

METU Mathematics Department

MATH 112: Exercise Set III

Problem 1. If two integers are selected at random and without replacement from $\{1, 2, \dots, 100\}$, what is the probability that the integers are consecutive?

Problem 2. If we select six numbers without replacement from $\{1, 2, \dots, 100\}$, what is the probability that the second smallest number is 5?

Problem 3. A die is biased (loaded) so that the probability of a given number to appear is proportional to that number. (For example, getting 4 is twice as likely as getting 2, and getting 3 is three times as likely as getting 1.) When the die is rolled once, what is the probability that the outcome is (a) even, (b) prime, (c) even or prime, (d) even and prime, (e) even given that it is prime?

Problem 4. Assume that there are 365 days in a year.

(a) What is the probability that in a group of n people ($2 \leq n \leq 365$) at least two people have the same birthday?

(b) By using a calculator, show that if there are 23 or more people in a group, that two people have the same birthday is more likely than that all people have different birthdays.

Problem 5. There are 3 cards of the same size; one red on both sides, one blue on both sides, and one red on one side and blue on the other side. You close your eyes and a friend of yours picks one of the sides of these three cards randomly and puts it on the table. You open your eyes and see a blue card lying on the table. What is the probability that the other face of the card is also blue?

Problem 6. Assume E and F are independent events. Prove that E and \bar{F} are independent events.

Problem 7. The cards in a standard deck of 52 cards are dealt randomly to four people A, B, C, D. What is the probability that

(a) A receives all the aces?

(b) A receives no aces?

(c) A and B receive all the aces and the kings together?

(d) A has exactly 2 aces, if B has none?

(e) A and B each has 2 hearts, if C and D together have 9 hearts?

Problem 8. Three coins are tossed. Find all the independent pairs of events among the following: all three are same, at least two heads, at most two heads.