2018-2019 Fall MATH 117 QUIZ 4 Section 13-14 Friday 08.40-10.30

Surname Name and Student ID: KEY

1. Given the function y and its first derivative below

$$y = \frac{x^2-1}{x^2}, \quad y' = \frac{2}{x^3}$$

determine the intervals of increase and decrease and also asymptotes of the function y.

$y' = \frac{2}{x^3}$ $x=0 \Rightarrow y'$ undefined

		0	
y'	$-$		$+$
y	$\searrow$		$\nearrow$

y is decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$

$\lim_{x \to 0} \frac{x^2-1}{x^2} = -\infty$ $x=0$ is VA (If $\lim_{x \to a} f(x) = \infty$ (or $-\infty$) ($x \to a^+$ OR $x \to a^-$) $x=a$ is VA)

$\lim_{x \to \infty} \frac{x^2-1}{x^2} = 1$ ($x \to -\infty$) $y=1$ is H.A (If $\lim_{x \to \infty} f(x) = b$ (OR $x \to -\infty$) $y=b$ is HA)

2018-2019 Fall MATH 117 QUIZ 4 Section 23-24-31 Friday 10.40-12.30

Surname Name and Student ID: KEY.

1. Given the function y and its second derivative below

$$y = \frac{x^2-4}{x-1}, \quad y'' = \frac{-6}{(x-1)^3}$$

determine the concavity intervals and inflection points(if exists) of the function y. Find asymptotes of the graph of the function y.

$y'' = -\frac{6}{(x-1)^3}$ $x=1$ undefined

		1	
y''	+	ϕ	−
y	∪		∩

at $x=1$ concavity changes but $f(x)$ is not continuous so graph has no inflection.

y is concave up on $(-\infty, 1)$, concave down on $(1, \infty)$

$\lim_{x\to 1} \frac{x^2-4}{x-1} = \infty$ $x=1$ VA. (If $\lim_{x\to a} f(x) = \infty$ (or $-\infty$) (OR $x\to a^+$ OR $x \to a^-$) $x=a$ VA)

$\lim_{x\to\infty} \left(\frac{x^2-4}{x-1} - (x+1)\right) = \lim_{x\to\infty} -\frac{3}{x-1} = 0 \rightarrow y = x+1$ Oblique Asymptote

(
$x^2-4 \mid x-1$
$x^2-x \quad\ x+1$
$+x-4$
$x-1$
-3
$\quad \frac{x^2-4}{x-1} = x+1-\frac{3}{x-1}$)

(If $\lim_{x\to\infty} (f(x)-(ax+b)) = 0$ OR $(x\to-\infty)$ $y=ax+b$ is O.A)

2018-2019 Fall MATH 117 QUIZ 5 Section . . . Friday 10.40-12.30

Surname Name and Student ID: KEY

a) Evaluate $L(f, P_n)$ (Lower Riemann Sum) and $U(f, P_n)$ (Upper Riemann Sum) of $f(x)$ for $n = 4$ on $[0, 1]$ if $f(x)$ is a continuous, increasing, function defined on $\mathbb{R}$ such that $f(0) = 0$, $f(1/4) = 2$, $f(1/2) = 4$, $f(3/4) = 50$ and $f(1) = 120$.

b) Using part (a) show that $14 \le \int_0^1 f(x)dx \le 44$

! Note: We don't have any info about graph of f except f(x) is increasing (concavity, inflection etc) So we can not draw graph!

a) Solution: Riemann sum for $n=4$: $\sum_{i=1}^{4} f(x_i^*)\Delta x_i$

Since we are given $f(0), f(1/4), f(1/2), f(3/4), f(1)$ use this as candidate for our partition

$P_4 = \{0, 1/4, 1/2, 3/4, 1\}$ (x_0, x_1, x_2, x_3, x_4)

(0 — 1/4 — 1/2 — 3/4 — 1: 4 intervals)

$\Delta x_i = \Delta x = \frac{1}{4}$

$x_i = x_0 + i\Delta x_i = \frac{i}{4}$

For Upper sum: Since f is given increasing, pick $x_i^* \in [x_{i-1}, x_i]$ as $x_i^* = x_i$ (right end point of each subinterval) (since not in terms of i)

$U_f(P_n) = \sum_{i=1}^{4} f(x_i)\cdot\Delta x = \Delta x \sum_{i=1}^{4} f(x_i) = \Delta x \cdot \sum_{i=1}^{4} f(\frac{i}{4})$

$= \frac{1}{4}\cdot(f(\frac{1}{4}) + f(\frac{2}{4}) + f(\frac{3}{4}) + f(\frac{4}{4}))$

$= \frac{1}{4}\cdot(2 + 4 + 50 + 120) = \frac{176}{4} = 44$

For Lower Sum: Since f is increasing it takes its max on $[x_{i-1}, x_i]$ at the point x_{i-1} so pick x_i^* of $[x_{i-1}, x_i]$ as $x_i^* = x_{i-1}$ for all i

$L_f(P_n) = \sum_{i=1}^{4} f(x_{i-1})\cdot\Delta x = \Delta x \sum_{i=1}^{4} f(x_{i-1}) = \frac{1}{4}\cdot\sum_{i=1}^{4} f(\frac{i-1}{4})$

$= \frac{1}{4}(f(0) + f(\frac{1}{4}) + f(\frac{2}{4}) + f(\frac{3}{4})) = \frac{1}{4}(0+2+4+50) = 14$

b) $L(f, P_n) \le \int_a^b f(x) \le U(f, P_n)$ for any Partition. So by part (a) $14 \le \int_0^1 f(x)dx \le 44$

(Since f(x) is integrable (in this example f is continuous on [01] so integrable))

2018-2019 Fall MATH 117 QUIZ 5 Section . . . Friday 08.40-10.30

Surname Name and Student ID: Key

Evaluate $L(f, P_n)$ (Lower Riemann Sum) and $U(f, P_n)$ (Upper Riemann Sum) for $f(x) = \cos x$ for $n = 6$ on $[0, \pi]$.

$f'(x) = -\sin x < 0$ for $x \in [0,\pi]$

(so $f(x)$ is decreasing on $[0,\pi]$

a riemann sum for $n=6$: $\sum_{i=1}^{6} f(x_i^*) \cdot \Delta x_i$

$\Delta x_i = \Delta x = \frac{\pi - 0}{6} = \frac{\pi}{6}$

equal length partition

$x_i = x_0 + i\,\Delta x_i = \frac{i\pi}{6}$, $x_0 = 0$

Pick $x_i^* \in [x_{i-1}, x_i]$ s.t. $x_i^* = x_{i-1}$ for upper riemann sum because on that subinterval f takes its max there (since its decreasing)

$U(f, P_6) = \sum_{i=1}^{6} f(x_{i-1}) \cdot \Delta x$

$= \sum_{i=1}^{6} \cos\left(\frac{\pi(i-1)}{6}\right) \cdot \frac{\pi}{6} = \frac{\pi}{6} \cdot \sum_{i=1}^{6} \cos\left(\frac{\pi(i-1)}{6}\right)$

$= \frac{\pi}{6} \cdot \left(\cos 0 + \cos\frac{\pi}{6} + \cos\frac{2\pi}{6} + \cos\frac{3\pi}{6} + \cos\frac{4\pi}{6} + \cos\frac{5\pi}{6}\right)$

$= \frac{\pi}{6} \cdot \left(1 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0 + \left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\right) = \frac{\pi}{6}$

$\sum_{i=1}^{6} f(x_i^*) \Delta x_i$

Pick $x_i^* \in [x_{i-1}, x_i]$ s.t. $x_i^* = x_i$ for Lower Riemann Sum because on that subinterval f takes its min at x_i (since its decreasing)

$L(f, P_6) = \sum_{i=1}^{6} f(x_i) \Delta x = \sum_{i=1}^{m} \cos\left(\frac{\pi i}{6}\right) \frac{\pi}{6} = \frac{\pi}{6} \cdot \sum_{i=1}^{6} \cos\frac{\pi i}{6}$

$= \frac{\pi}{6} \cdot \left(\cos\frac{\pi}{6} + \cos\frac{2\pi}{6} + \cos\frac{3\pi}{6} + \cos\frac{4\pi}{6} + \cos\frac{5\pi}{6} + \cos\frac{6\pi}{6}\right) = \frac{\pi}{6} \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2} + 0 + \left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) + (-1)\right) = -\frac{\pi}{6}$