Lecture Notes

EE209
Fundamentals of Electrical and Electronics Engineering

Department of Electrical and Electronics Engineering

Middle East Technical University (METU)

Preface

These partially complete lecture notes were prepared with the purpose of helping the students to follow the lectures more easily and efficiently. We included important results, properties, comments and examples, but left out most of the mathematics, derivations and solutions of examples, which we do on the board and expect the students to write into the provided empty spaces in the notes.

These lecture notes were prepared from the lecture notes prepared by Prof. Özlem Aydın Çivi and were typed into latex by Safa Celik.

This is the first version of the notes. Therefore the notes may contain errors and we are open to corrections, feedback and comments, especially from the students taking the course.

Fatih Kamışlı
May 30^{th}, 2021.

Contents

Chapter 1

Fundamental Concepts

Contents

1.1 Introduction

Definition of electric circuit: An electric circuit is a collection of electrical elements interconnected in some specified way.

An example of an electric circuit that consist of two terminal elements is shown as

- Examples of two terminal cicuit elements are :
- There are circuit elements with more than two terminals, such as transistors and operational amplifiers (OP-AMPs).

SI Units:
In defining any scientific quantity we need to have a standard system of units so that when a quantity is described by measuring it, we can all agree on what the measurement means. This system, which we shall use, is the International System of Units (SI) adopted in 1960 by the General Conference on Weights and Measures.

- There are 6 basic units in SI, and all other units are derived from them: meter, kilogram, second, coulomb, kelvin and candela.
- Some of the derived units are
  - unit of force, 'newton (N)'
    $1\ \text{N} = 1\ \text{kg m/s}^2$ (force required to accelerate a 1 kg mass by 1 meter per second per second)
  - unit of energy, 'joule (J)'
    $1\ \text{J} = 1\ \text{Nm}$ (work done by a constant 1 N force applied through a 1 m distance)
  - unit of power, 'watt (W)'
    $1\ \text{W} = 1\ \text{J/s}$ (the rate at which work is done or energy is expended)

Prefixes
Due to orders of magnitude difference in electrical quantities we generally use certain

prefixes instead of exponential notation.

Prefixes in the SI

Multiple	Prefix	Symbol
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	k
10^{-3}	Mili	m
10^{-6}	Micro	μ
10^{-9}	Nano	n
10^{-12}	Pico	p

1.1.1 Charge

- There are two kinds of electric charges, positive and negative.
- Unlike charges attract and like charges repel.
- Unit of charge is coulomb (C).
- We usually denote charge by q or Q.
  → Q: is usually used for denoting constant charges
  → q: usually indicates the time varying charge q(t)

1.1.2 Current

- The primary purpose of an electric circuit is to move or transfer charges along specified paths. This motion of charge constitutes an 'electric current', denoted by i or I. Formally, the current is the time rate of change of charge.

- The unit of current is 'ampere (A)'.
- According to the Benjamin Franklin's convention, current is assumed as the movement of positive charges. In fact, in metal conductors the current is the movement of electrons that have been pulled loose from the orbits of atoms of the metal, which is known as 'electron' current.
  We use the convention that motion of positive charges gives rise to a positive amount of current.

- Current is specified with its arrow. Arrow does not indicate the actual direction of flow. It is a part of convention.
- There are several types of current in common use.

1.1.3 Voltage

- Voltage across on element is the work done in moving a unit charge (+1 C) through the element from one terminal to the other. It is denoted by v or V.
- The unit of voltage (or potential difference) is Volt (V).
  1 V = 1 J/C
- To represent voltage we use + − polarity convention as shown below.

  - Terminal A is V_o volts positive with respect to terminal B
  - or terminal A is at a potential of V_o volts higher than terminal B
  - or voltage drop of V_o volts occurs in moving from A to B
  - or voltage rise of V_o volts occurs in moving from B to A.

1.1.4 Energy and Power

Energy: Energy is defined as the capacity of doing work against a possible resisting force. It is denoted by W and the unit of energy is joule (J).

- In transfering charge through an element work is being done, or energy is being supplied. To know whether energy is supplied to element or by the element to the rest of the circuit, we must know both
  → the polarity of the voltage across the element
  → the direction of the current through the element.
- If a positive current enters the positive terminal, then an external force must be driving the current, i.e. energy is delivered to the element. (The element is absorbing energy in this case.)

Power: The time rate of change of energy is power and denoted by P. The unit of power is 'watt' (W).

- Instantaneous power:
- The average power is defined within an interval t_1 to t_2 as
- For any two terminal electrical component, the absorbed instantaneous power is given by

∗ If it turns out that the absorbed power becomes negative, then this simply means that the component supplies power rather than absorbing it.

Ex: The voltage and current in an electric network are as follows for $t > 0$:

i) Sketch the instaneous power
ii) What is the total energy dissipated during the first second?

__**Solution**__:

1.1.5 Passive and Active Elements

A circuit element is said to be passive if the total energy delivered to it from the rest of the circuit (or total energy absorbed by it from the rest of the circuit) is always non-negative.

Ex: resistors, capacitors and inductors

An active element is the one which is not passive.

Ex: batteries and generators.

1.1.6 Independent Sources

Independent current source: is a two terminal element through which a specified current flows. The current is completely independent of the voltage across the element.

Independent voltage source: is a two terminal element that maintains a specified voltage between its terminals. The voltage is completely independent of current through the element.

The symbol

• Independent sources are usually meant to deliver power to the external circuit. Thus if

v is the voltage across the source and its current i is directed out of the positive terminal, then the source is delivering power, given by $p = vi$ to the external circuit. Otherwise, it is absorbing power.

1.1.7 Dependent Sources

• Dependent sources are very important in circuit theory, particularly in electronic circuit. These sources have terminal characteristics that are controlled by a current or voltage at some remote part of the circuit.

There are four types of controlled (dependent) sources.

1.1.8 Circuit Analysis

If an electric circuit is subjected to an input or excitation, such as voltage or current provided by an independent source, then an output or response is produced.

Circuit Analysis is concerned with finding the output for given input and network.

Circuit Synthesis is concerned with finding the circuit for given input and output.

We shall develop systematic methods of analysis that can be applied to any circuit of type we consider in next lectures.

Computer aided circuit analysis: SPICE (Simulation Program with Integrated Circuit Emphasis)

Electrical engineering systems are concerned with the design, analysis and operation of human made systems involving electrical signals. These systems could be divided into four general groups.

1. Communication systems: In communication systems, electrical engineers are concerned with the generation, transmission and distribution of information via electrical signals.

2. Computer systems: use of electrical signals to carry out computations

3. Control systems: use electrical signals to control processes.

4. Power systems: use electrical signals in generation transmission and distribution of large blocks of power.

There is considerable interaction between these general types of systems.

Electrical engineering is intimately connected to many other engineering disciplines.

The aim of this course is to introduce the non-electrical engineering student to those aspects of electrical engineering that are likely to be most relevant to his/her professional career.

Chapter 2

Resistive Circuits

Contents

2.1 Resistive Circuits

When currents flow in conductors, electrons which make up the current collide with the lattice of atoms. This resists the motion of the electrons. The larger the number of the collisions, the greater the resistance of the conductor.

<u>Resistor</u>: An ideal resistor is two terminal device, the voltage across which is directly proportional to the current flowing through it.

• The constant of proportionality is the resistance value of the resistor in ohms (Ω) when voltage is in V and current in A.

When R is constant, the resistor is called as a 'linear resistor'. There are 'nonlinear resistors' whose resistances do not remain constant for different terminal currents. For such a resistor the resistance is a function of current.

• The instantaneous power of a resistor is

Since $R > 0$, $p(t) > 0 \ \forall t$. So, the resistor always absorbs power (which is dissipated as heat)

Equivalently, we can characterize a resistor by

where G is the conductance of the resistor measured in mhos ($\mho$) (Siemens)

- Short circuit: An ideal conductor between two points.
  - It is a resistance of zero ohms.
  - It can carry any current but the voltage across it is always zero.
- Open circuit: Open circuit is a break in the circuit through which no current can flow.
  - It may be considered to be an infinite resistance.
  - It may have any voltage.

2.2 Passive Sign Convention

• Assignment of reference polarity for voltage and reference direction for current is entirely arbitrary.

• However, whenever the reference direction for the current in an element is in the direction of the voltage drop across the element, use a positive sign in the expression that relates the voltage to the current. Otherwise, use a negative sign.

Ex:

Ex: (Ohm's law)

Ex: Determine the power absorbed by each element.

<u>Solution:</u>

Ex: The current entering a terminal is given by

$$i = 6t^2 - 2t\ A$$

Find total charge entering the terminal between $t = 1$ & $t = 3$ s.

Solution:

2.3 Kirchhoff's Laws

• Kirchhoff's laws together with the terminal characteristics for the various circuit elements, permit systematic methods of solution for any electrical network.

• There are two Kirchhoff's laws: Kirchhoff's Voltage Law (KVL) & Kirchhoff's Current Law (KCL).

• The elements of circuits are connected by electrical perfect conductors which have zero resistance.

• Perfect conductors are zero resistance wires which allow current to flow freely but accumulate no charge and no energy.

• In this case, the energy can be considered to reside, or be lumped, entirely within each circuit element, and thus the network is called as lumped-parameter-circuit.
• <u>Node:</u> A point of connection of two or more circuit elements.

An example of a circuit with 3 nodes:

Note that every element has a node at each of its ends.

2.3.1 Kirchhoff's Current Law (KCL)

KCL : The algebraic sum of the currents entering any node is zero = The sum of the currents entering any node equals the sum of the currents leaving the node

$\sum_{n=1}^{N} i_n = 0$, where i_n is the n^{th} current entering the node and N is the number of node currents.

This law is based onthe principle of conservation of charge.

Ex:

Note that $i = -5$A entering the node is equivalent to 5A leaving the node. Therefore it is not necessary to guess the correct current direction prior to solving the problem.

Before stating Kirchhoff's voltage law, we shall define path and closed path.

Suppose that we start at one node in a network and move through a simple element to the node at the other end, then continue from that node through a different element to the next node, and continue this movement until we have gone through as many elements as we wish. If no node was encountered more than once, then the set of nodes and elements that we have passed through is defined as a path. If the node at which we started is the some of the node on which we ended, then the path is a closed path or loop. Branch is a single path in a network, composed of one simple element and the nodes at each end of that element.

2.3.2 Kirchhoff's Voltage Law (KVL)

KVL : The algebraic sum of the voltage drops taken in a fixed direction around a closed path is zero
(Take the algebraic sign as positive when going from + to - (from higher to lower potential) and negative when going from - to + (from lower to higher potential).)

A lumped-parameter cct is a consequitve system, which means that the work required to move a charge around any closed path is zero.

Ex:

Ex:

Ex:

2.3.3 Generalizations of KCL and KVL

KCL can be generalized from being applied at a single node to a closed surface as illustrated by the following case.

Then the generalized KCL requires that

$$i_1 + i_2 - i_3 + i_4 - i_5 + i_6 = 0 \tag{2.1}$$

as if we treat the closed surface S like a single node.

Generalized KCL: The algebraic sum of the currents entering any closed surface is zero.

Generalized KVL: We can apply KVL to any closed path and this path need not follow the physical current paths.

Ex:

Solution:

2.4 Series Resistance and Voltage Division

Series connection: Elements are said to be connected in series when they carry the same current.

Voltage division: The potential of source v is divided between resistors. R_1 and R_2 in direct proportion to their resistances. This is called ‘principle of voltage division’.

The voltage division can be generalized to N resistors in series connection as

Ex:

Solution:

2.5 Parellel Resistance and Current Division

Parallel connection: Elements are connected in parallel when the same voltage is common to each of them.

Current division: The current of the source i divides between conductances G_1 & G_2 in direct proportion to the their conductance.

- The current divides in inverse proportion to the resistances.
- This is called ‘principle of current division’.

The power absorbed by the parallel combination is

Now, we can generalize the current division to N resistors connected in parallel.

Ex:

Ex:

Solution:

Ex: (Q4, HW1, Spring 1995)

Solution:

Ex:

Solution:

2.6 Ammeters, Voltmeters, Wattmeters, Ohmmeters

An ideal ammeter measures the current flowing through its terminals. It has a zero voltage across its terminals.

- It has a zero internal resistance.
- It does not absorb or deliver power ($p = v\,i$ since $v = 0$).

An ideal voltmeter measures the voltage across its terminals. It has a terminal current of zero.

- It has infinite internal resistance.
- It does not absorb or deliver power ($p = v\,i$ since $i = 0$).

Practical ammeter & voltmeter

If you know the internal resistances, you can measure the required currents and voltages accurately. (look at the related problems in reference book)

Ex:

A wattmeter measures the power dissipated by a circuit element.

An ideal ohmmeter measures the resistance connected between its terminals and delivers zero power to the resistance.

Other version of Page 26

An ideal ammeter measures the current flowing through its terminals and has a zero voltage across its terminals.

An ideal voltmeter measures the voltage across its terminals and has a terminal current of zero.

An ideal ohmmeter measures the resistance connected between its terminals and delivers zero power to the resistance.

Note : Practical instruments are different than ideal.

D’Arsonval Meter:

This device is constructed by suspending an electrical coil between the poles of a permanent magnet. A DC current passing through a coil causes a rotation of the coil. The movement is designed so that deflection of the pointer is directly proportional to the current in the coil.

D’Arsonval meters are characterized by their full-scale current, I_{FS} which is the current that will cause the meter to read its greatest value.

Common I_{FS} are $10\,\mu A$ to $10\,mA$.

Equivalent circuit for D’Arsonval meter:

Now let’s see how this D’Arsonval meter is used or ammeter, voltmeter abd ohmmeter.

DC Ammeter Circuit: D’Arsonval meter can not measure currents more than I_{FS}. But by connecting a parallel resistance, R_P, current I_{FS} can be measured

DC Voltmeter Circuit: Full scale voltage $v = v_{FS}$ when the meter current is I_{FS}.

Current sensitivity: of a voltmeter, is the value obtained by dividing the resistance of the voltmeter by its full-scale voltage.

Ohmmeter circuit: Battery E causes a current i to flow when R_x is connected to a circuit.

Chapter 3

Operational Amplifiers (OP-AMPs)

Contents

3.1 Operational Amplifiers

• An Op-amp is an integrated circuit containing tens of semiconductor components such as transistors, diodes and resistors inside. Op-amps are used in many control and instrumentation systems to perform tasks like voltage regulators, oscillators, logarithmic amplifiers, peak detectors, voltage comparators and special purpose amplifiers for audio applications and etc.

• Other connections of an op-amp (DC power supply connections, etc.) are not shown. Hence, in circuits containing op-amps, the generalized KCL must be used with care!

The equivalent circuit model of an op-amp is as follows

For practical op-amps,

- R_i (input resistance) :
- R_o (output resistance) :
- A (voltage gain) :

- For an ideal op-amp,
  - R_i (input resistance) :
  - R_o (output resistance) :
  - A (voltage gain) :

- With these resistance and gain values, following circuit constraints are valid for an op-amp:
  - The input currents are zero.
  - The voltage difference between the input terminals is zero.
    (These are the main constraints of op-amps that we will use in circuit analysis in this course!)

3.2 Amplifier Circuits with Op-Amps

3.2.1 Voltage Controlled Voltage Source

1. Since there is no voltage across the input terminals of the op-amp
2. KVL for loop abca

3. Applying KCL at node b and noting that the current into the negative terminal of the op-amp is zero, we have

The v_2, output voltage of op-amp is a function only of the input voltage v_1 and two resistors.

An equivalent circuit is

• As the gain is positive it is called as a non-inverting amplifier.

• A special case of non-inverting amplifier is the case $R_2 = 0$ (short circuit) and $R_1 \to \infty$ (open circuit)

• This circuit is called a voltage follower, i.e. v_2 follows v_1.

• It is also called a buffer amplifier, because it may be used to isolate one circuit from another. (The voltages at the two pairs of terminals are the same, but no current can flow from one pair to the other.)

3.2.2 Inverter

This circuit is called an inverter because the polarity of v_2 is opposite that of v_1.

Input current $i_1 = \frac{v_1}{R_1}$

An equivalent circuit is

Another equivalent circuit

We may obtain dependent current sources from this inverter circuit

Note: The resistor R_2 used in inverting and non-inverting amplifiers is called the "feedback resistance". A practical op-amp is a very high gain device and is generally never used without feedback, o.w. the output voltage will be very large leading to saturation. In cases when the feedback is to one input terminal rather than to both, it must always be the inverting terminal to achieve non-saturated operation.

Ex: (Summer circuit)

Solution:

Ex: (Differential Amplifier)

Solution:

Chapter 4

Nodal and Mesh Analysis

Contents

We shall consider two systematic ways of formulating and solving the equations that arise in the analysis of more complicated circuits.

• The first one is primarily based on KCL and leads equations in which the unknowns are voltages. This method is known as "nodal analysis".

• The second method is based on KVL leading to equations in which the unknowns are currents. This technique is called as "mesh analysis".

4.1 Nodal Analysis

We shall consider methods of circuit analysis in which voltages are the **unknowns** to be found. A convenient choice of voltages for many networks is the set of **node voltages**.

- For a systematic analysis, one node in the network is selected as a reference node. Frequently, the reference node is chosen to be the node to which the largest number of branches are connected.
- Most of the cases, the reference node is referred to as ground.
- Once such a reference node is chosen, every other node is assigned a voltage with respect to this reference node.
- It is common practice to select polarities so that the node voltages are positive relative to the reference node.
- For a circuit containing N nodes, there will be $N-1$ node voltages. We have to write $N-1$ independent equation.
- Since the circuit unknowns are to be voltages, the describing equations are obtained by applying KCL at nodes.
- The currents in the elements are proportional to the element voltages, which are themselves either a node voltage or the difference of two node voltages.

Consider the following circuit:

Super Nodes :

If we have a voltage source (dependent or independent) having no connection to the reference node, then we shall have difficulty in writing the current through this voltage source. We use super-nodes for such cases. Consider the following circuit:

We have three unknowns v_2, v_3 and v_4, hence we need three independent equations.

To apply KCL we need to express the current through the source v_b. To avoid this we enclosed the voltage source by dashed lines and form the so-called super node. Generalized KCL now holds for this super node.

Apply generalized form of KCL to the super node enclosing the voltage source:

Express these currents in terms of unknown voltages using Ohm's law.

We can represent these three equations in a matrix form

This matrix equation can be solved by e.g. using Cramer's rule.

Cramer's Rule: explicit formula for solution of a system of a linear equations with as many equations as unknowns (valid whenever system has a unique solution)

Ex: Use the nodal analysis to find the power delivered by the $3A$ current source.

Solution:

Ex: (Sample MT Q.)

Solution:

Ex: Find all node voltages.

Solution:

4.2 Nodal Analysis for Op-Amp Circuits

For circuits containing op-amps, the nodal analysis technique is very convenient. In electronic circuits the reference node is usually shown as grounded and all other elements connected to reference node are often shown individually grounded. Thus nodes are easily identified for the nodal analysis but the loops are not so easily visualized.

Ex:

Solution:

<u>Note:</u>

- Usually, we avoid writing nodal equations at output nodes of op-amp because it is difficult to find the current of an op-amp. Although input currents of an op-amp are zero, the output current of an op-amp is not zero due to the other terminals not shown.
- We also avoid writing a node equation (KCL) at the ground node, since there may be additional currents into the ground from unshown terminals.

Ex:

For the ideal op-amp circuit find R_1 and R_2 so that $v_0 = v_{s2} - \frac{v_{s1}}{4}$.

Solution:

4.3 Mesh Analysis

Nodal analysis method is completely general and can always be applied to any electrical network. However, mesh analysis is applicable only to those networks which are planar. If it is possible to draw the diagram of circuit on a plane surface in such a way that no branch passes over or under any orher branch, then that circuit is said to be a planar circuit

Mesh: A mesh is a certain chosen closed path in a circuit passing through a node or element only once. A mesh is a loop that contains no elements within it.

We define mesh current as the current which flows around a mesh. (The mesh current may constitute the entire current in an element of the mesh or it may be only a portion of the element current.)

The advantage of using mesh currents is that we can express the six branch currents only using three mesh currents. This is since the branch currents are linearly dependent as they must satisfy KCL at nodes.

Ex:

1. Identify the meshes and allocate the associated mesh currents.
2. It is a common practice to use the clockwise direction for all of them.
3. Then write down KVL in terms of these currents in each mesh.

3 meshes $\Rightarrow$ we have to solve for the three unknown mesh currents.

Super Mesh :

In the mesh analysis, we mainly use KVL but the voltage across a current source is not readily known. For this reason in writing KVL equations we must avoid including current sources to our KVL paths. We do this by using a super mesh that encircles this current source. (Note that if the current source is included by a single mesh as in previous circuit then we do not need to use a super mesh.)

Ex:

Solution:

Ex:

Solution:

Ex: (Sample MT Q.)

Solution:

4.4 Wheatstone Bridge

- Used to measure resistance (or other parameters related to resistance)
- R_1, R_2, R_3 known, R_x unknown to be determined.

Wheatstone Bridge and Op-Amp - Strain Gauges

Question: How could you measure bending in a mechanical component?

- Can you directly measure bending angle? No.
- We use a "transducer" which converts bending into an electrical form by measuring change in length.
- We can use gauges on both sides so that
  One will increase in resistance, the other will decrease.

Now: How can we measure the change in resistance? Ohmmeter? Not much luck. (too small a change)

- We use a wheatstone bridge with strain gauges on the branches.
- We measure the voltage difference between the two legs of the bridge.
- We use an op-amp to amplify this voltage.

Applications:

- Designing structures that bend, twist, stretch when subjected to external forces.
- Example: Car chassis, aircraft frame, wings
- You need to properly orient and bend the gages
- You need to use your circuit knowledge to measure change in electrical properties

.

Chapter 5

Network Theorems

Contents

5.1 Linearity

A linear system is characterized by the following input-output relations:

If

then as the system ζ is linear it satisfies the following input-output relation

for all kinds of excitations (inputs) x_1 & x_2, and for all a_1, a_2.

• A linear circuit element is described by

The element is linear if multiplying x by constant K results in the multiplication of y by the same constant K. This is called proportionality property.

• The proportionality property of a single linear element also holds for a linear circuit. In the sense that if all the independent sources of the circuit are multiplied by a constant K, then all the currents and voltages of the remaining elements are multiplied by this same constant K. For example, a loop equation is of the form

f: algebraic sum of the voltages of the independent sources in the loop

v: voltages of the remaining loop elements

a: 0 or $\pm$ 1

Multiply (1) by K

Ex:

5.2 Superposition Principle

Linear systems have the simplicity of enabling the use of the superposition principle. Superposition principle tells us that in order to find the response for two or more independent sources in any linear circuit, any circuit voltage (or current) may be calculated as the algebraic sum of all the individual voltages (or currents) caused by each independent source acting alone, i.e. with all other independent sources "killed".

First, we choose an electrical quantity (certain current or voltage) as our response (or output) and consider the contribution to this quantity from each independent source one by one. In calculating the contribution of a source we must kill all other independent sources.

To kill an independent current source: Replace independent current source by an open circuit, then $i = 0$.

To kill an independent voltage source: Replace independent voltage source by an short circuit, then $v = 0$.

Note: Always keep dependent sources in the circuit as they are. Never kill them.

Ex: Use the principle of superposition to find the value of I needed to cause $V_2 = 0$ in the circuit.

Since there are two independent sources, V_2 has two contributions, one from 10 V voltage source and one from the current source of value I.

- First, kill current source find V_2' due to voltage source

Let's use nodal analysis:

- Second, kill 10 V voltage source to find V_2''

Let's use nodal analysis:

Ex: Apply the principle of superposition to find the current I_2 shown in the following circuit.

- First, kill current source and find I_2'

- Second, kill 20 V source, find I_2''

Note that power is not a linear expression, therefore superposition will not apply to obtain power directly.

Note: Superposition is not a particularly attractive method of circuit analysis since a circuit with N sources requires N different circuit analyses to obtain the final result. But, superposition is still an important property of linear circuits because it is often used as a conceptual tool to develop other circuit analysis techniques. (Ex: Thevenin's Theorem)

5.3 Thevenin's and Norton's Theorems

Either of these theorems enables us to replace an entire circuit seen at a pair of terminals by an equivalent circuit made up of a single resistor and a single source. (Thus, we may determine the voltage or current of a single element of a relatively complex circuit by replacing the rest of the circuit by an equivalent resistor and source and analyzing the resuşting simple circuit.)

We shall assume that the circuit can be seperated into two parts.

We want to simply circuit A without affecting i_0 and v_0.

Thevenin's Theorem: At a pair of terminals ab, linear circuits can be represented by an equivalent circuit composed of a voltage source in series with a resistance. The V_{oc} is the voltage measured across the open circuited terminals ab. The resistance value R_{th} is the ratio of the open-circuited voltage V_{oc} to the short-circuited current I_{sc}. Alternatively, R_{th} can be determined by killing all independent sources in A and determining the equivalent resistance between a & b.

Ex: Find the Thevenin and Norton equivalent of the following circuit as viewed by the resistor R_4.

V_{oc}: Open circuit the terminals a and b, solve the resultant circuit to find the open circuit voltage $V_{oc} = V_{ab}$

I_{sc}: Short circuit the terminals a and b, solve the resultant circuit to find the short circuit current I_{sc} flowing from terminal a to b.

Alternatively, since there are no dependent sources, we can find R_{th} by killing all independent sources and find the input resistance seen from terminals a and b.

Ex: Find the Thevenin equivalent of the circuit external to 2Ω resistor and use the result to find i.

Solution:

Ex: (Previous MT Q.)
Find the Thevenin equivalent circuit as viewed by the resistor R.

Solution:

Note: The presence of dependent source prevents us from determining R_{th} directly for the inactive network through resistance combination.

Ex:

Solution:

Practical Sources

A practical voltage source has an internal drop in voltage when current flows through its terminals, and this internal drop diminishes the voltage at the terminals.

For a given practical voltage source, the load resistance R_L determines the current drawn from the terminals.

- Practical current source:

5.4 Network Simplification

Very often network analysis can be greatly simplified by changing voltage source in series with resistor to current source in parallel with resistor (using Thevenin/Norton equivalents).

Ex:

We may combine source to obtain equivalent sources.

5.5 Maximum Power Transfer

There are many applications in circuit theory where it is desirable to obtain maximum possible power that a given practical source can deliver.

v_g & R_g are fixed, thus P_L is a function of R_L. To maximize P_L we can make $\frac{dP_L}{dR_L} = 0$ and solve for R_L.

Also note that

The maximum power that the practical voltage source is capable of delivering to the load is

In the case of practical current source,

→ Maximum power is obtained from a linear circuit at a given pair of terminals when the terminals are loaded by the Thevenenin resistance of the circuit.

Ex: (Previous MT Q.)

By using source conversion and network simplification, obtain the Thevenin's equivalent to the left of the terminal pair $a - b$ in the following circuit and calculate the current through 1Ω load resistor.

Ex: (Previous MT Q.)

Ex:

Ex:

Chapter 6

Energy Storage Elements

Contents

6.1 Capacitors

A capacitor is two terminal device that consists of two conducting bodies that are seperated by a nonconducting material. It primarily stores electric energy but in electronic circuits it can have a variety of applications.

Charge-voltage relationship of a capacitor is given by

then $i - v$ relationship is given as

Note that if v is constant, then the current i is zero. Therefore; a capacitor acts like an open circuit to a DC voltage.

The power absorbed by a capacitor is

Energy stored in a capacitor

Stored energy depends only on the final value of voltage $v(t)$.

The ideal capacitor, unlike the resistor, cannot dissipate any energy: The energy which is stored in the device can thus be recovered. Consider, for instance, a $1\,F$ capacitor which has a voltage of $10\,V$: The stored energy is

Suppose the capacitor is not connected in a circuit; then no current can flow, and the charge, voltage and energy remain constant. If we now connect a resistor across the capacitor, current flows until all the energy is absorbed as heat by the resistor and the voltage across the combination is zero.

Ex: A $1\,\mu F$ capacitor has a voltage of $v(t=0) = 1\,V$ at $t = 0$ across its terminals. A current $i(t)$ is applied between $t = 0$ to $t = 4$ seconds. Find $v(t)$ and the energy supplied between $t = 1$ to $t = 3$.

Note that v and i do not necessarily have the same shape. The maximum and minimum values of v and i do not necessarily occur at the same time, unlike the case for the resistor.

Ex:

Note that voltage across a capacitor is continuous even though the current is discontinuous. Instantaneous charges in the voltage across a capacitor are not possible. (unless the current includes an impulse function)

Series and Parallel Capacitors :

KVL

KCL

Ex: Find the equivalent capacitances

6.2 Inductors

An inductor is a two terminal energy storage element. It stores magnetic energy.

The $i - v$ relationship

• If i is constant, then voltage is zero. Therefore, an inductor acts like a short circuit to DC current.

If we integrate both sides of the $i - v$ relationship from t_o to t

Instantaneous power absorbed by an inductor is

Energy stored within the inductor

The stored energy in an inductor depends only on the final value of the current.

Ex: The current through a 500 μH inductance is

Note that voltage across on inductor is discontinuous even though the current is continuous. Instantaneous charges in the current through an inductor are not possible. (unless the voltage includes an impulse function)

An ideal inductor does not dissipate any power. Therefore, energy stored in the inductor can be recovered. For example, $2\,H$ inductor is carrying a $5\,A$ current. Stored energy is

Suppose that inductor, by means of an external circuit, is connected in parallel with a resistor. In this case, a current flows through the inductor-resistor combination until all the energy previously stored in the inductor is absorbed by the resistor and the current is zero.

Series and Parallel Inductors :

KVL

Ex: Find the equivalent inductance

Ex:

Solution:

Ex:

Solution:

Chapter 7

Sinusoidal Steady State Circuit Analysis

Contents

7.1 Transient and Steady State Response

• The response of a physical system to an applied excitation (input) is determined by three factors:

(i) The excitation (input)

(ii) Elements of the system, their interconnection

(iii) Past history of the dynamic elements (e.g. initial voltage or current in C and L's)

• In analyzing circuits having dynamical elements such as capacitors and inductors; we obtain an equation having differentiation and integration due to $i - v$ characteristics of these elements. In addition to this, we must also know the initial voltage across a capacitor and the initial current through an inductor.

• As an example consider the following parallel RC circuit excited by a sinusoidal current source as the switch closes at $t = 0$. The initial voltage across the capacitor at $t = 0$ is $v_c(t = 0) = V_o$ Volts.

This is an ordinary first order differential equation for $v_c(t)$ with an initial condition $v_c(t = 0) = V_o$.

The solution is composed of two parts:

1) First consider the homogenous solution. Hence, set $i_s(t) = 0$. For $v_{c,h}(t)$ try the function Ae^{st} where A and s are to be determined. Insert Ae^{st} for $v_c(t)$ in the differential equation.

Note that, as time goes on, the homogenous part decays to zero and about $t = 5/RC$ it reaches nearly zero.

2) Now consider the particular solution due to forcing function $i_s(t) = I_s \cos wt$.

We try for $v_{c,p}(t)$ a sinusoidal function of the general form. $\underline{v_{c,p}(t) = D\cos wt + E\sin wt}$ where D and E are to be determined. Insert $v_{c,p}(t)$ in the differential equation

Equate the coefficients of sine and cosine functions on both sides.

Hence

Then the total response is

3) Finally, we determine the A using initial condition $v_c(t = 0) = V_o$ V,

We write the total response as the sum of transient response and the steady state response. The transient response is the transitory portion of the complete response which approaches zero as time increases. Since the transient response decays in time generally it is not the main interest in electronic circuits.

• The steady-state response becomes equal to the total response as the transient part dies out.

• For sinusoidal excitations if we are interested only in the steady-state solutions, then we can obtain this without solving differential equations. The only additional cost is that we have to work with complex numbers.

Properties of Sinusoids:

In electrical engineering, sinusoidal functions are extremely important. For example, the carrier signals generated for communication purposes are sinusoids, in the electric power industry the sinusoid is the dominant signal. Indeed, almost every useful signal in electrical engineering can be resolved into sinusoidal components.

A sinusoidal function is given as

A sinusoid is a periodic function defined by

where T is the period, i.e. the function goes through a complete cycle, or period, which is repeated every T seconds.

The relation between angular frequency & frequency is

More general sinusoidal waveform can have a phase difference ϕ,

The argument of a trigonometric function must be either in radians or in degrees. But we sometimes denote the phase term in degrees and wt term in radians.

E.g.

Assume that we have to sinusoids with same arg. frequency w

If v_1 reaches its peak $\phi_1 - \phi_2$ radians earlier than v_2, then we use the terminology that v_1 leads v_2 by $\phi_1 - \phi_2$ rad., or equivalently v_2 lags v_1 by $\phi_1 - \phi_2$ rad.

As an example, consider $v_1 = 2\sin(2t + 45°)$ and $v_2 = \sin(2t - 23°)$, then v_1 leads v_2 by $45° - (-23°) = 68°$.

Note that $\cos(wt - \frac{\pi}{2}) = \sin wt$ or $\sin(wt + \frac{\pi}{2}) = \cos wt$

The trigonometric functions are connected to complex exponentials via Euler's identity, Euler's identity greatly facilitates sinusoidal circuit analysis, leading us to manipulations requiring complex number.

Summary of complex Numbers:

The complex number z şs written in rectangular form as

Euler's identity:

The trigonometric functions can be represented using complex exponentials.

Alternative Way to obtain Steady-State Solution of Previous Parallel RC Circuit using Complex Numbers:

Complex exponentials are mathematically easier to handle as excitations then sinusoids.

For steady-state solution try $v_c(t) = Ae^{jwt}$ where A is to be determined.

Now, relate the response to $I_s e^{jwt}$ to the response to $I_s \cos wt$. ($I_s \cos wt = \Re\{I_s e^{jwt}\}$). Hence, we do the same for the

Note that this is identical to the steady-state response due to sinusoidally excited (i.e. $i_s(t) = I_s \cos wt$) parallel RC circuit we found previously.

CONCLUSION: For the sinusoidal steady-state response, we can first treat the circuit excited by a complex exponential rather than the sinusoidal excitation and to obtain the actual result take the real part of the obtained response.

The original steady-state response to sinusoidal excitation is recovered from the complex excitation response by taking the real part

7.2 Phasors

- To solve the complex exponential excitation easily we can use phasors.
- Let $v_o(t) = V_m \cos(wt + \theta)$, then $v_1(t) = V_m\, e^{j(wt+\theta)} = V_m\, e^{j\theta}\, e^{jwt}$.

If w is known, then v is completely specified by its amplitude V_m and its phase θ. These quantities are displayed in a related complex number

Ex:

→ We have chosen to represent sinusoids and their related phasors on the basis of cosine functions.

Replace the excitation

In general, the real solutions are time-domain functions and their phasors are frequency domain functions. Thus, to solve the time domain problem, we may convert to phasors and solve the corresponding frequency-domain problems, which are generally much easier. Finally, we convert back to the time domain by finding the time function from its phasor representation.

7.3 I-V Relations of R, L and C

Let the voltage across the component (R,L or C) be $v(t) = V\, e^{jwt}$, then find the current through the component using its $i - v$ relation.

The frequency domain relation for the resistor is exactly like the time domain relation.

Ex:

$\rightarrow$ Sinusoidal voltage and current for a resistor have the same phase angle. They are said to be <u>in phase</u>.

<u>Inductor:</u>

Capacitor:

Hence, we can establish an Ohm's law like relation for the inductor and capacitors in phasor (frequency) domain.

7.4 Impedance & Admittance

Ratio of the phasor voltage to the phasor current is defined as the impedance.

Impedance

Admittance

Impedance Z in rectangular form

Note that even though

In general $Z = Z(jw)$ is a complex function of jw but $R(w)$ and $X(w)$ are real functions of w.

We can summarize the impedances/admittances for R,L,C as in the following table:

7.5 Kirchhoff's Law and Impedance Combinations

• Kirchhoff's laws hold for phasors as well as for their corresponding time-domain voltages or currents.

• In circuits having sinusoidal excitations with a common frequency w, if we are interested only in steady-state response, we may find phasor voltages or currents of every element and use Kirchhoff's laws to complete the analysis.

KVL

• Series and parallel combination formulas that were used for resistors can also be used for the impedances.

• Furthermore voltage and current division rules hold for phasor circuits with impedances and frequency domain quantities.
• If circuit has sinusoidal excitations (sources) with different angular frequencies (e.g. w_1 and w_2), then we can use superposition if the circuit is linear.

Ex:

Apply the following procedure in solving for the sinusoidal steady-state response using phasor technique.

$\star$ All the sources must be at the same frequency w, otherwise we must use superposition in time domain.

1) Express the sources in phasor representation. As a convention, before we transform to phasor domain, all the time-domain expressions must be represented in the form of a 'cosine' function.

2) Replace the inductors and capacitors with the corresponding impedances or admittances evaluated at the excitation frequency w.

3) Using circuit analysis tools the required current or voltage phasor can be calculated.

KVL and KCL are still applicable to phasor voltages and currents, we just need to work with complex numbers. Series and parallel combinations of impedances can also be simplified as in resistive circuits.

4) Obtain the time-domain representation for the phasors. For example, for the phasor in the polar form $V_o\angle\theta$, the corresponding time-domain expression is given by

Summary of Phasor Domain Analysis:

For the steady-state response of a sinusoidally excited circuit we use the phasor domain techniques. The method is basically one in which the time functions are transformed to the phasor representations of the sinusoids. In the case of circuits having more than one AC sources but with different frequencies we must apply the principle of superposition (for linear circuits) and then use the phasor technique for each part seperately.

The time-domain waveform for an independent voltage source

has the phasor representation $V = V_m \angle \theta$ in polar form.

The time-domain waveform for an independent current source

is transformed to phasor domain as $I = I_{rms} \angle \theta$. Note that we prefer the rms values in the phasor domain as it is convenient in power calculations. These phasors which are in essence complex numbers can be shown in the complex plane using vectors.

7.6 RMS Values

• A sinusoidal waveform is characterized by its maximum value, period and its phase. However, we need to introduce new quantity to express the strength of the waveform. The maximum value is not very suitable for this purpose as it is attained only instantaneously in a period.

• We introduce rms or effective value of a periodically varying waveform $i(t)$ as

This is valid for any periodic waveform.

The rms value of a constant is simply the constant itself.

Ex:

• The historical aim in introducing the rms value for a time varying current was to find an effective value that could be used in power calculation just as if it were a DC value.

• The rms values are usually more common than the peak values of the sinusoidal waveform. Actually 220 V domestic electricity supply is also an rms value.

• We sometimes can use rms values of voltage & current in phasor representation.

7.7 Nodal and Mesh Analysis

Once we transform an AC circuit to phasor domain, we can use the systematic analysis tools: nodal & mesh analyses just like in resistive circuits.

Ex: Find voltage across $R = 0.5\,\Omega$ resistor by using nodal analysis.

Solution:

Ex: (Q1 of MT1, Fall 1995) Use nodal analysis to find the steady-state voltage $v(t)$ and the equivalent impedance Z_{eq} that is viewed by the independent source

Solution:

Ex: Find the steady-state voltage $v(t)$ if $V_s(t) = 5\cos 3t$ V.

<u>Solution:</u>

Ex: Find steady-state voltage $v(t)$ using mesh analysis.

Solution:

SUPERPOSITION:

• If a circuit has two or more inputs with same source frequency, we may find the phasor currents or voltages due to each input acting alone (i.e. with others dead) and add the individual corresponding time-domain responses to obtain the total.

• If the sources have different frequencies, we must use superposition, because the definition of impedance $Z(jw)$ allows us to use only one frequency at a time, and thus we cannot even construct a phasor circuit. If the source frequencies are different, apply superposition in time domain.

Ex:

Solution:

Ex:

Solution:

7.8 Thevenin and Norton Equivalent Phasor Circuits

Procedure is identical to that for resistive circuits. The only change is that V_{oc} and i_{sc}, are replaced by their phasor representations. V_{oc} and I_{sc}, and R_{th} is replaced by Z_{th}.

Ex:

Solution:

7.9 Phasor Diagrams

• Since phasors are complex numbversi they may be represented by vectors in the complex plane, where operations such as addition of phasors, may be carried out geometrically. Such a sketch is called a phasor diagram.

Since the current I is common to all elements take it as our reference phasor: $I = |I|\angle 0°$

We have taken the angle of I arbitrarily to be zero, since we want I to be our reference.

The voltage phasors are

Assume that

If current is fixed, then the real component of V_ρ is fixed and is $R|I|$. In this case, the possible location of V_ρ on the phasor diagram, i.e. the locus of the phasor V_ρ is the dashed line of following figure.

Ex: Find I, using phasor diagram. Show the phasor representation of $i_R(t)$, $i_C(t)$ and $i_L(t)$ with the phasor representation of the source voltage as reference.

Chapter 8

AC Steady-State Power

Contents

8.1 Instantaneous and Average Power

• Instantaneous power, is the rate at which energy is absorbed by an element and it varies as a function of time.

• Instantaneous power, is an important quantity since its maximum value, (i.e. peak power) must be limited for all physical devices. For this reason, the maximum instantaneous power or peak power is a commonly used specification for characterizing electrical devices. In an electronic amplifier, if the specified peak power at the input is exceeded, the output signal will be distorted and greatly exceeding this input rating may even permanently damage the amplifier.

In linear networks, which have inputs that are periodic functions of time, the steady-state current and voltages produced are periodic, each having identical periods.

Let v and i be periodic of period T (or have frequency $\omega = \frac{2\pi}{T}$). Then instantaneous power $p(t) = v(t).i(t)$

Therefore, the instantaneous power is also periodic of period T.

The fundamental period T_1 of p (the minimum time in which p repeats itself) is not necessarily equal to T, however T must contain an integral number of periods T_1.

Ex:

Solution:

Average power : time integral of instantaneous power over a complete period, divided by the period

We may obtain the average power by integrating over the period of v or i.

Let us consider the general two terminal device

If a two terminal device is resistor R, then $\theta = 0$.

If $i = I_{dc}$ a constant (dc) current, then $\omega = \theta = \phi = 0$.

- If two port device is an inductor, $\theta = 90^\circ$ $P_{av} = 0$
- If two port device is an capacitor, $\theta = -90^\circ$ $P_{av} = 0$

Therefore, an inductor or a capacitor, or any network composed entirely of ideal inductors and capacitors, in any combination dissipates zero average power.

Ideal inductors and capacitors are called lossless elements. Physically, lossless elements store energy during part of the period and release it during the other part, so that the average delivered power is zero.

For the passive loads, the average power is nonnegative.

This requires that

Ex: Find the average power delivered by source.

P_R is equal to the average power delivered to Z since the capacitor absorbs no average power.

8.2 Superposition and Power

Consider the following circuit with two sources

By superposition $i = i_1 + i_2$ due to v_{g1} & v_{g2}

In general $2R\, i_1\, i_2 \neq 0 \Longrightarrow p \neq p_1 + p2$, and superposition may not apply for instantaneous power.

In the case of p periodic with period T, the average power

Since we are assuming $i = i_1 + i_2$ is periodic of period T

Hence, if $m = n$ superposition does not apply (except $cos(\phi_1 - \phi_2) = 0$ cases), but if $m \neq n$ superposition does apply for average power.

For the periodic sinusoid with any number of sinusoidal components of different frequencies, the average power due to the sum of the components is the sum of the average powers due to each component acting alone.

Also, superposition of average power holds for sinusoids whose frequencies are not integral multiples of sum frequency ω, provided we generalize the definition of average power to

Ex:

8.3 Complex Power

Using phasor representations of the voltage and current, we define a new quantity, the complex power S

All the quantities S, V and I are in general complex numbers.

S: complex power measured in voltamperes (VA)
P: average real (active) power, measured in Watts (W)
Q: reactive power measured in voltampere reactive (VAr)

Q represents an exchange of energy between the source and reactive part of load, hence no net power is gained or lost in the process, average reactive power is zero.

A circuit in which the current lags the voltage (inductive circuit, RL) is said to have a lagging power factor. A circuit in which the current leads the voltage (capacitive circuit, RC) is said to have a leading power factor.

$Q < 0 \Longleftrightarrow$ leading power factor, a capacitive load (RC combination)

$Q > 0 \Longleftrightarrow$ lagging power factor, an inductive load (RL combination)

Ex: 10 kVA load at 0.8 power factor leading means:

In the case of purely resistive loads, the voltage and current are in phase, hence $\theta = 0$, $Pf = 1$. In a purely reactive load, $\theta = \pm 90°$, $Pf = 0$.

The complex power delivered by the source to the interconnected loads is the sum of that delivered to each individual load.

In practice, the power factor of a load is very important. In industrial applications, load may require thousands of watts to operate and power factor greatly affects the electric bill.

Ex: A mill consumes 100 kW from a 220 V rms line. At $Pf = 0.85$ lagging, Generating station must generate larger current in the case of the lower Pf. since the transmission lines supplying the power have resistance, the generator must produce a larger average power to supply the 100 kW to the load.

If the resistance is 0.1 Ω, then power generated by the source is

8.4 Power Factor Compensation

Industrial loads usually have a lagging power factor (i.e. inductive loads) due to windings of electrical motors. The lagging power factor results in large amounts of current requirements from the generators. The desirable case is to have $Pf = \cos\theta = 1$ at the load end so that load will look like purely resistive.

We may change the power factor of load having an impedance $Z = R + jX$, by connecting an impedance Z_1 in parallel with Z.

For this connection load voltage does not change. Since Z is fixed, I does not change. Hence the power delivered to the load is not affected. But the current I_1 supplied by generator changes.

Problem: Select Z_1 so that

(1) Z_1 absorbes zero average power

(2) Z_T satisfies the desired power factor Pf, i.e $Pf = \cos\left[\tan^{-1}\left(\frac{Im\, Z_T}{Re\, Z_T}\right)\right]$

Solution:

The complex power to uncorrected load Z is

Complex power to Z_1 (in parallel with Z)

From conservation of complex power, for the composite load

Note that addition Z_1 affects the net reactive power only.

Ex: Given a load which requires 2 kW at a 0.75 Pf lagging at a voltage of 220 V. Calculate the reactive power to be supplied by a parallel connected capacitor to increase the Pf to 1. Also determine the impedance X_c of the capacitor.

Solution:

Ex: Repeat the previous example for a lot of 2 kW at 0.75 Pf lagging which is desired to increase to 0.95 Pf.

8.5 The Volt-Ampere Method

The volt-ampere method is used to the analyze the circuits involved in power distribution. It is based on the conservation of complex power. When it is applied, the active power and reactive power related to every circuit element are computed and their respective sums are equated to the active power and the reactive power supplied by source(s).

Ex: Two generators supply a 10 kW load at 0.8 power factor (lagging) through a distribution system. The generator V_2 supplies 5 kW at a 0.6 pf (lagging). Find the generator voltages V_1 and V_2. The voltamperes ($|S|$) and power factor of generator V_1.

Solution:

Let's start from load side and progress towards the generators.

The voltamperes of the load $VA = |S_L| = \sqrt{P_L^2 + Q_L^2} = 12500\,VA$

$|S_L| = |V_L||I_L| \Longrightarrow |I_L| \cong 28.4\,A.$

$P_{R_2} = 0.6\,|I_L|^2 = 484\,W \qquad (S = (R + jX_L)I_L.I_L^* = R|I_L|^2 + jX|I_L|^2)$

$Q_{X_2} = 0.7|I_L|^2 = 565\,VAr$

To the right of the generator V_2 we can simplify as

From the power specifications of generator V_2

Calculate the net active and reactive power for the right part of the circuit including generator V_2.

Knowing $|I_b|$ we can compute P_{R_1} and Q_{X_1}

The power factor of generator V_1 $(Pf)_{S1} = \frac{P_{S1}}{|S_{S1}|} = 0.965$ (lag)

Ex: An electrical load of 10 kVA, 0.8 Pf (lagging) is operating at 220 V. The cable connecting load to the source has an impedance of $0.1 + j\,0.2\ \Omega$. Find

a) source voltage

b) power factor at the source

8.6 Power Factor Correction

We have seen that reactive powers in inductive and capacitive loads are opposite in sign.

Inductors are assumed to consume reactive power, capacitances are assumed to generate reactive power.

In power systems, the use of inductance is a must because they create magnetic fields (i.e. in transformers). Practically, almost all the loads are of inductive in nature since they contain coils & windings. Since the power company charges for the reactive power consumed, it may be desirable for the user to by a large capacitance and to connect it in parallel with the inductive load to supply some of the reactive power. The main purpose of the capacitance is to improve the power factor of the combined load.

Benefits for the user: gets rid off penalty charges.
Benefits (for TEK):

i) Released generation and distribution capacity (i.e. avoiding of unnecessary wage of expensive power apparatus such as generators, lines, and transformers for reactive power transmission. Note that all these equipment are rated in S (VA), not in MW).

ii) Reduced system losses, owing to smaller line currents.

iii) Improved system voltage regulation due to reduced voltage drops.

In this lecture, power factor correction problem will be examined from a mathematical point of view. Detailed considerations to practical problems such as the location of capacitors, their ratings, switching means and protection will be treated later on.

Ex:

a) Find the Pf at the source.

b) Find the impedance that is equivalent to the three parallel loads.

Solution:

Ex: Assume that the active and reactive energy consumption of a large plant in a month is 5×10^6 kWhr and 5×10^6 kVArhr, respectively. If the energy costs are: 5000 TL/kVArhr and 8000 TL/kWhr. Find the total (monthly) energy bill. The utility company applies penalty charges on reactive energy if the average Pf is less than 0.9 lag.

Assume that a shunt capacitor bank of 4500 kVAr is connected continuously during the month and the cost of compensation equipment is 2,000,000 TL/kVAr.

i) What will be the new power factor?

ii) What will be the new energy bill and savings per month?

Solution:

8.7 Maximum Power Transfer (Impedance Matching)

The problem is to find the load impedance that will maximize the real (average) power absorbed by the load.

The real power absorbed by the load:

In practical circuits, in many cases, matched load impedance can be provided by using transformers.

8.8 Three Phase Balanced Circuits

Due to economic and operational advantages, the electric energy is generated and distributed in the form of three phase, rather than a single phase that we have been considering so far. The weight of the conductors and other components in 3ϕ system is much lower than in a single phase system delivering the same amount of power. Also, 3ϕ system can deliver a constant power, while the generating power in single phase systems has a pulsating nature.

A 3ϕ AC supply has three equal magnitude and frequency voltage sources but having $120°$ phase shift among one another.

In practical applications these three voltage generators are connected in two different ways: Y-connection, Δ-connection.

8.8.1 Y-Connection

For a Y-connection the magnitude of line voltages are $\sqrt{3}$ times that of the phase voltages. There is again 120° phase shift among the line voltages.

Balanced system:

1) V_{an}, V_{bn}, V_{cn} form a balanced set.

2) Transmission lines are balanced, $Z_{l,a} = Z_{l,b} = Z_{l,c}$.

3) Load is balanced, $Z_a = Z_b = Z_c$.

If the system is balanced, all phase currents are equal in magnitude but out of phase by 120° among each other.

The current in the neutral line under balanced conditions is zero. So, the neutral line connection is redundant in physical applications. However, in mathematical calculation we still retain this neutral line to obtain and equivalent single phase circuit.

The above Y-Y balanced system is like three identical single phase circuits with currents and voltages having a phase shift of 120° among each other. We only need to solve one of these single phase circuits to determine phase voltages .

Ex:

Find the average power delivered to $(2 + j4)\,\Omega$.

Solution: In this topology we cannot use equations referring impedances from one side to the other. We must solve this circuit using our circuit analysis tools, such as mesh analysis.