

Calculus and Analytical Geometry

Final

Code : <i>Math 119</i>	Last Name:
Acad. Year : <i>2007-2008</i>	Name : Student No.:
Semester : <i>Fall</i>	Department: Section:
Date : <i>9.1.2008</i>	Signature:
Time : <i>9:00</i>	6 QUESTIONS ON 6 PAGES
Duration : <i>150 minutes</i>	TOTAL 103 POINTS
1	2
3	4
5	6

1. (5+5+5=15 points) Compute the following integrals

(a) $\int \frac{x^3 + 1}{x^2 - x} dx$

(b) $\int \sin^4 x \cos^3 x dx$

(c) $\int_0^1 \frac{1}{2x - 1} dx$

2. (7+7+7+7=28 points) Compute the following indefinite integrals

(a) $\int 3x^2 \sin x \, dx$

(b) $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$

(c) $\int \frac{x}{\sqrt{3 - 2x - x^2}} \, dx$

(d) $\int \sin 4x \cos 5x \, dx$

3. (10 points) Find the volume of the infinite solid obtained by rotating the area between $y = \frac{1}{x^3}$ and $y = 0$ for $x \geq 1$ around the x -axis.

4. (10 points) Find an approximation to the area of the region bounded by $y = 2^{-x^2}$, $y = 0$, $x = -\frac{1}{2}$, and $x = \frac{7}{2}$ using the Midpoint rule with $n = 4$.

5. (10+10=20pts.) Let $f(x) = e^{-x^2}$

(a) Graph the function f by finding its domain, asymptotes, local extremum(s), concavity and inflection points. **Note:** For full credit, show all the necessary work to obtain the graph of f .

(b) Suppose that a rectangle has its base on the x-axis and two vertices on the graph of the function $y = e^{-x^2}$. Show that the rectangle has maximum area when the two vertices are the points of inflection of the curve.

6. (5+5+5+5=20pts.) Compute the following:

(a) $\frac{d}{dx} \left[\ln \left(\sqrt[4]{\frac{x \tan x}{(x^2 + 1) e^x}} \right) \right]$

(b) $\frac{d}{dx} [\cos(\tan^{-1}(\sqrt{x}))]$

(c) $\lim_{x \rightarrow \infty} \left(\frac{2x + 3}{2x + 1} \right)^x$

(d) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{x^2}{2}}{x^3}$