METU Mathematics Department
MATH 112: Exercise set VII

1. Find a recurrence relation for the number of bit (0 and 1) sequences of length n with an even number of 0's.

2. Determine the number of n-digit of quaternary $(0, 1, 2$ and $3)$ sequences in which there is never a 1 anywhere to the left of 0.

3. Find a linear homogeneous recurrence relation with constant coefficients that has the following general solution
$$a_n = c_1(-2)^n + c_2 5^n + c_3$$
where c_1, c_2, c_3 are constants.

4. Consider the recurrence relation
$$a_n - 4a_{n-2} = 2^n, \quad n \geq 2, \quad a_0 = 4, a_1 = 5.$$
   (a) Find $a_n^{(h)}$ by finding a solution to the **homogenous** equation
$$a_n - 4a_{n-2} = \mathbf{0}, \quad n \geq 2.$$
   (b) Verify that $a_n^{(p)} = n2^{n-1}$ is a solution of
$$a_n - 4a_{n-2} = 2^n, \quad n \geq 2,$$
   but it does **not** satisfy $a_0^{(p)} = 4, a_1^{(p)} = 5$.

   (c) Use part (a) and (b) to solve the original recurrence relation.

5. Solve each of the following first degree recurrence relations:

   (a) $a_{n+1} - a_n = n^3 + 1, \quad n \geq 0, \quad a_0 = 0.$

   (b) $a_{n+1} + a_n = n, \quad n \geq 0, \quad a_0 = 1.$

   (c) $a_{n+1} - 5a_n = 5^n, \quad n \geq 0, \quad a_0 = 2.$

6. Solve each of the following second degree recurrence relations:

   (a) $a_{n+2} + 3a_{n+1} - 10a_n = 3^n, \quad n \geq 0, \quad a_0 = 0, a_1 = 1.$

   (b) $a_{n+2} - 2a_{n+1} + a_n = 1, \quad n \geq 0, \quad a_0 = 1, a_1 = 2.$

7. The solution of a first degree recurrence relation
$$a_{n+1} - a_n = f(n+1), \quad n \geq 0, \quad a_0 = 0$$
is given by $a_n = n^5/5 + n^4/2 + n^3/3 - n/30$ for all $n \geq 0$.

   (a) Determine $f(n)$.

   (b) What can you say about $\sum_{i=1}^{n} f(i)$?