METU, Spring 2014, Math 504.

Homework 5

(due April 30)

1. If $u \in F$ is algebraic over of odd degree over K, then show that $K(u) = K(u^2)$.

2. Consider the extension $\mathbf{Q}(u)$ of $\mathbf{Q}$ generated by a real root u of $x^3 + 2x - 2$. Express each of the following elements in terms of the basis $\{1, u, u^2\}$:

$$\frac{u^5}{2}, \frac{u-1}{u}, \frac{u}{u-1}, \frac{14}{u^2-2}.$$

3. In the field $K(x)$, let $u = x^3/(x+1)$. Show that $K(x)$ is a simple extension of the field $K(u)$. What is $[K(x) : K(u)]$?

4. Show that a regular pentagon can be sketched by ruler and compass constructions.

5. Find the number of subfields of $K = \mathbf{Q}(\sqrt[5]{2}, \zeta_5)$ where $\zeta_5 = \exp(2\pi i/5)$ is a fifth root of unity. Illustrate the correspondence of the fundamental theorem of Galois theory by giving a nontrivial example.