M E T U
Department of Mathematics

Group	Discrete Mathematics Midterm 1	List No.

Code : *Math 112* Acad. Year : *2010* Semester : *Spring* Instructor : *A. B. , K. Z., Ö. K.* Date : *April 1, 2010* Time : *17:40* Duration : *90 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 5 PAGES 60 TOTAL POINTS

1	2	3	4	5	6		

1. (10pts) Consider a polynomial $(x + 2y + 3z + t^{-2} + 2)^{32}$,

a. How many terms in the expansion of the polynomial have y-part exactly y^8 (such as $x^{24}y^8$, $x^{12}y^8z^{12}$, ...)?

The terms are $x^{n_1}(2y)^8(3z)^{n_2}(t^{-2})^{n_3}2^{n_4}$

$n_1 + n_2 + n_3 + n_4 + 8 = 32$ $\quad n_i \in \mathbb{Z}, n_i \geq 0 \quad i = 1, 2, 3, 4$

$n_1 + n_2 + n_3 + n_4 = 24$

Number of solutions $C(4+24-1, 24)$

Thus, number of terms $C(27, 24)$

b. What is the coefficient of the term $x^8y^{10}z^{12}$ in the expansion of the polynomial?

$$\frac{32!}{8!\,10!\,12!\,2!}(x)^8(2y)^{10}(3z)^{12}(2)^2 = \frac{32!}{8!\,10!\,12!\,2!}2^{10}3^{12}x^8y^{10}z^{12}2^2$$

The coefficient: $\dfrac{32!}{8!\,10!\,12!\,2!}2^{12}3^{12}$

2. **(10pts)** Twenty balls numbered $1, 2, \ldots, 20$ a placed in an urn. A player selects m balls at random. A player wins if there are two balls in the selection that add up to 21.

a. What minimum number of balls must be selected so that a player always wins?

Divide balls into pairs (ten pairs)

$(1,20)$; $(2,19)$; $(3,18)$; $(4,17)$; $(5,16)$; $(6,15)$; $(7,14)$; $(8,13)$; $(9,12)$; $(10,11)$

If we select 11 balls there must be two balls from the same pair, they add up to 21. Thus, 11 balls must be selected.

b. What is the probability of winning if 10 balls are selected?

Sample space $S' = \{$ selections of 10 balls $\}$ $\quad \# S' = C(20,10)$.

$A = \{$ selections with 2 balls that add up to 21 $\}$

$\bar{A} = \{$ sel. where no 2 balls add up to 21 $\} = \{$ sel. with exactly 1 ball from each pair above $\}$

$\# \bar{A} = 2^{10}$ $\qquad p(A) = 1 - p(\bar{A}) = 1 - \dfrac{2^{10}}{C(20,10)}$

3. **(10pts)** How many integers between 1 and 10 000 have distinct digits?

Digits : $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Integers with:

1 digit : x_1 - 9

2 digits : $x_1 x_2$ - $9 \cdot 9$ $\qquad$ first place 9 possib. (can not use 0)
second place 9 possib. (one digit used)

In the same way

3 digits : $x_1 x_2 x_3$ - $9 \cdot 9 \cdot 8$

4 digits : $x_1 x_2 x_3 x_4$ - $9 \cdot 9 \cdot 8 \cdot 7$

Totaly : $9 + 9 \cdot 9 + 9 \cdot 9 \cdot 8 + 9 \cdot 9 \cdot 8 \cdot 7$.

4. (10pts)

a. In how many distinct ways can r boys and s girls sit in a row?

Total number of people is $r+s$.

$(r+s)!$

b. In how many distinct ways can r boys and s girls sit in a row if boys must sit together and girls must sit together?

There are two groups and they can be organized in two different ways. Boys and girls permute within themselves

$2!\,r!\,s!$

c. In how many distinct ways can r boys and s girls sit in a row if girls must sit together?

Consider all girls as a single person then there are $r+1$ people and $(r+1)!$ different permutations Girls permute within themselves. So the answer is

$(r+1)!\,s!$

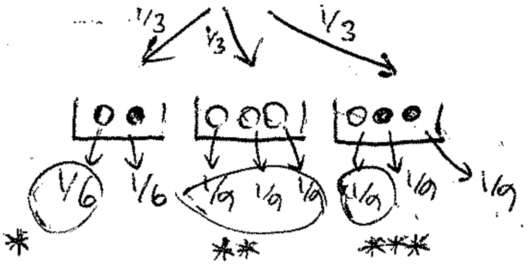
5. (10 pts) A pair of dice is loaded (unfair, biased). The probability that 6 appears on the first die is $\frac{3}{8}$, and the probability that 1 appear on the second die is $\frac{3}{8}$. Other outcomes for each die appear with probability $\frac{1}{8}$. What is the probability that the sum equals 7 if these two dice are rolled?

First die	Second die	
6	1	$\rightarrow 3/8 \cdot 3/8$
5	2	$\rightarrow 1/8 \cdot 1/8$
4	3	$\rightarrow 1/8 \cdot 1/8$
3	4	$\rightarrow 1/8 \cdot 1/8$
2	5	$\rightarrow 1/8 \cdot 1/8$
1	6	$\rightarrow 1/8 \cdot 1/8$
		+
		$14/64$

The probability that the sum is equal 7 is $7/32$.

6. (10pts) Three cups stand before me. The first cup contains one white and one black ball; the second cup contains three white balls; the third cup contains one white and two black balls. I select one of the three cups at random (equal probability for each cup), then I select one of the balls from the cup at random (equal probability for each ball).

a. What is the probability that I select a white ball?



$$\underbrace{1/6}_{1^{st}\text{ cup}} + \underbrace{3\cdot 1/9}_{2^{nd}\text{ cup}} + \underbrace{1/9}_{3^{rd}\text{ cup}} = 11/18$$

b. What is the probability that I select cup 1 and a white ball?

probability of selecting white from 1st cup.

$$1/3 \cdot 1/2 = 1/6$$

probability of selecting 1st cup

c. Suppose I reveal to you that the ball selected is white. What is the conditional probability that I selected cup 1? What is the conditional probability that I selected cup 2? What is the conditional probability that I selected cup 3?

$$\text{Cup 1} \longrightarrow \frac{(1/6)^{*}}{11/18} = \frac{3}{11}$$

$$\text{Cup 2} \longrightarrow \frac{(3\cdot 1/9)^{**}}{11/18} = \frac{6}{11}$$

$$\text{Cup 3} \longrightarrow \frac{(1/9)^{***}}{11/18} = \frac{2}{11}$$

Note that they add up to 1.