4) Find the length of the curve

$x = t^3 \quad y = 3t^2/2 \quad 0 \le t \le \sqrt{3}$

We were finding $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\, \Delta x$

or

$= \sqrt{1 + \left(\frac{\Delta x}{\Delta y}\right)^2}\, \Delta y$

Here x & y are given in terms of t

so we will use $\Delta s = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2}\, \Delta t$

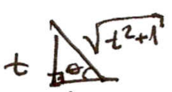
So we get formula for length $= \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Solution: $x = t^3 \Rightarrow \frac{dx}{dt} = 3t^2 \quad y = \frac{3t^2}{2} \Rightarrow \frac{dy}{dt} = 3t$

$0 \le t \le \sqrt{3}$

Length of given curve is $\int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + (3t)^2}\, dt$

$$= \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\, dt = \int_0^{\sqrt{3}} \sqrt{9t^2(t^2+1)}\, dt = \int_0^{\sqrt{3}} 3t\sqrt{t^2+1}\, dt$$



$\tan\theta = t$

$\sec^2\theta\, d\theta = dt$

$\cos\theta = \frac{1}{\sqrt{t^2+1}} \Rightarrow \sqrt{t^2+1} = \sec\theta$

$$= \int_{t=0}^{t=\sqrt{3}} 3 \cdot \tan\theta\, \sec\theta \cdot \sec^2\theta\, d\theta$$

$\left(\begin{matrix} \sec\theta = u \\ \sec\theta\tan\theta\, d\theta = du \end{matrix}\right) \longrightarrow \quad = \int_{t=0}^{t=\sqrt{3}} 3 \cdot u^2\, du = u^3 \Big|_{t=0}^{t=\sqrt{3}}$

$\left(\begin{matrix} \text{back substitution} \\ u = \sec\theta \end{matrix}\right) \rightarrow \quad = (\sec\theta)^3 \Big|_{t=0}^{t=\sqrt{3}}$

$\left(\begin{matrix} \text{back subs.} \\ \sec\theta = \sqrt{t^2+1} \end{matrix}\right) \rightarrow \quad = \left(\sqrt{t^2+1}\right)^3 \Big|_{t=0}^{t=\sqrt{3}} = \left(\sqrt{3+1}\right)^3 - \left(\sqrt{0+1}\right)^3$

$= 8$ //

6) a) $(r,\theta) = (-4, \frac{2\pi}{3}) \Rightarrow (x,y) = ?$

b) $(x,y) = (-1,-1) \Rightarrow (r,\theta) = ?$

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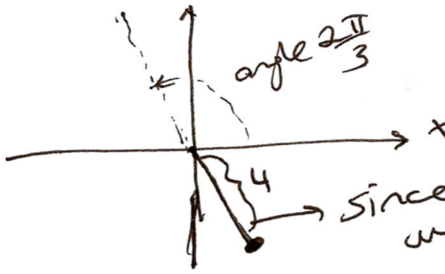
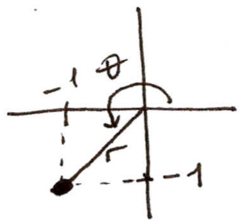
a) $(r,\theta) = (-4, \frac{2\pi}{3})$



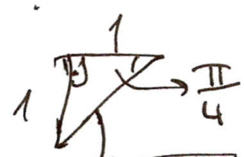
since r is -4 (negative) we plot in negative direction.

b) $(x,y) = (-1,-1)$ $\quad (r,\theta) = ?$



$\theta = ?$

$r = ?$



$\Rightarrow \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

So $(r,\theta) = (\sqrt{2}, \frac{5\pi}{4})$

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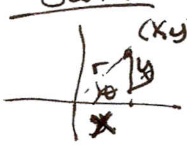
7) a) Convert $2x - 5x^3 = 1 + xy$ into polar coordinates

b) Convert $r = -8\cos\theta$ into cartesian coordinates

Soln: **a)**



$x = r\cos\theta$, $y = r\sin\theta$ } subs. into eqn.

$2 \cdot r\cos\theta - 5 \cdot (r\sin\theta)^3 = 1 + r\cos\theta \cdot r\sin\theta$

$2r\cos\theta - 5r^3\sin^3\theta = 1 + r^2 \sin 2\theta$ //

b) $r = -8\cos\theta$

$x = r\cos\theta$, $y = r\sin\theta$ } $x^2+y^2 = r^2$, $r = \sqrt{x^2+y^2}$

$\cos\theta = \frac{x}{r}$ $\quad \sin\theta = \frac{y}{r}$

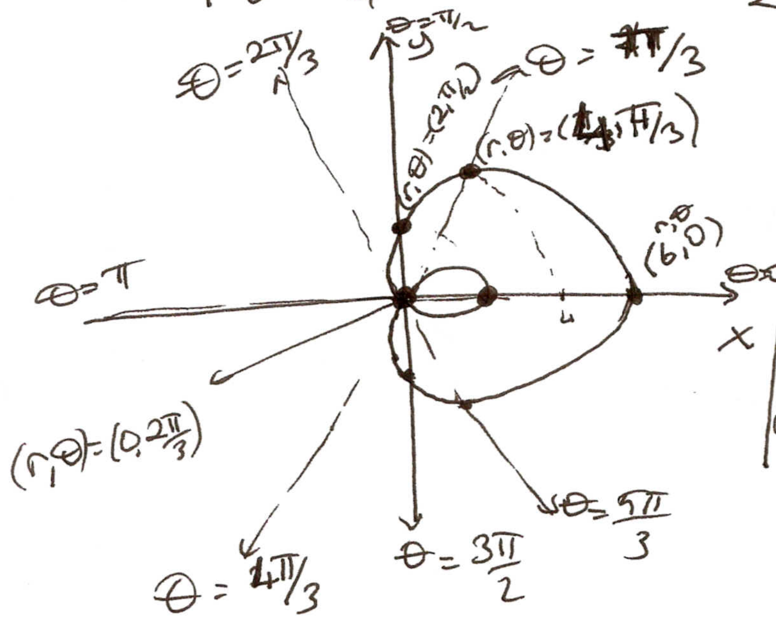
$\sqrt{x^2+y^2} = -8\frac{x}{r}$, $r \to \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = -8\frac{x}{\sqrt{x^2+y^2}}$ } $x^2+y^2 = -8x$

$x^2+y^2+8x = 0$ //

8) Draw the graph of $r = 2 + 4\cos\theta$ (7)

θ	0	$\pi/3$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	$5\pi/3$	2π
r	6	4	2	0	-2	0	2	4	6



Since $r(-\theta) = r(\theta)$ graph is symmetric wrt x axis so we can draw for $\theta \in [0, \pi]$ and ~~[illegible]~~ we may take symmetric of it wrt x axis without using $\theta \in [\pi, 2\pi]$ part

between $\frac{\pi}{2} \leq \theta \leq 2\pi/3$

$-\frac{1}{2} \leq \cos\theta \leq 0$

$-2 \leq 4\cos\theta \leq 0$

$0 \leq 2 + 4\cos\theta \leq 2$

$\boxed{\text{So } 0 \leq r \leq 2 \text{ for } \theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$

for $\frac{2\pi}{3} \leq \theta \leq \pi$

$-1 \leq \cos\theta \leq -\frac{1}{2}$

$-2 \leq r = 2 + 4\cos\theta \leq 0$

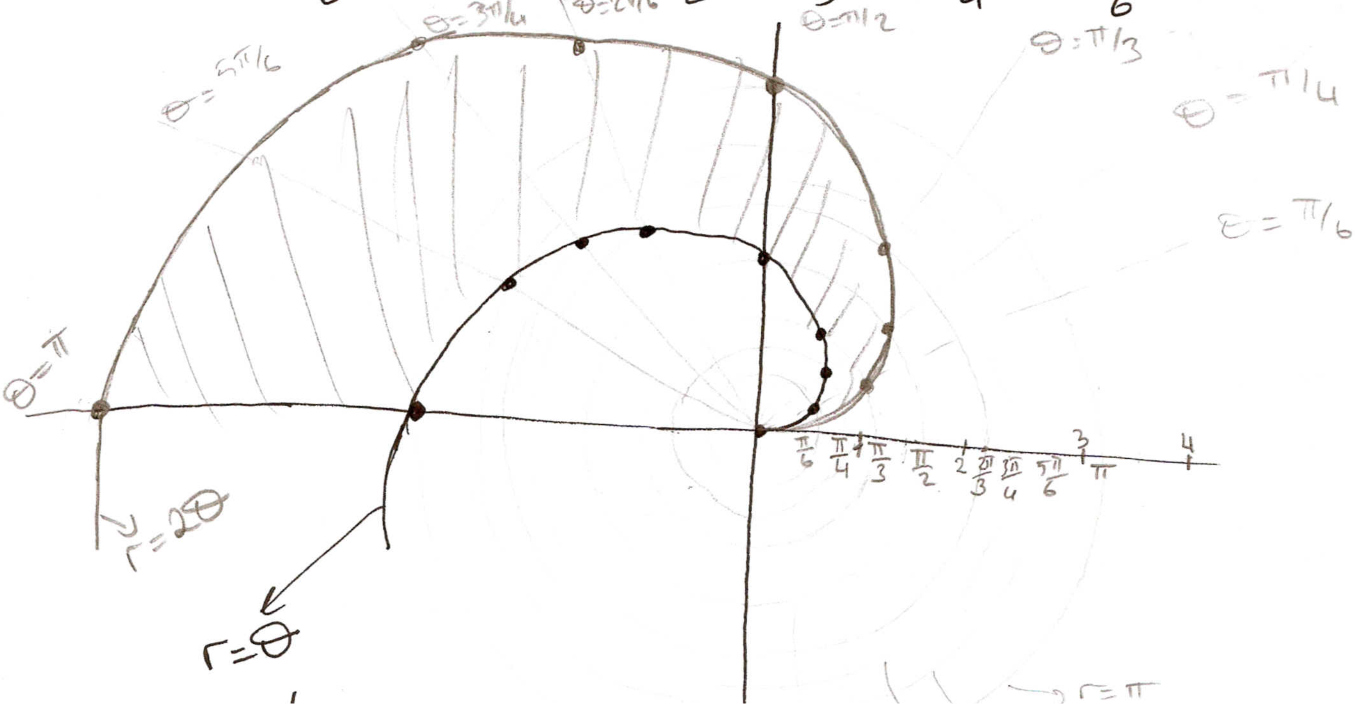
10) $r=\theta$, $r=2\theta$, $\theta=[0.\pi]$

(8)

Find area between them & horizontal axis.

$r=\theta$: θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π

$r=2\theta$: θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	$0.$	$2.\frac{\pi}{6}$	$2.\frac{\pi}{4}$	$2.\frac{\pi}{3}$	$2.\frac{\pi}{2}$	$2.2\frac{\pi}{3}$	$2.\frac{3\pi}{4}$	$2.\frac{5\pi}{6}$	$2.\pi$



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