Target Tracking: Lecture 5
Multiple Target Tracking: Part I

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Lecture Outline

- What is an hypothesis?
- What is
  - Single Hypothesis Tracking (SHT)?
  - Multiple Hypothesis Tracking (MHT)?
- Single Hypothesis Tracking
  - Global nearest neighbor (GNN)
  - Joint probabilistic data association (JPDA)

What is a hypothesis?

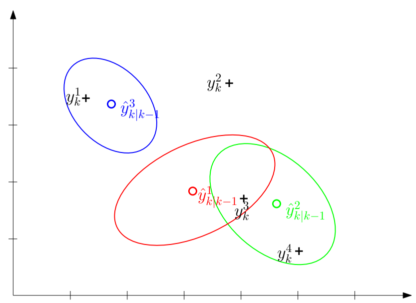
Definition

An (association) **hypothesis** is a partitioning of a set of measurements according to the their origin.

- At each time step, a single hypothesis tracking algorithm keeps only a single hypothesis about all of the measurements received in the past.
  - Global nearest neighbor algorithm does this by selecting the best hypothesis according to a criterion.
  - Joint probabilistic data association filter (JPDAF) combines all possible current hypotheses into a single one to form a single composite hypothesis. For this reason it can also be called as a "composite hypothesis tracker".
- A multiple hypothesis tracker, on the other hand, keeps multiple hypotheses about the origin of the received data and has much more computation and memory requirements.

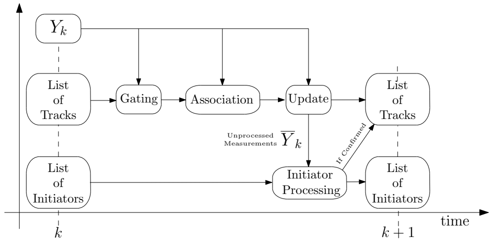
Basic Scenario Considered in the Lecture

- All the past is summarized by a 3 track hypothesis and possibly some tentative tracks.
- Tentative track processing is the same as what we learned in Lecture-3.
- Using single target tracking methods for each target gives only locally optimal results.
- The global picture must be taken into account for targets sharing measurements in their gates or possibly some other measurement-to-target association conflict.



Single hypothesis tracking

- All the past is summarized by a single hypothesis.
- In this single hypothesis, we have n_T tracks and n_I initiators (or tentative tracks). Generally, the initiation procedure is separated from the main logic.
- When a set of new measurements arrives, one first gates the measurements with the existing (confirmed) targets.
- Using the gating results, association is carried out.
- Using association results, confirmed tracks are updated.
- Unprocessed remaining measurements are sent to the initiator logic.

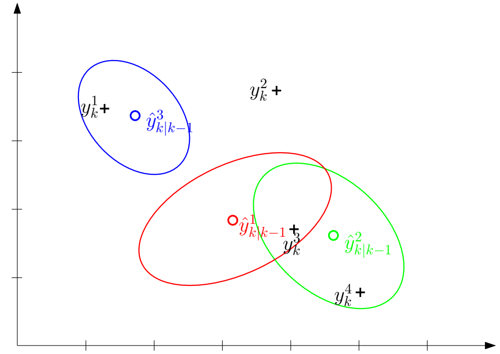


Gating

- Suppose there are $n_T = 3$ (confirmed) tracks in the hypothesis summarizing the past. Once we get the measurements $Y_k = \{y_k^1, \dots, y_k^4\}$, using the gate criteria we can prepare the following matrix to facilitate hypothesis generation.

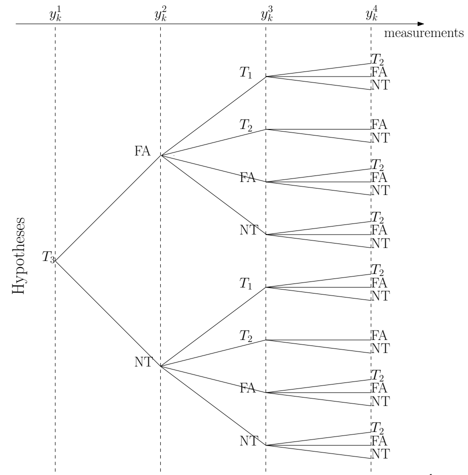
	T_1	T_2	T_3
1	0	0	1
2	0	0	0
3	1	1	0
4	0	1	0

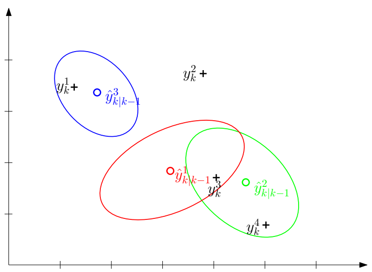
Such a matrix is called as **validation matrix**.



Association Hypotheses

Iterate over measurements





Validation Matrix

	T_1	T_2	T_3
1	0	0	1
2	0	0	0
3	1	1	0
4	0	1	0

Repeat the procedure above for $y_k^1 =$ FA and $y_k^1 =$ NT.

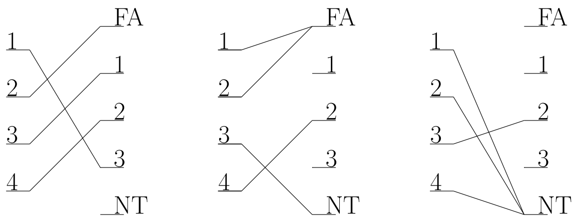
Association Hypotheses

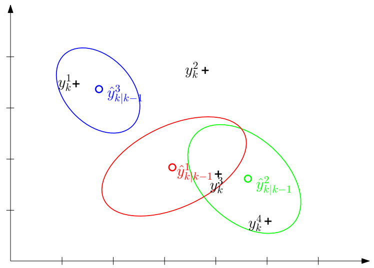
We can define an association hypothesis θ_k formally as a mapping

$$\theta_k(\cdot) : \{1, 2, \dots, m_k\} \to \{\text{FA}, 1, 2, \dots, n_T, \text{NT}\}$$

- where m_k is the number of measurements in Y_k i.e., $Y_k = \{y_k^1, \dots, y_k^{m_k}\}$
- n_T is the number of targets formed in the past.

Example Hypotheses with $m_k = 4$, $n_T = 3$





Probability of a Hypothesis

Suppose we are at time k at an intermediate stage of tracking. We have $j = 1, \ldots, n_T$ targets established previously and have just received $Y_k = \{y_k^1, \ldots, y_k^{m_k}\}$
Suppose $\theta_k(\cdot)$ is an arbitrary hypothesis about the origin of Y_k.

- Number of false alarms m_k^{FA} in S, the surveillance region is distributed as $P_{FA}(m_k^{FA})$;
- False alarm spatial density is $p_{FA}(y)$
- Number of new targets in S, the surveillance region is distributed as $P_{NT}(m_k^{NT})$;
- New target spatial density is $p_{NT}(y)$;
- Detection probability of the jth target: P_D^j;
- Gate probability of the jth target: P_G^j;
- Predicted measurement density of jth target: $p_{k|k-1}^j(y)$.

Standard Settings

- $P_{FA}(m_k^{FA}) = \frac{(\beta_{FA} V_S)^{m_k^{FA}} \exp(\beta_{FA} V_S)}{m_k^{FA}!}$
- $p_{FA}(y) = 1/V_S$ when $y \in V_S$.
- $P_{NT}(m_k^{NT}) = \frac{(\beta_{NT} V_S)^{m_k^{NT}} \exp(\beta_{NT} V_S)}{m_k^{NT}!}$
- $p_{NT}(y) = 1/V_S$ when $y \in V_S$.

Fundamental Theorem of TT

Theorem: Suppose θ_k is an association hypothesis about the current measurement set Y_k. Then the posterior probability of θ_k is given as

$$P(\theta_k | Y_{0:k}) \propto \beta_{FA}^{m_k^{FA}} \beta_{NT}^{m_k^{NT}} \left[\prod_{j \in \mathcal{J}_D} P_D^j p_{k|k-1}^j \left(y_k^{\theta_k^{-1}(j)} \right) \right] \left[\prod_{j \in \mathcal{J}_{ND}} (1 - P_D^j P_G^j) \right]$$

where

- $\mathcal{J}_D$ is the set of indices of detected targets, i.e., indices of targets which were assigned a measurement by θ_k.
- $\mathcal{J}_{ND}$ is the set of indices of non-detected targets i.e., indices of target that were not assigned a measurement by θ_k.
- $\theta_k^{-1}(j)$ is the index of the measurements that is assigned to target when $j \in \mathcal{J}_D$.

Fundamental Theorem of TT

Since there is a single hypothesis for the past, the term

$$\prod_{j=1}^{n_T} (1 - P_D^j P_G^j) = \prod_{j \in \mathcal{J}_D} (1 - P_D^j P_G^j) \prod_{j \in \mathcal{J}_{ND}} (1 - P_D^j P_G^j)$$

is constant for all hypotheses. Then, we have

$$P(\theta_k | Y_{0:k}) \propto \beta_{FA}^{m_k^{FA}} \beta_{NT}^{m_k^{NT}} \prod_{j \in \mathcal{J}_D} \frac{P_D^j p_{k|k-1}^j \left(y_k^{\theta_k^{-1}(j)} \right)}{(1 - P_D^j P_G^j)}$$

Taking the logarithm, we have

$$\log P(\theta_k | Y_{0:k}) = m_k^{FA} \log \beta_{FA} + m_k^{NT} \log \beta_{NT} + \sum_{j \in \mathcal{J}_D} \log \frac{P_D^j p_{k|k-1}^j \left(y_k^{\theta_k^{-1}(j)} \right)}{(1 - P_D^j P_G^j)}$$

Fundamental Theorem of TT

We can write the summed elements in the log-probability in a matrix given as

$\mathcal{A}$	T_1	T_2	T_3	FA$_1$	FA$_2$	FA$_3$	FA$_4$	NT$_1$	NT$_2$	NT$_3$	NT$_4$
y_k^1	$\times$	$\times$	ℓ_{13}	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$	$\times$	$\times$
y_k^2	$\times$	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$	$\times$
y_k^3	ℓ_{31}	ℓ_{32}	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$
y_k^4	$\times$	ℓ_{42}	$\times$	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$

where

- $\times$ represents $-\infty$.
- $\ell_{ij} \triangleq \log \frac{P_D^j p_{k|k-1}^j(y_k^i)}{(1-P_D^j P_G^j)}$.
- This matrix is called **assignment matrix**.

- Finding the optimal association hypothesis then corresponds to finding the column indices $\{j_1, j_2, j_3, j_4\}$, $j_{i_1} \neq j_{i_2}$ for $1 \leq i_1 \neq i_2 \leq 4$ such that the sum $\sum_{i=1}^4 \mathcal{A}_{ij_i}$ is maximized.

Assignment Problem

We can make a formal definition of the problem as follows

- We are given the matrix $\mathcal{A} \in \mathbb{R}^{m\times n}$ with $n \geq m$.
- Define the auxiliary matrix $Z = [z_{ij}]$ where $z_{ij} \in \{0, 1\}$.

Problem Definition

$$\text{Maximize} \sum_{i=1}^{m} z_{ij}\mathcal{A}_{ij}$$

subject to

$$\sum_{j=1}^{n} z_{ij} = 1 \quad \forall i \text{ and } \sum_{i=1}^{m} z_{ij} \leq 1 \quad \forall j$$

This problem is called as **assignment problem** in optimization literature.

Assignment Problem

- Considered first in economics.
- Possible equivalents are assigning personnel to jobs or assigning delivery trucks to locations.
- Earlier methods used linear programming techniques, like Hungarian method which is computationally costly.
- Less computationally expensive methods appeared later when different applications were found.
  - Munkres algorithm
  - JVC algorithm (by Jonker and Volgenant)
  - Auction algorithm (by Bertsekas): Explained thoroughly in the book.

Global Nearest Neighbor (GNN) Algorithm

$\mathcal{A}$	T_1	T_2	T_3	FA$_1$	FA$_2$	FA$_3$	FA$_4$	NT$_1$	NT$_2$	NT$_3$	NT$_4$
y_k^1	$\times$	$\times$	ℓ_{13}	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$	$\times$	$\times$
y_k^2	$\times$	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$	$\times$
y_k^3	ℓ_{31}	ℓ_{32}	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$	$\times$
y_k^4	$\times$	ℓ_{42}	$\times$	$\times$	$\times$	$\times$	$\log\beta_{FA}$	$\times$	$\times$	$\times$	$\log\beta_{NT}$

- Choose the best (largest probability) association hypothesis.
- The measurements associated to targets in the best association hypothesis are processed by target KFs.
- The measurements associated to FA and NT are handled by the initiator logic.

GNN Algorithm

- Therefore, we combine the FA and NT cases into the single category of **external sources** (EX).
- The external sources become Poisson with density $\beta_{EX} = \beta_{FA} + \beta_{NT}$.

$\mathcal{A}$	T_1	T_2	T_3	FA_1	FA_2	FA_3	FA_4	NT_1	NT_2	NT_3	NT_4
y_k^1	$\times$	$\times$	ℓ_{13}	$\log \beta_{FA}$	$\times$	$\times$	$\times$	$\log \beta_{NT}$	$\times$	$\times$	$\times$
y_k^2	$\times$	$\times$	$\times$	$\times$	$\log \beta_{FA}$	$\times$	$\times$	$\times$	$\log \beta_{NT}$	$\times$	$\times$
y_k^3	ℓ_{31}	ℓ_{32}	$\times$	$\times$	$\times$	$\log \beta_{FA}$	$\times$	$\times$	$\times$	$\log \beta_{NT}$	$\times$
y_k^4	$\times$	ℓ_{42}	$\times$	$\times$	$\times$	$\times$	$\log \beta_{FA}$	$\times$	$\times$	$\times$	$\log \beta_{NT}$

$\mathcal{A}$	T_1	T_2	T_3	EX_1	EX_2	EX_3	EX_4
y_k^1	$\times$	$\times$	ℓ_{13}	$\log \beta_{EX}$	$\times$	$\times$	$\times$
y_k^2	$\times$	$\times$	$\times$	$\times$	$\log \beta_{EX}$	$\times$	$\times$
y_k^3	ℓ_{31}	ℓ_{32}	$\times$	$\times$	$\times$	$\log \beta_{EX}$	$\times$
y_k^4	$\times$	ℓ_{42}	$\times$	$\times$	$\times$	$\times$	$\log \beta_{EX}$

Global Nearest Neighbor (GNN) Algorithm

GNN

- Time $k = 0$: Send all measurements to initiation logic.
- Time $k > 0$: Suppose we have m_k measurements and n_T targets
  - Form the assignment matrix $\mathcal{A} \in \mathbb{R}^{m_k \times (n_T + m_k)}$
  - `auction(`$\mathcal{A}$`)`
  - Process the targets with their associated measurements.
  - Send the measurements associated to external sources (EX) to initiation logic.
  - Process the initiators (tentative tracks with EX associated measurements).
  - Add any confirmed tentative track to the confirmed track list.

PDA vs. JPDA

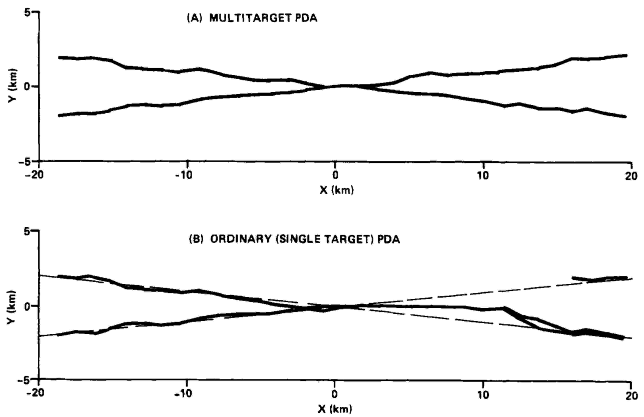


Figure 3. Comparative Performance Against Crossing Targets (Perfect Resolution).

Figure taken from:
R.J. Fitzgerald, "Development of Practical PDA Logic for Multitarget Tracking by Microprocessor," American Control Conference, pp.889–898, Jun. 1986.

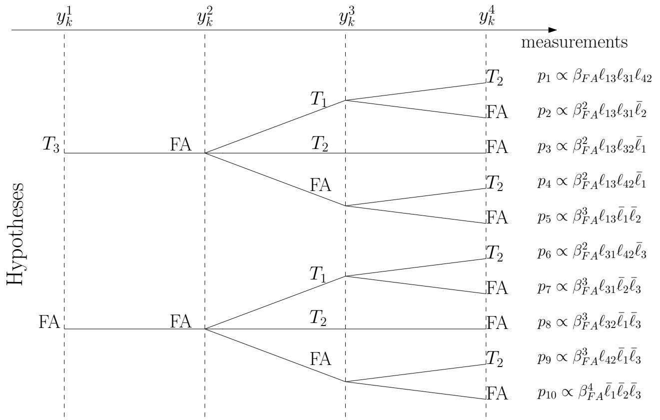
Joint Probabilistic Data Association (JPDA) Filter

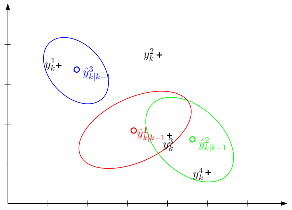
- Soft decision equivalent of GNN in the way that PDA is a soft version of NN.
- All past is again summarized with a single hypothesis (n_T confirmed targets n_I tentative targets).
- The number of targets is assumed fixed in the association, hence no NT associations in the possible hypotheses.
- For each previously established target, we need to calculate

$$P(T_j \leftrightarrow y_k^i) \quad \text{and} \quad P(T_j \leftrightarrow \phi) \tag{1}$$

  for y_k^i that are in the gate of the target. The update is then made with PDA update formulas by using these probabilities instead.
- A separate track initiation logic must run along with JPDAF to detect and initiate new tracks.

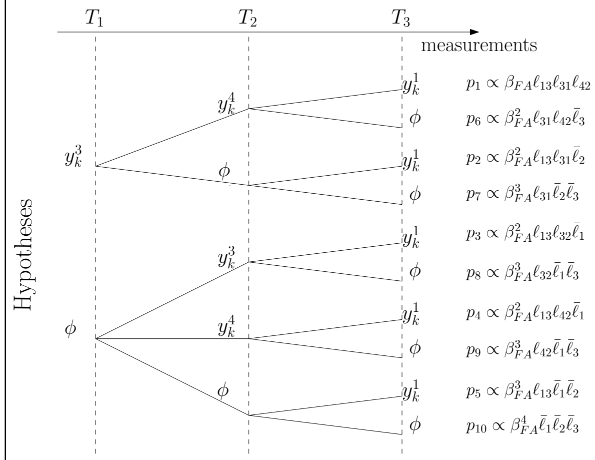
JPDAF





$\ell_{ij} = P_D^j p_{k|k-1}^j(y_k^i)$ $\bar{\ell}_j = 1 - P_D^j P_G^j$

JPDAF



$P(T_1 \leftrightarrow y_k^3) = \frac{1}{c}\sum_{i\in\{1,6,2,7\}} p_i$

$P(T_1 \leftrightarrow \phi) = \frac{1}{c}\sum_{i\in\{3,4,5,8,9,10\}} p_i$

$P(T_2 \leftrightarrow y_k^3) = \frac{1}{c}\sum_{i\in\{3,8\}} p_i$

$P(T_2 \leftrightarrow y_k^4) = \frac{1}{c}\sum_{i\in\{1,6,4,9\}} p_i$

$P(T_2 \leftrightarrow \phi) = \frac{1}{c}\sum_{i\in\{2,7,5,10\}} p_i$

$P(T_3 \leftrightarrow y_k^1) = \frac{1}{c}\sum_{i=1}^{5} p_i$

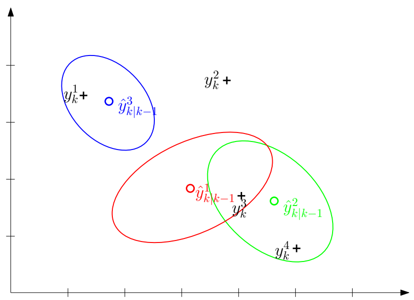
$P(T_3 \leftrightarrow \phi) = \frac{1}{c}\sum_{i=6}^{10} p_i$

$\ell_{ij} = P_D^j p_{k|k-1}^j(y_k^i)$ $\bar{\ell}_j = 1 - P_D^j P_G^j$

$c = \sum_{i=1}^{10} p_i$

JPDAF

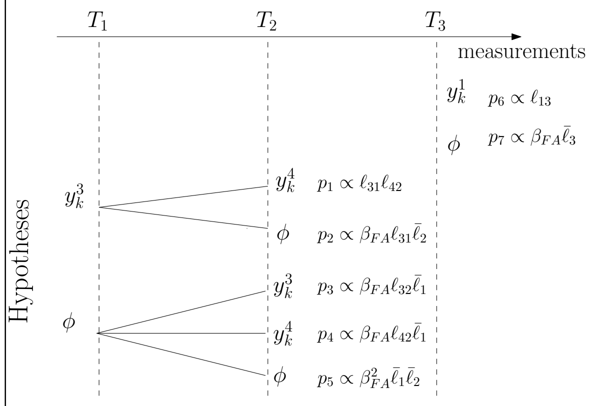
- Probability calculations show that the calculations can be done separately for the clusters of targets that does not share gated measurements.
- In other words our previous scenario can be grouped into two clusters $T_1 \& T_2$, T_3 and probability calculations can be done separately for the corresponding hypothesis trees.



The clustering can be done using the validation matrix

	T_1	T_2	T_3
1	0	0	1
2	0	0	0
3	1	1	0
4	0	1	0

JPDAF



$P(T_1 \leftrightarrow y_k^3) = \frac{1}{c_1}(p_1 + p_2)$

$P(T_1 \leftrightarrow \phi) = \frac{1}{c_1}(p_3 + p_4 + p_5)$

$P(T_2 \leftrightarrow y_k^3) = \frac{1}{c_1}p_3$

$P(T_2 \leftrightarrow y_k^4) = \frac{1}{c_1}(p_1 + p_4)$

$P(T_2 \leftrightarrow \phi) = \frac{1}{c_1}(p_2 + p_5)$

$P(T_3 \leftrightarrow y_k^1) = \frac{1}{c_2}p_6$

$P(T_3 \leftrightarrow \phi) = \frac{1}{c_2}p_7$

$\ell_{ij} = P_D^j p_{k|k-1}^j(y_k^i)$ $\bar{\ell}_j = 1 - P_D^j P_G^j$

$c_1 = \sum_{i=1}^{5} p_i$

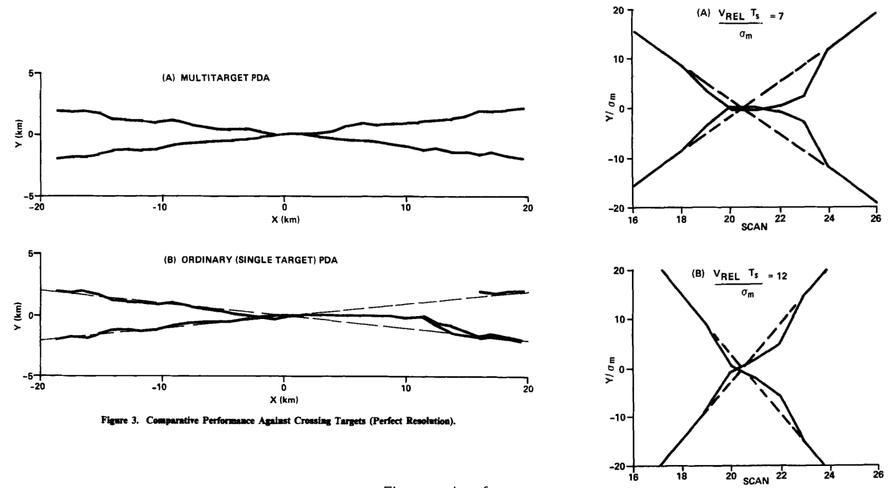
$c_2 = p_6 + p_7$

JPDAF

JPDAF

- Time $k = 0$: Send all measurements to initiation logic.
- Time $k > 0$: Suppose we have m_k measurements and n_T targets
  - Form the validation matrix.
  - Group the targets into clusters in which targets share gated measurements.
  - For each cluster, calculate PDA probabilities for each target in the cluster by using a hypothesis tree.
  - Update targets with the weighted equivalent measurements as PDA.
  - Send the unprocessed measurements and possibly gated extra measurements to initiation logic.
  - Process the initiators.
  - Add any confirmed tentative track to the confirmed track list.

Track Coalescence Problem of JPDAF



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References

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