M E T U
Department of Mathematics

Abstract Algebra	
Final Exam	
Code : *Math 367* Acad. Year : *2015* Semester : *Fall* Instructor : *Küçüksakallı* Date : *Jan 19, 2016* Time : *13:30* Duration : *120 minutes*	Last Name : Name : Student No. : Signature : 7 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4	5	6	7	8		

1. (25pts) For each of the following polynomials, determine whether it is an irreducible element of the indicated integral domain.

- $a(x) = 2x + 2 \in \mathbb{Z}[x]$.

- $b(x) = x^2 + 2x + 4 \in \mathbb{Z}_5[x]$.

- $c(x) = x^3 + 4x^2 + 6x + 4 \in \mathbb{Q}[x]$.

- $d(x) = x^4 + x^3 + x^2 + x + 1 \in \mathbb{Q}[x]$

- $e(x) = x^5 + x + 1 \in \mathbb{Z}_2[x]$.

2 (18pts) Let $n \geq 2$ be an integer and $I_n = \{f \in \mathbb{Z}[x] \mid f(0) \text{ is divisible by } n\}$.

- Show that $I_n = \langle x, n \rangle$ in $\mathbb{Z}[x]$.

- If I_n is a prime ideal of $\mathbb{Z}[x]$ then show that n is prime in $\mathbb{Z}$.

- If n is prime in $\mathbb{Z}$ then show that I_n is a prime ideal of $\mathbb{Z}[x]$.

3. (7pts) Show that $\mathbb{Z}[x]/\langle x^2+1 \rangle$ and $\mathbb{Z}[\sqrt{2}]$ are not isomorphic as rings.

4. (13pts) Find all maximal ideals in $\mathbb{Z}_{360}$.

5a. (6pts) What is the smallest positive integer n such that there are exactly **three** nonisomorphic Abelian groups of order n. Name the three groups.

$n =$ $A_1 =$ $A_2 =$ $A_3 =$

5b. (6pts) What is the smallest positive integer n such that there are exactly **four** nonisomorphic Abelian groups of order n. Name the four groups.

$n =$ $A_1 =$ $A_2 =$ $A_3 =$ $A_4 =$

6. (13pts) Show that every Euclidean domain is a principal ideal domain.

7. (12pts) Consider the binary operation $*$ on the set of integers defined by $a*b = a+b-4$.

- Show that $(\mathbb{Z}, *)$ is a group.

- Show that the groups $(\mathbb{Z}, *)$ and $(\mathbb{Z}, +)$ are isomorphic.