$x^2 \quad x^2+1$

27.10.2017

Section 33 Quiz 1 Key

Consider $f(x) = \begin{cases} 3x+2 & \text{if } x \neq 1 \\ 6 & \text{if } x = 1. \end{cases}$

a) Show that $\lim_{x \to 1} f(x) = 5$ (Proof by ε-δ)

b) Is f continuous at $x=1$?

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a) (We need to show that $\forall \varepsilon > 0, \exists \delta > 0$ satisfying:
$0 < |x-1| < \delta \Rightarrow |f(x) - L| < \varepsilon$)

Consider $|f(x) - 5| = |3x+2-5| = |3x-3| = 3|x-1|$

Stratch work.

$|x-1| \neq 0$, $x \neq 1$

we want this to be less than ε when $|x-1| < \delta$ so choose $\delta \leq \frac{\varepsilon}{3}$

Given $\varepsilon > 0$, choose $\delta = \frac{\varepsilon}{3}$ then;

if $0 < |x-1| < \delta \Rightarrow |3x+2-5| = |3x-3| = 3\underbrace{|x-1|}_{<\delta} < 3 \cdot \frac{\varepsilon}{3} = \varepsilon$

$\boxtimes$

b) $\lim_{x \to 1} f(x) = 5$ (by part a) $f(1) = 6$.

for continuity we need $\lim_{x \to 1} f(x) = f(1)$

but $\lim_{x \to 1} f(x) = 5 \neq 6 = f(1)$ so f is not cont

at $x = 1$

Section 33 Quiz 2 Key

03.11.2017

Using the definition of derivative compute the derivative of $\frac{x^2+2}{x+1} = f(x)$

$$\left(\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \overset{\text{if limit exists}}{=} f'a \quad \Big| \quad \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \overset{\text{if limit exist.}}{=} f'(x) \right)$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{(x+h)^2+2}{x+h+1} - \frac{x^2+2}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2+2)(x+1) - (x+h+1)\cdot(x^2+2)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+2x^2h+xh^2+\cancel{2x}+\cancel{x^2}+2xh+h^2+\cancel{2} - \cancel{x^3}-x^2h-\cancel{x^2}-\cancel{2x}-2h-\cancel{2}}{(x+h+1)\cdot h\cdot(x+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x^2+xh+2x+h-x^2-2)}{(x+h+1)\cdot\cancel{h}\cdot(x+1)} = \frac{x^2+2x-2}{(x+1)^2}$$

Section 33 Quiz3 Key

17.11.2017

Show that $f(x) = \arctan x + e^x$ is <u>invertible</u> on $\mathbb{R}$. Find the derivative of inverse, at $x=1$.

Soln: $f'(x) = \dfrac{1}{1+x^2} + e^x > 0 \quad \forall x$ so $f(x)$ is increasing. Then $f(x)$ is one-to-one $\Rightarrow$ f is invertible.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{2}$$

$f^{-1}(1) = a \Rightarrow f(a) = 1$
$\Rightarrow \arctan a + e^a = 1$
$a=0$ satisfies this eqn. & f is 1-1 so $a=0$ only soln

$f^{-1}(1) = 0$

$f'(0) = \dfrac{1}{1+0} + e^0 = 2$