Section: 161

Math 120 Spring 2019-2020
Quiz no.: 01
Date: 21.02.20
Time Limit: ~10 Minutes

KEY

Name & Surname: ____________________

ID Number: ____________________

Grade:

Declaration of Honesty: By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

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1. Find the sum of the series $\sum_{k=1}^{\infty}(-\frac{119}{120})^k$.

Geometric series with $r=-\frac{119}{120}$, $\left|-\frac{119}{120}\right|<1$ so it is convergent.

$$\sum_{k=1}^{\infty}\left(-\frac{119}{120}\right)^k = \sum_{k=0}^{\infty}\left(-\frac{119}{120}\right)^{k+1} = \sum_{k=0}^{\infty}\left(-\frac{119}{120}\right)^k \cdot \left(-\frac{119}{120}\right)^{+1}$$

$$= \frac{1}{1-\left(-\frac{119}{120}\right)} \cdot \left(-\frac{119}{120}\right) = \frac{1}{\frac{239}{\cancel{120}}} \cdot \left(-\frac{119}{\cancel{120}}\right) = -\frac{119}{239}$$

OR $\sum_{k=1}^{\infty}\left(-\frac{119}{120}\right)^k$ Geometric series with the first term $-\frac{119}{120}$ & $r=-\frac{119}{120}$ $\left|-\frac{119}{120}\right|<1$

So convergent. and $\sum_{k=1}^{\infty}\left(-\frac{119}{120}\right)^k = -\frac{119}{120} \cdot \frac{1}{1-\left(-\frac{119}{120}\right)} = -\frac{119}{\cancel{120}} \cdot \frac{1}{\frac{239}{\cancel{120}}} = -\frac{119}{239}$

$$\sum_{n=0}^{\infty} a \cdot r^n = a \cdot \frac{1}{1-r}$$

OR

$$\sum_{n=1}^{\infty} a \cdot r^{n-1} = a \cdot \frac{1}{1-r}$$

Section: 62

Name & Surname: Key

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1. Find the sum of the series $\sum_{k=2}^{\infty} e^{-k}$.

Geometric series with $r = \frac{1}{e}$, $\left|\frac{1}{e}\right| < 1$ so it is convergent.

$$\sum_{k=2}^{\infty} e^{-k} = \sum_{k=2}^{\infty}\left(\frac{1}{e}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{e}\right)^{k+2} = \sum_{k=0}^{\infty}\left(\frac{1}{e}\right)^k \cdot \left(\frac{1}{e}\right)^2$$

$$= \frac{1}{1-\frac{1}{e}} \cdot \frac{1}{e^2} = \frac{1}{\frac{e-1}{\cancel{e}}} \cdot \frac{1}{e^{\cancel{2}}} = \frac{1}{e^2 - e}$$

OR $\sum_{k=2}^{\infty} e^{-k}$ Geometric series with the first term $\frac{1}{e^2}$ & $r = \frac{1}{e}$ $\left|\frac{1}{e}\right| < 1$

$$\left(\sum_{k=2}^{\infty} e^{-k} = \frac{1}{e^2} + \frac{1}{e^3} + \cdots \right)$$

first term $\frac{1}{e}$ common ratio $r = \frac{1}{e}$

So convergent. and $\sum_{k=2}^{\infty} e^{-k} = \frac{1}{e^2} \cdot \frac{1}{1-r} = \frac{1}{e^2(1-\frac{1}{e})} = \frac{1}{e^2 - e}$

$$\boxed{\begin{array}{l} \sum_{n=0}^{\infty} a \cdot r^n = a \cdot \frac{1}{1-r} \\ \text{OR} \\ \sum_{n=1}^{\infty} a \cdot r^{n-1} = a \cdot \frac{1}{1-r} \end{array}}$$

Section:62

Name & Surname: ____________

Math 120 Spring 2019-2020
Quiz no.: 02
Date: 28.02.20
Time Limit: ~10 Minutes

ID Number: ____________

Grade:

Key

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1. Determine whether the series $\sum_{n=1}^{\infty} \underbrace{\frac{(-1)^n}{\sqrt{n+3}}}_{a_n}$ is absolutely convergent, conditionally convergent or divergent.

Check $\sum |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}}$

$\sqrt{n+3} < \sqrt{n+3n}$

$\frac{1}{\sqrt{n+3}} > \frac{1}{2\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ $p = \frac{1}{2} < 1$ by p test

$\frac{1}{2}\sum \frac{1}{\sqrt{n}}$ is divergent, then $\sum \frac{1}{\sqrt{n+3}}$ is divergent by comparison.

i) $a_n \cdot a_{n+1} = \frac{(-1)^{2n+1}}{\sqrt{n+3}\cdot\sqrt{n+4}} < 0$

ii) $\sqrt{n+3} < \sqrt{n+4}$

$\Rightarrow \frac{1}{\sqrt{n+3}} > \frac{1}{\sqrt{n+4}}$ $|a_n| > |a_{n+1}|$

iii) $\lim |a_n| = \lim_{n\to\infty} \frac{1}{\sqrt{n+3}} = 0$

Then by AST. $\sum a_n$ is conv.

$\sum a_n$ is conditionally convergent (Because $\sum |a_n|$ is divergent but $\sum a_n$ is convergent)

<u>Recall:</u> if $\sum |a_n|$ is convergent $\sum a_n$ is absolutely convergent

Section: 161

Name & Surname Solution Key

Math 120 Spring 2019-2020
Quiz no.: 02
Date: 06.03.20
Time Limit: ∼10 Minutes

ID Number:

Grade:

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1. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n}$.

1st way

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{for } |x|<1$$

integrate term by term in the interval of convergence

$$\int_0^x \sum_{n=0}^{\infty} t^n \, dt = \int_0^x \frac{1}{1-t}\, dt, \quad \text{for } |x|<1 \ (\text{so } |t|<1)$$

$$\sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1}\Big|_0^x = -\ln(1-t)\Big|_0^x \qquad |x|<1$$

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln(1-x) \qquad |x|<1$$

$\Downarrow$ Shifting index

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x) \qquad |x|<1$$

put $x = -\frac{1}{3}$, $\left(\left|-\frac{1}{3}\right| = \frac{1}{3} < 1\right)$

$$\sum_{n=1}^{\infty} \left(-\tfrac{1}{3}\right)^n \cdot \frac{1}{n} = -\ln\left(1-\left(-\tfrac{1}{3}\right)\right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n} = -\ln\left(\frac{4}{3}\right) = \ln\frac{3}{4}$$

2nd way

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{for } |x|<1$$

$$\sum_{n=0}^{\infty} (-x)^n = \frac{1}{1+x} \quad \text{for } |x|<1$$

integrate term by term in the interval of convergence

$$\int_0^x \sum_{n=0}^{\infty} (-1)^n t^n \, dt = \int_0^x \frac{1}{1+t}\, dt, \quad \text{for } |x|<1 \ (\text{so } |t|<1)$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{t^{n+1}}{n+1}\Big|_0^x = \ln(1+t)\Big|_0^x \qquad |x|<1$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \ln(1+x) \qquad |x|<1$$

$\Downarrow$ Shifting index

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = \ln(1+x) \qquad |x|<1$$

put $x = \frac{1}{3}$, $\left(\frac{1}{3} < 1\right)$

$$\sum_{n=1}^{\infty} \left(-\tfrac{1}{3}\right)^n \cdot \frac{1}{n} = -\ln\left(1+\tfrac{1}{3}\right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n} = -\ln\left(\frac{4}{3}\right) = \ln\frac{3}{4}$$

Recall: $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x|<1;\quad e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!},\ x\in\mathbb{R};\quad \sin(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!},\ x\in\mathbb{R};\quad \cos(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!},\ x\in\mathbb{R}$