## METU Mathematics Department
MATH 112: Exercise set X

1. Find all the nonisomorphic complete bibartite graphs $G = (V, E)$ with $|V| = 6$.

2. Prove that any subgraph of a bibartite graph is bibartite.

3. Let m, n be integers with $m \geq n \geq 2$.

   (a) How many cycles of length 4 are there in $K_{m,n}$?

   (b) How many different paths of length 3 are there in $K_{m,n}$?

4. Let G_n be a convex polygon with n vertices and n edges. For which values of n, is G_n bibartite?

5. Let $n \geq 4$ be an integer. How many subgraphs of K_n is isomorphic to the complete bibartite graph $K_{1,3}$.

6. Determine which of the following graphs are planar.

(a)

(b)

(c)

(d)
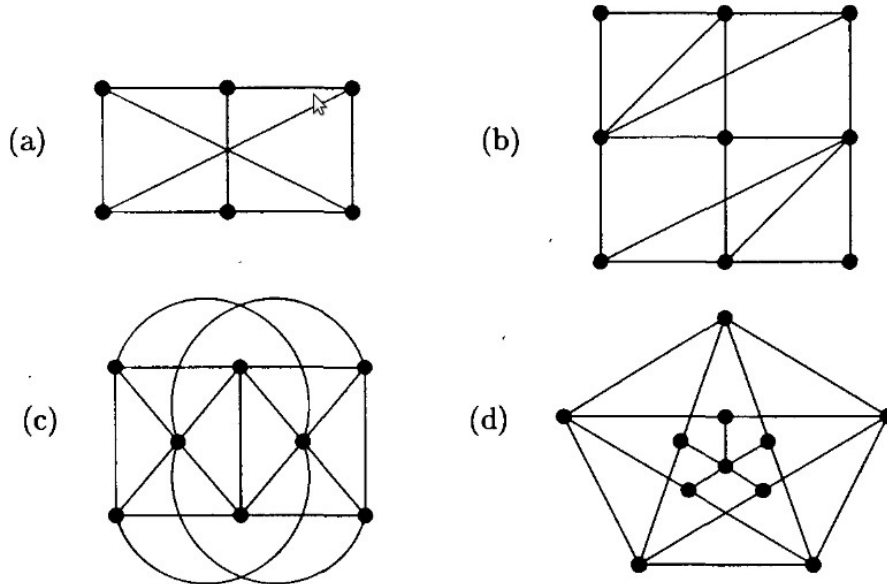
7. Prove that every loop-free connected planar graph has a vertex with degree strictly less than 6.

8. Let $G = (V, E)$ be a loop-free connected 4-regular planar graph. If $|E| = 16$, how many regions are there in a planar depiction of G.

9. Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?

10. Which of the nonplanar graphs K_5, K_6, $K_{3,3}$, $K_{3,4}$ have the property that the removal of **any** vertex and all edges incident with that vertex produces a planar graph.