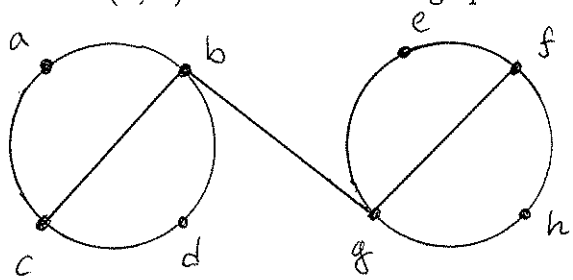
MATH 112 Discrete Mathematics.
Suggested Exercise set 8

1. Let $G = (V, E)$ be the undirected graph



How many path are there in G from a to h? How many of these path have length 5?

Problem 2. e Let $G = (V, E)$ be an undirected graph. define are relation $\mathcal{R}$ on V by $a\mathcal{R}b$ if and only if $a = b$ or there is a path from a to b. Prove that $\mathcal{R}$ is an equivalence relation. Describe the partion of V induced by $\mathcal{R}$.

Problem 3. n cities are connected by a network of k highways (a highway is a road between two cities that does not go through the intermediate cities). Show that if $k > \frac{1}{2}(n-1)(n-2)$ then one can always travel between any two cities trough connecting highways (there is at most one highway between two any cities).

Problem 4. Vertices of K_6 graph are painted either red or blue. Show that we can always find at least two subgraphs K_3 with monochromatic (same color) vertices. Color of the vertices of one of the K_3 can be different from color of the vertices of the other K_3.

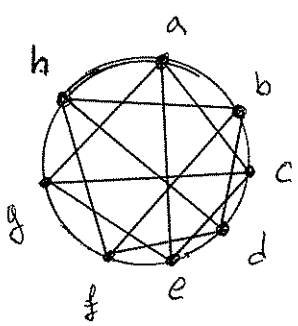
Problem 5. If two graphs have same number of vertices same number of edges and each vertex is incident with exactly two edges does it mean that the graphs are isomorphic.

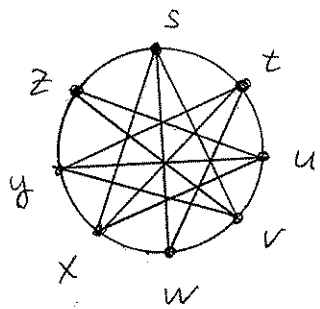
Problem 6. Let $G(E, V)$ be an undirected graph with k connected components. Prove that $|V| \geq |E| - k$.

Problem 7.

a. If G_1, G_2 are (loop-free) undirected graphs, prove that G_1, G_2 are isomorphic if and only if $\bar{G}_1$, $\bar{G}_2$ are isomorphic.

b. Determine if the following graphs are isomorphic.





Problem 8. Let $G = (V, E)$ be a loop-free undirected graph. G is called self-complemantary if if G and $\bar{G}$ are isomorphic. Prove that if G is self-complementary then

a. G is connected;

b. $|V| = 4m$ or $|V| = 4m + 1$ for some $m \in \mathbb{N}$.

Problem 9. Find all loop-free nonisomorphic undirected graphs with four vertices. how many of thes graphs are connected?