M E T U
Department of Mathematics

	Math 112: Discrete Math Final Exam	
Code : *Math 112* Acad. Year: *2009-2010* Semester : *Spring* Instructor : *KZ, AB, ÖK* Date : *09.06.2010* Time : *16.30* Duration : *120 minutes*	Last Name : Name : Student No.: Department: 6 QUESTIONS ON 4 PAGES TOTAL 80 POINTS	

1	2	3	4	5	6		

PLEASE GIVE SOLUTIONS AND REASONS, NOT ONLY ANSWERS!
ALL GRAPHS ARE LOOP-FREE AND THEY HAVE NO MULTIPLE EDGES.

1. (12 pts) A committee of 7 is to be chosen from 8 men and 9 women. One of the women is Ms White; one of the men is Mr Black.
(a) What is the probability that the committee contains at least 6 women?

$S = \{$all committees$\}$ $\quad \# S = C(17,7)$

$A = \{$at least 6 women in a committee$\}$

$A = \{$exactly 6 women in a committee$\} \cup \{$exactly 7 women...$\}$

$\# A = C(9,6) \cdot C(8,1) + C(9,7)$

$$P(A) = \frac{\# A}{\# S} = \frac{C(9,6)\cdot C(8,1) + C(9,7)}{C(17,7)}$$

(b) What is the probability that the committee contains either Ms White or Mr Black, but not both?

$B = \{$committee with Ms. W. or Mr. B. but not both$\}$

$B_1 = \{$committee with Ms. W.$\}$

$B_2 = \{$committee with Mr. B.$\}$

$B = (B_1 \cup B_2) \setminus (B_1 \cap B_2)$

$\# B = \# B_1 + \# B_2 - 2\# B_1 \cap B_2 = C(16,6) + C(16,6) - 2C(15,5)$

$$P(B) = \frac{\# B}{\# S} = \frac{2 \cdot C(16,6) - 2C(15,5)}{C(17,7)}$$

2. (12 pts) A child is making a tower using blocks of one or two units tall.
(a) How many towers of height $3n$ can be made using exactly $2n$ blocks?

x - number of 2-unit blocks $\quad x+y=2n$ (number of blocks)
y - number of 1-unit blocks $\quad 2x+y=3n$ (height of a tower)

Permuting n identical 1-unit blocks and
n identical 2-unit blocks

we get $\dfrac{(2n)!}{n!\,n!}$ different towers

(b) How many towers of height n can be made using any number of blocks? (Give either a function of n or a recurrence relation with initial conditions.)

a_n - number of towers of height n

last block is 1 unit block $\quad \underbrace{\qquad}_{n-1}\underbrace{\;}_{1}$ - a_{n-1}

last block is 2 unit block $\quad \underbrace{\qquad}_{n-2}\underbrace{\;}_{2}$ - a_{n-2}

$a_n = a_{n-1} + a_{n-2}$

$n=1$ (one 1-unit block) $a_1 = 1$

$n=2$, (one 2-unit block or two 1-unit blocks)

$a_2 = 2$ $\qquad \underline{a_n = a_{n-1} + a_{n-2} \quad a_1 = 1,\ a_2 = 2}$

3. (7 pts) Let G be a 5-regular planar graph with 30 edges. How many regions are there in a planar drawing of G? Justify your answer.

v- number of vertices $\qquad 2e = \sum \deg(x_i) = 5\cdot v$
e- number of edges $\qquad 60 = 5v$
r - number of regions $\qquad v = 12$

By Euler formula $\quad v - e + r = 2$

$12 - 30 + r = 2$

$\underline{r = 20}$ $\quad$ Twenty regions

4 (15 pts) (a) How many subgraphs of K_n, $n \geqslant 5$, are isomorphic to the complete bipartite graph $K_{2,3}$?

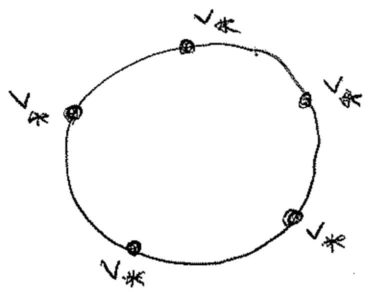
Choose 5 vertices, out of n, to construct K_5 as a subgraph of K_n. Then choose 2 of these vertices to construct $K_{2,3}$ as a subgraph of K_5.

Answer: $\binom{n}{5}\binom{5}{2}$

(Alternatively $\binom{n}{2}\binom{n-2}{3}$)

(b) How many cycles of length 5 are there in K_n, $n \geqslant 5$? (Note that the cycles $v_1 \to v_2 \to v_3 \to v_4 \to v_5 \to v_1$ and $v_2 \to v_3 \to v_4 \to v_5 \to v_1 \to v_2$ are the same.)

Choose 5 vertices $\{v_1, v_2, v_3, v_4, v_5\}$ out of n vertices.



There are $\frac{5!}{5 \cdot 2} = 4!/2 = 12$ different cycles. (similar to 5 keys on a keyring)

Answer: $\binom{n}{5} \cdot 12$

5. (10 pts) True or False? If G is an n-regular graph with $2n+2$ vertices, then the complement of G has a Hamilton cycle.

Let $G = (V, E)$. If G is n-regular, then

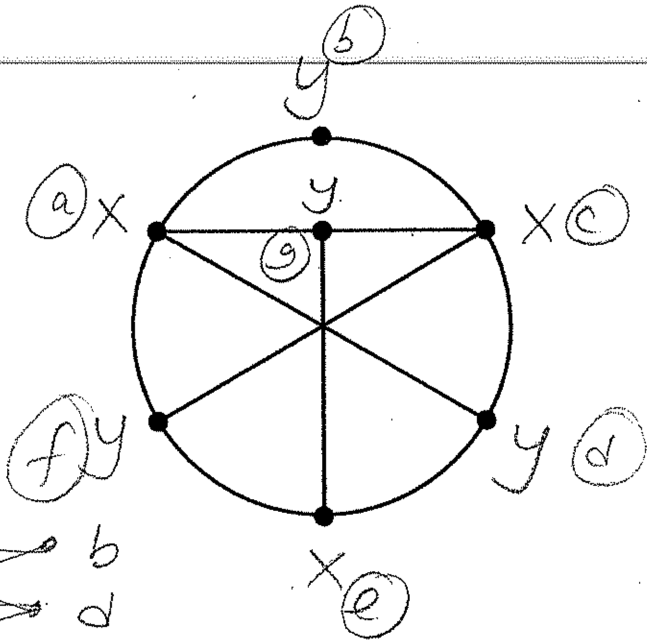
$$\deg_G(x) = n \quad \forall x \in V$$

It follows that $\deg_{\bar{G}}(x) = 2n+1-n = n+1$ for all x in V. Now we have

$$\deg_{\bar{G}}(x) + \deg_{\bar{G}}(y) = 2n+2 \geqslant 2n+2 = |V|$$

for all $x, y \in V$. Therefore $\bar{G}$ has a Hamilton cycle.

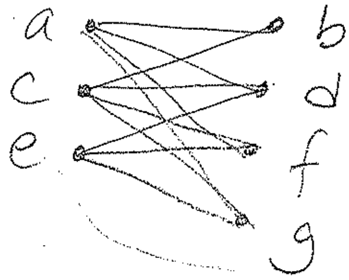
6. (6+6+6+6 pts) Let G be the following graph.
(Do not forget to justify your answers.)



(a) Is G bipartite?

G is bipartite since the vertex ~~set~~ can be divided into two disjoint sets.

OR G is isom to



is a subgraph of $K_{3,4}$.

(b) Does G have a Hamilton cycle?

No since $3 \neq 4$
G does not have a Hamilton cycle

OR G has 7 vertices, so if G has a Ham. cycle then it is of length 7. But in a bipartite graph, cycles are of even length.

(c) Does G have an Euler circuit?

No, since $\deg f = 3$ odd.

(d) Is G planar? No, since the subgraph with vertices a, c, e, d, f, g is isom. to $K_{3,3}$. By Kuratowski, G is not planar.