M E T U
Department of Mathematics

Abstract Algebra	
Midterm 1	
Code : *Math 367* Acad. Year : *2015* Semester : *Fall* Instructor : *Küçüksakallı* Date : *Nov 9, 2015* Time : *17:40* Duration : *100 minutes*	Last Name : Name : Student No. : Signature : 6 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4	5	6		

1. (24pts) Consider $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 6 & 1 & 2 & 7 & 3 & 5 \end{pmatrix}$ and $\tau = (123)(345)(678)$ in S_8.

- Express σ as a product of transpositions.

- Express τ as a product of disjoint cycles.

- Find σ^{100}.

- Is it possible to find a permutation $\gamma \in S_8$ such that $\gamma\sigma\gamma^{-1} = \tau$? If your answer is yes, then find such a permutation.

2. **(24pts)** Let $G = \{(x, y) \in \mathbb{R}^2 \mid x^2+4y^2 = 4\}$. Set $(x_1, y_1)\star(x_2, y_2) = \left(\frac{x_1x_2}{2} - 2y_1y_2, \frac{x_1y_2 + x_2y_1}{2}\right)$.

- Consider $P = (6/5, 4/5)$. Is P an element of G? Is $P \star P$ an element of G?

- Show that $\star$ is a binary operation on G.

- Show that $(G, \star)$ is a group.

- Let $H = G \cap \mathbb{Q}^2$. Show that $H \leq G$.

3. (16pts) Let G be a group. Let H be the subgroup of G generated by the squares of elements in G, i.e. $H = \langle \{g^2 | g \in G\} \rangle$.

- Show that $H \trianglelefteq G$.

- Show that G/H is commutative.

4. (10pts) Show that the groups $(\mathbb{C}, +)$ and $(\mathbb{C} - \{0\}, \times)$ are not isomorphic.

5. (16pts) If a cyclic subgroup C of G is normal in G, then show that every subgroup of C is normal in G.

6. (10pts) Let $n \geq 3$ be an integer. Consider the map $f : S_n \to S_n$ defined by the formula $f(\sigma) = \sigma^2$. Show that f is <u>not</u> a homomorphism.