On Towers of Function Fields Over Finite Fields

Seher Tutdere

Institute of Applied Mathematics, METU

Let $\mathcal{F} = (F_n)_{n\geq 0}$ be a tower of function fields over a finite field $\mathbb{F}_q$, and let $r \geq 1$ be an integer. Then the limit

$$\beta_r(\mathcal{F}) := \lim_{n\to\infty} (\text{the number of places of } F_n/\mathbb{F}_q \text{ of degree } r\ )/(\text{genus of } F_n)$$

exists. We call the numbers $\beta_r(\mathcal{F})$, with $r \in \mathbb{N}$, the invariants of the tower $\mathcal{F}$. The case $r = 1$ is extensively studied by Ihara, Serre, Garcia, Stichtenoth, et al. Especially, towers with $\beta_1 > 0$ are quite useful in coding theory and cryptography. Towers with many positive invariants are helpful to obtain both good algebraic geometric codes and bounds for the bilinear complexity of multiplication in finite fields. Moreover, the function fields in such towers have large asymptotic class number.

In this talk we first briefly introduce the theory of towers of function fields over finite fields. More precisely, we discuss recursive (explicit) towers of function fields whose construction was first given by A. Garcia and H. Stichtenoth in 1995. We then describe a method for constructing towers over any finite field with finitely many invariants being positive.