METU, Spring 2014, Math 504.

Homework 3

(due March 26)

1. Let V be a vector space over a division ring D. Prove that V is both a projective and an injective D-module.
2. Using the definition, show that a nonzero finite abelian group A is neither a projective nor an injective $\mathbf{Z}$-module, i.e. give examples of diagrams so that a suitable h does not exist.
3. Show that every torsion-free divisible abelian group D is a direct sum of copies of the rationals $\mathbf{Q}$.
4. Show that $\mathbf{Q}/\mathbf{Z} \otimes \mathbf{Q}/\mathbf{Z} = 0$.
5. Let $I = (2, x)$ be the ideal generated by 2 and x in the ring $R = \mathbf{Z}[x]$.
   - Show that $2 \otimes x \neq x \otimes 2$ in $I \otimes_R I$.
   - Show that the element $2 \otimes 2 + x \otimes x$ in $I \otimes_R I$ is not a simple tensor, i.e. cannot be written as $a \otimes b$ for some $a, b \in I$.