EE793 Target Tracking: Computer Exercise 3

Due: 02.05.2013, 23:59

1 Interacting Multiple Model (IMM) Filter

This short computer exercise aims that you learn how to implement an IMM filter and observe the related quantities that was illustrated in class. There is an additional document that the instructor has prepared for you to learn how to derive and implement the IMM filter and it is highly recommended that you read that before starting this exercise.

a) Download the true target data from the web address `http://www.eee.metu.edu.tr/~umut/ee793/files/trueTarget.mat`. This is the same target data that you used in Exercise 1.
When you load this mat file in Matlab, it is going to load into your workspace a 3×151 size matrix named trueTarget. The first, second and third rows of this matrix contain the time stamps, true x-positions and true y-positions of the target you will consider in the following parts. Plot the $\mathsf{x} - \mathsf{y}$ positions and observe the true target trajectory.

b) Now assume that you measure the x and y positions of the target with Gaussian measurement noise with standard deviation $\sigma_e = 20$m. Generate your measurement data and observe the measured position values on top of the $\mathsf{x} - \mathsf{y}$ trajectory of the target. Note that in this exercise we are only going to consider the case $P_D = 1$.

c) Implement three different KFs using nearly CV model (the same state model as the one given in Exercise 1-b) with different process noise covariance matrices.

$$Q_1 = 0.1^2 I_2 \qquad Q_2 = 1^2 I_2 \qquad Q_3 = 10^2 I_2 \tag{1}$$

where I_2 denotes the 2×2 identity matrix. You can use a single point initialization for your filters with $v_{\max} = 50$m/s. Observe the estimation errors by plotting estimated positions on top of true target trajectory. Plot also the estimation errors.

Note that you do not have to do gating in this simplified exercise, i.e., just implement your filters with $P_G = 1$ which corresponds to an infinite gate threshold γ_G. However, in order to see how different filters result in different gates, plot $P_G = 0.99$ gates over the estimated positions. As a quantitative description of the gate size plot also the gate volumes (area in 2D) which can be calculated for this example with the formula

$$V_G^k = \pi \gamma_G \sqrt{|S_k|} \tag{2}$$

where γ_G is the gate threshold, S_k is the innovation covariance used for calculating the gate and the sign $|\cdot|$ denotes the determinant operation for a matrix.

d) Implement an IMM filter that uses two nearly CV models that are the same except for the process noise covariances. The first model uses $Q(1) = 0.1^2 I_2$ and the second one should use $Q(2) = 10^2 I_2$. Choose the transition probability matrix Π of the JMLS as

$$\Pi \triangleq \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix} \tag{3}$$

You can initialize both KFs of the IMM filter in the same way as in the previous part. Initial mode probabilities can be assumed to be equal to 0.5 both. Plot the estimation errors, gate ellipsoids, mode probabilities and gates volumes. Comment on your results.
Hints:

- The JMLS formulation in the given additional document only considers mode dependent A, B, C, D matrices for different models. For our exercise, these matrices are constant, i.e., not mode dependent. The result for mode dependent Q matrices can be obtained easily by replacing $B(i)QB^T(i)$ terms in the derivation with $B(i)Q(i)B^T(i)$.
- In order to calculate the gates of the IMM filter one would need a single measurement prediction $\hat{y}_{k|k-1}$ and innovation covariance S_k. You can obtain these by the following formulae

$$\hat{y}_{k|k-1} \triangleq \sum_{i=1}^{N_r} \mu^i_{k|k-1} \hat{y}^i_{k|k-1} \tag{4}$$

$$S_k \triangleq \sum_{i=1}^{N_r} \mu^i_{k|k-1} \left[S^i_k + (\hat{y}^i_{k|k-1} - \hat{y}_{k|k-1})(\hat{y}^i_{k|k-1} - \hat{y}_{k|k-1})^T \right] \tag{5}$$

  where the predicted mode probabilities $\mu^i_{k|k-1}$ are given as

$$\mu^i_{k|k-1} = \sum_{j=1}^{N_r} \pi_{ji} \mu^j_{k-1}. \tag{6}$$

e) Change the TPM of the above IMM filter to

$$\Pi_1 = \begin{bmatrix} 0.999 & 0.001 \\ 0.001 & 0.999 \end{bmatrix} \quad \text{and} \quad \Pi_2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \tag{7}$$

and observe and plot the mode probabilities when you run the filters. How does different TPMs affect the estimated mode probabilities $\{\mu^i_k\}_{i=1}^2$? How would such changes in mode probabilities affect estimates, gates etc. Comment on your results.