METU Mathematics Department
MATH 112: Answers to Exercise Set IV

1. (a)
$$\frac{\binom{6}{1} \cdot \frac{8!}{3!} + \binom{6}{2} \cdot \frac{8!}{2!2!}}{6^8}$$

   (b)
$$\frac{\binom{4+7}{7}}{6^8} = \frac{\frac{8!}{7!\cdot 1!} + \frac{8!}{6!\cdot 1!\cdot 1!} + \frac{8!}{6!\cdot 2!} + \frac{8!}{5!\cdot 2!\cdot 1!} + \frac{8!}{4!\cdot 4!}}{6^8}$$

2. Place P's to two of four suitable spots. This replaces two letters and they can be moved to any of remaining four spots. Only two special choices make it possible to find different options for the last two letters. So the probability that none of the letters is in its original position is

$$\frac{\binom{4}{2} \cdot \binom{4}{1} \cdot \binom{3}{1} + \binom{4}{2} \cdot 2}{\frac{6!}{2!}} = \frac{84}{360}$$

   We can verify this answer by writing all 14 permutations of LAPTOP starting with PP. Then we multiply 14 by $6 = \binom{4}{2}$.

3.

4.

5.

6.

7. $2m + 1$

8.

9.

10. $4, 8, 12, 16, 3, 7, 11, 15, 2, 6, 10, 14, 1, 5, 9, 13$. No.

11.

12.