Quiz 5 Answer Key.

a) Write an Upper Riemann Sum that approximates area under the curve $\boxed{y = x^4+1}$ between -1 and 3 for $n=4$ with equal length partition

b) Same question but write Lower R. sum

c) Evaluate ~~under~~ area under curve $y = x^4+1$ from -1 to 3 by using integral

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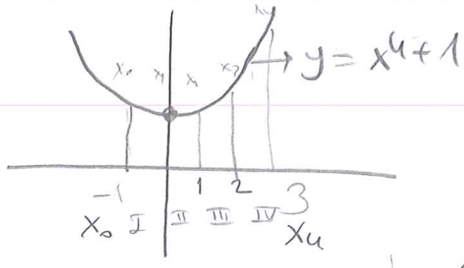
Solution:



$\star$ for upper sum we should pick $x_i^* \in [x_{i-1}, x_i]$ such that $f(x_i^*)$ must be max on $[x_{i-1}, x_i]$

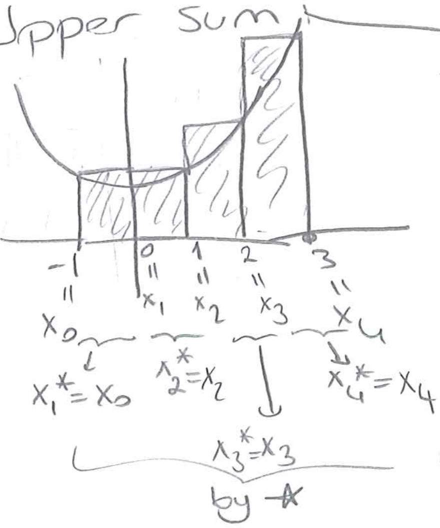
$$\left.\begin{aligned} x_0 &= -1 \\ x_4 &= 3 \end{aligned}\right\} \quad \Delta x = \frac{3-(-1)}{4} = \frac{4}{4} = 1$$

$$x_i = x_0 + i\Delta x = -1+i$$

$$x_1 = 1 \quad x_2 = 2 \quad x_3 = 3$$

$\star\star$ for lower sum we should pick $x_i^* \in [x_{i-1}, x_i]$ such that f takes its min at x_i^* on $[x_{i-1}, x_i]$

a) Upper sum



$$\sum_{i=1}^{4} f(x_i^*)\,\Delta x = f(x_0).\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

pick x_i^* so that f has max there on $[x_{i-1}, x_i]$

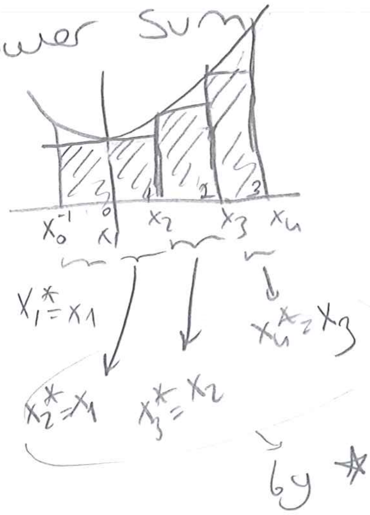
$$= f(-1).1 + f(1).1 + f(2).1 + f(3).1$$

$$= (1+1).1 + (1+1) + (16+1) + (81+1)$$

$$= 103 //$$

add up areas of rectangles below curve

b) Lower sum



$$\sum_{i=1}^{4} f(x_i^*)\Delta x = f(x_1)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

pick x_i^* so that f has min there on $[x_{i-1}, x_i]$

$$= f(0)\cdot 1 + f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1$$

$$= 1 + 1 + 2 + (16+1)$$

$$= 21 //$$

c) $$\int_{-1}^{3} (x^4+1)\,dx = \left.\frac{x^5}{5} + x\right|_{-1}^{3} = \frac{243}{5} + 3 - \left(-\frac{1}{5} - 1\right)$$

$$= \frac{258}{5} + \frac{6}{5} = \frac{264}{5}$$

Alternative Quiz 5 Answer Key.

a) Upper R. sum for $y = x^4$ from -1 to 3

b) Lower R. sum for $y = x^4$ from -1 to 3

c) evaluate area under $y = x^4$ from -1 to 3 by using integral

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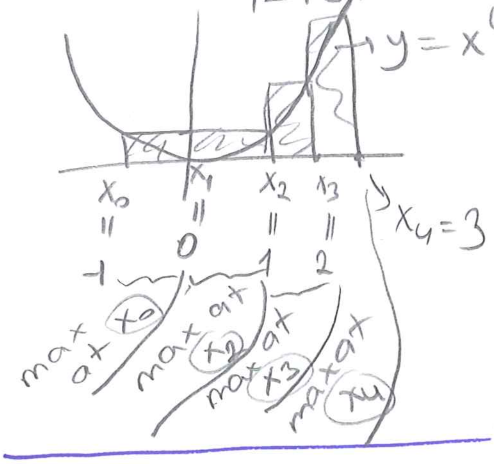
a) $\sum_{i=1}^{4} f(x_i^*) \cdot \Delta x = f(x_0)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$

If we pick x_i^* such that f takes its maximum at x_i^* on $[x_{i-1}, x_i]$ we get upper Riemann sum

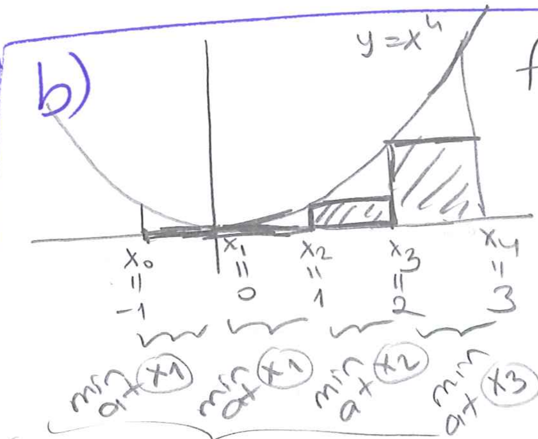
$\left(x_0 = -1 \quad x_4 = 4 \Rightarrow \Delta x = \frac{x_4 - x_0}{4} = \frac{4}{4} = 1 \right)$

$= f(-1) \cdot 1 + f(1)\, 1 + f(2) \cdot 1 + f(3) \cdot 1$

$= 1 + 1 + 16 + 81 = 99$ //



b)



$f(x_1)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$

$= f(0)\Delta x + f(0)\Delta x + f(1)\Delta x + f(2)\Delta x$

$= 0 \cdot 1 + 0 \cdot 1 + 1 \cdot 1 + 16 \cdot 1$

$= 17$ //

(for Lower sum pick x_i^* so that f takes its min at $x_i^* \in [x_{i-1}, x_i]$)

c) $\int_{-1}^{3} x^4 \, dx = \left. \frac{x^5}{5} \right|_{-1}^{3} = \frac{243}{5} - \left(-\frac{1}{5} \right) = \frac{244}{5}$ //

(observe that Lower Sum $\leq \int_{-1}^{3} x^4 dx \leq$ Upper Sum)