, $|CD|$ is decreasing at a rate of $3cm/min$ and the height h of the trapezoid is increasing at a rate of $1cm/min$. Find the rate at which the area of the trapezoid is changing at an instant when $|AB| = 5cm$, $|CD| = 7cm$ and $h = 3cm$.

7. (15 points) Consider the curves $x^n + y^n = 2xy$ where $n \geq 3$ is an integer. Show that all of these curves have the same tangent line through the point $(1, 1)$. Find an equation of this line.