Quiz 4 Key (30.11.2017–01.12.2017)

Let $f(x) = \dfrac{\sqrt{1-x^2}}{x}$

a) Find domain of $f(x)$
b) Find x, y intercepts
c) Find asymptotes if any
d) Find $f'(x)$
e) Find the intervals where $f(x)$ is increasing/decreasing.
f) Find $f''(x)$
g) Draw the graph of $y=f(x)$

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a) $\boxed{x\neq 0}$ & $\boxed{1-x^2 \geq 0}$

$1-x^2 \geq 0 \Rightarrow x^2 \leq 1$

$-1 \leq x \leq 1$

Domain: $[-1,0)\cup(0,1]$

b) x-intercept: $y=0 \Rightarrow \dfrac{\sqrt{1-x^2}}{x}=0$

$x=\pm 1$

$\boxed{(1,0)\ (-1,0)}$

y-intercept: $x=0 \Rightarrow$?

$x=0$ is not possible since 0 is not included to our domain so there is not a y-intercept

c) For vertical asymptote

$x=0$ is candidate (it makes denominator zero)

$$\lim_{x\to 0^+} \frac{\sqrt{1-x^2}}{x} = \infty$$

So $\boxed{x=0}$ is vertical asymptote

$$\left(\lim_{x\to 0^-} \frac{\sqrt{1-x^2}}{x} = -\infty\right)$$

For horizontal & oblique asymptote we will check limit at ∞ and $-\infty$ but our domain is bounded so we don't have horizontal or oblique asymptote

d) $$f'(x) = \frac{\frac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x)\cdot x - 1\sqrt{1-x^2}}{x^2}$$

$$= \frac{-x^2-1+x^2}{x^2\cdot\sqrt{1-x^2}} = -\frac{1}{x^2\sqrt{1-x^2}}$$

e)

x	-1		0		1
sign $f'(x)$		$-$		$-$	
Behavior $f(x)$		$\searrow$		$\searrow$	

$f(x)$ is decreasing on $[-1,0)$ and $(0,1]$

f) $$f''(x) = \frac{1}{x^4\cdot(\sqrt{1-x^2})^2}\cdot\left(2x\cdot\sqrt{1-x^2} - x^2\cdot\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\right)$$

$$= \frac{2x-2x^3-x^3}{x^4(1-x^2)\cdot\sqrt{1-x^2}} = \frac{\cancel{x}(2-3x^2)}{\cancel{x}\,x^3(1-x^2)^{3/2}}$$

$$= -\frac{3x^2-2}{x^3(1-x^2)^{3/2}} \rightarrow \text{root } x=\pm\sqrt{\tfrac{2}{3}}$$

g)

x	-1		$-\frac{2}{\sqrt{3}}$		0		$+\frac{2}{\sqrt{3}}$		1
sign $f'(x)$		$-$		$-$		$-$		$-$	
sign $f''(x)$		$+$		$-$		$+$		$-$	
Behavior of $f(x)$									

inflection points ($f''(x)=0$ & we can draw a tangent line)

$$f(-x) = \frac{\sqrt{1-(-x)^2}}{-x} = -\frac{\sqrt{1-x^2}}{x} = -f(x) \rightarrow f(x) \text{ odd, sym wrt origin.}$$