CVE 372 HYDROMECHANICS

OPEN CHANNEL FLOW I

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Overview

3.1 General Characteristics of Open Channel Flow
- 3.1.1 Classification of Open Channel Flows
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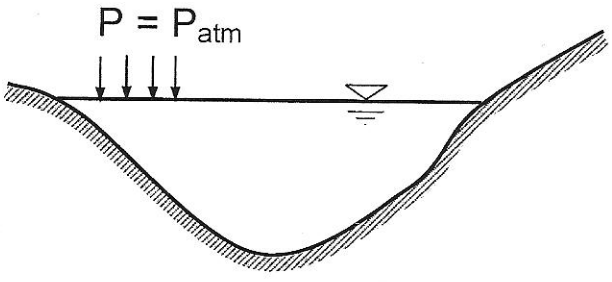
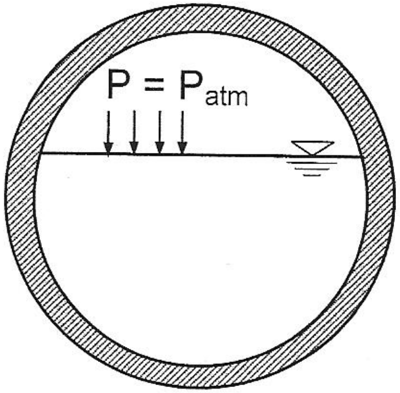
- 2.3.4 Friction Loss for Noncircular Conduits

3.2 Uniform Flow
- 3.2.1 Resistance in Open Channel Flow
- 3.2.2 Uniform Flow Equations
- 3.2.3 Composite and Compound Sections

3.3 Specific Energy Concept
- 3.3.1 Specific Energy and Alternate Depth
- 3.3.2 Critical Flow
- 3.3.3 Channel Transition and Chocking Problems

General Characteristics of Open Channel Flow



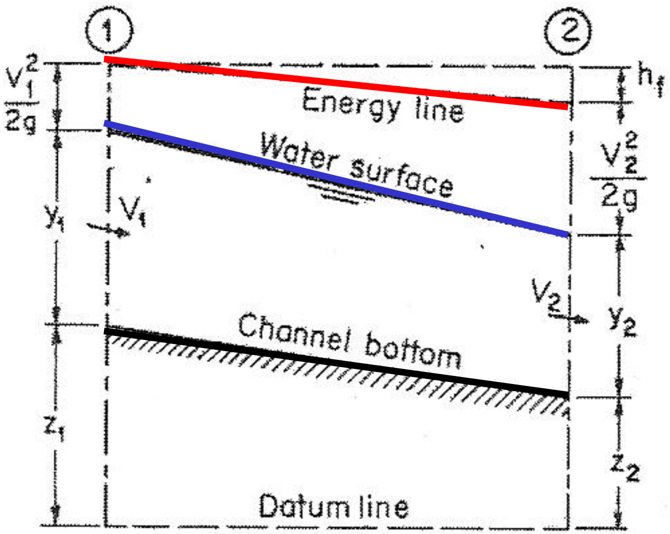


Open channel flow is a flow which has a free surface.

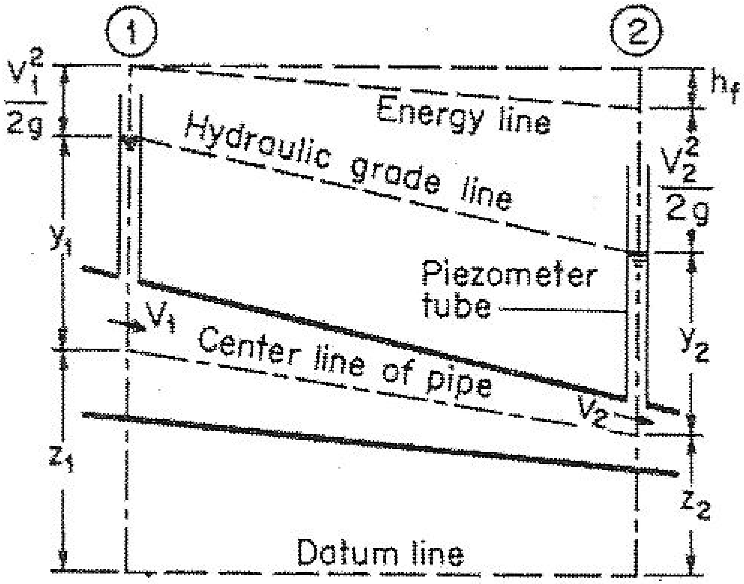
General Characteristics of Open Channel Flow

Comparison of Open Channel Flow and Pipe Flow

Open Channel Flow



Pipe Flow



S_o: channel bottom slope

S_w: slope of water surface

S_e: slope of energy grade line

General Characteristics of Open Channel Flow

Kinds of Open Channel Flow

An open channel is a conduit in which water flows with a free surface.

1. Canal is usually a long and mild-sloped channel built in the ground.

Kennet and Avon Canal, England

General Characteristics of Open Channel Flow

Kinds of Open Channel Flow

An open channel is a conduit in which water flows with a free surface.

1. Canal is usually a long and mild-sloped channel built in the ground.

Royal Canal, Ireland

Llangollen Canal, Denbighshire, Wales, UK

General Characteristics of Open Channel Flow

Kinds of Open Channel Flow

2. Flume is a channel usually supported on or above the surface of the ground to carry water across a depression.

Bull Run Hydroelectric Project diversion flume

White River diversion flume in Washington

General Characteristics of Open Channel Flow

Kinds of Open Channel Flow

3. Chute is a channel having steep slope.

4. Drop is similar to chute, but the change in elevation is affected in a short distance

5. Culvert is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.

6. Open-Flow Tunnel is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

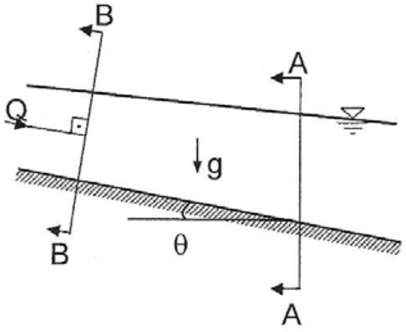
General Characteristics of Open Channel Flow

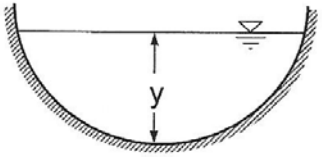
Channel Geometry

Prismatic Channel: A channel built with unvarying cross-section and constant bottom slope.

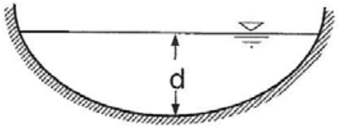
Non-prizmatic Channel: A channel bult with varying cross-section or bottom slope.

The Channel Section is the cross-section of a channel taken normal to the direction of the flow.





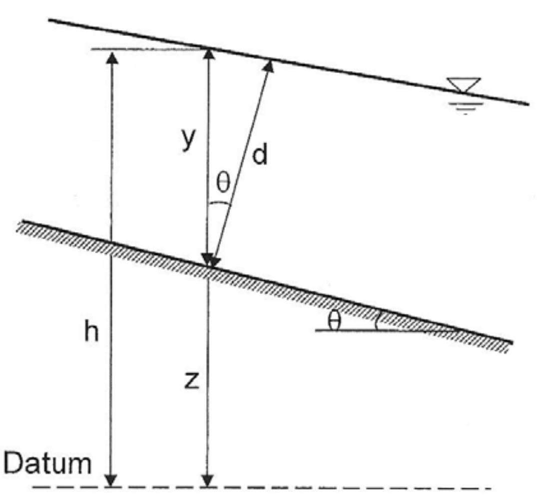
The vertical channel section (A-A)

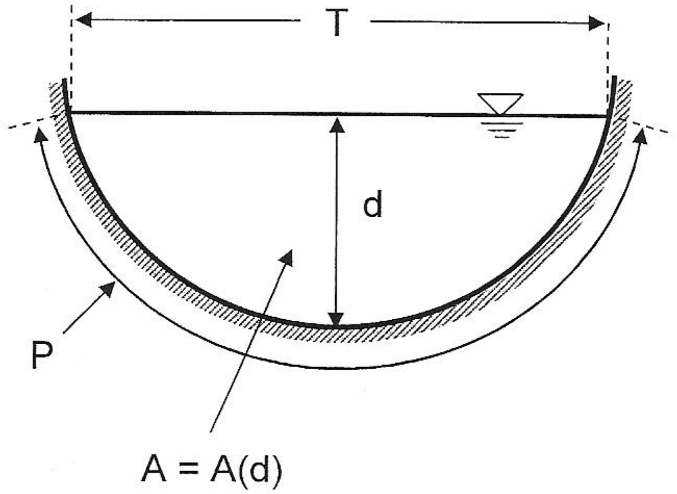


The channel section (B-B)

General Characteristics of Open Channel Flow

Geometric Elements of Channel Section





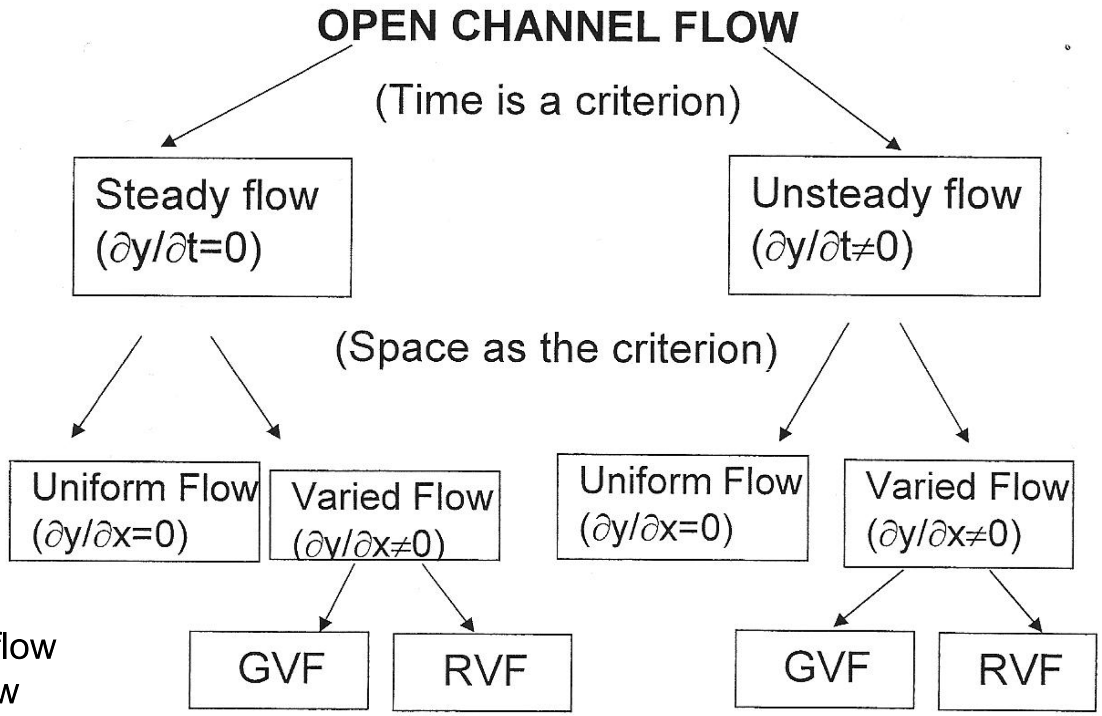
y: the depth of flow
d: the depth of flow section
h: the stage

T: the top width
A: the water area
P: the wetted perimeter
R (=A/P) the hydraulic radius
D (=A/T) the hydraulic depth

General Characteristics of Open Channel Flow

3.1.1 Classification of Open Channel Flows

According to the **change in flow depth** with respect to time and space



GVF: Gradually varied flow
RVF: Rapidly varied flow

General Characteristics of Open Channel Flow

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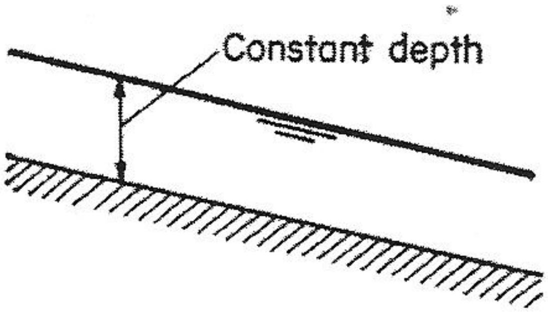
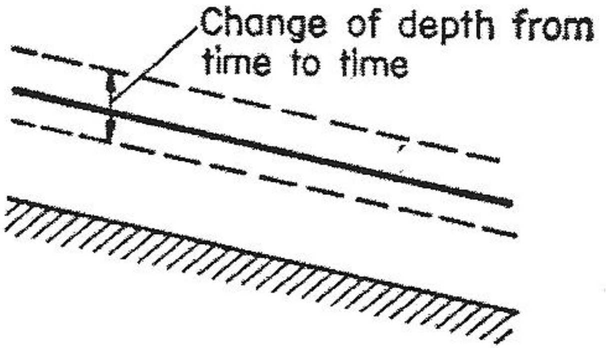


Uniform flow – Flow in a prismatic channel

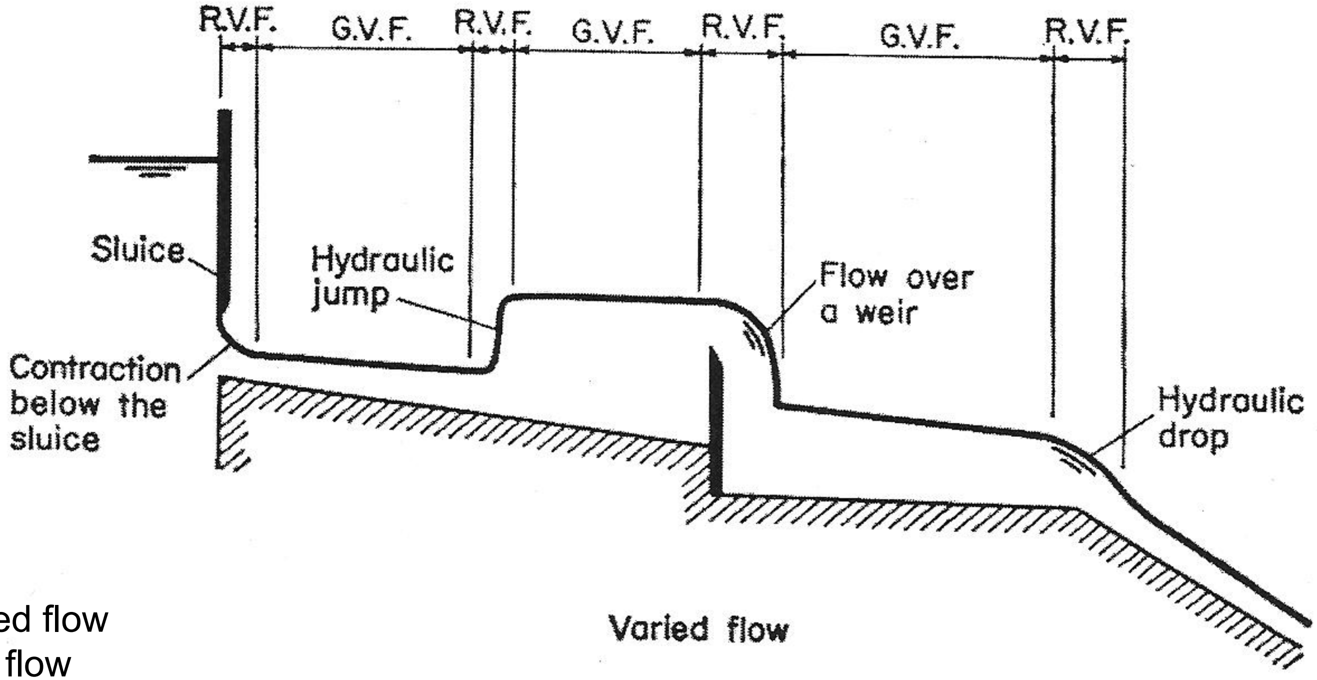


Unsteady uniform flow – Rare

General Characteristics of Open Channel Flow

3.1.1 Classification of Open Channel Flows

According to the **change in flow depth** with respect to time and space



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RVF: Rapidly varied flow

General Characteristics of Open Channel Flow

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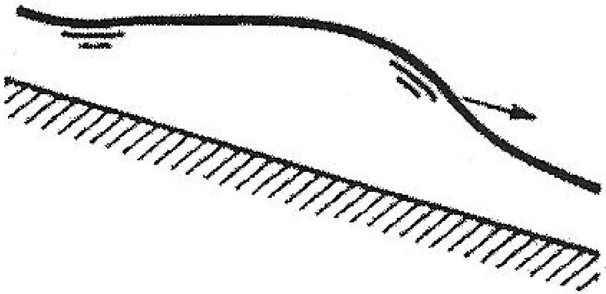
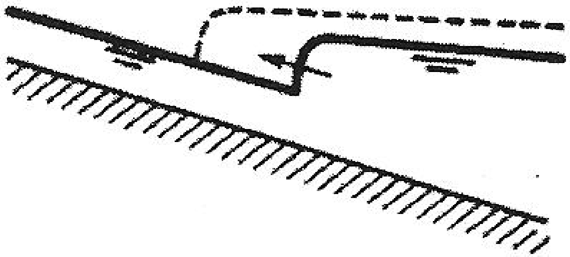


G.V.F. – Flood wave



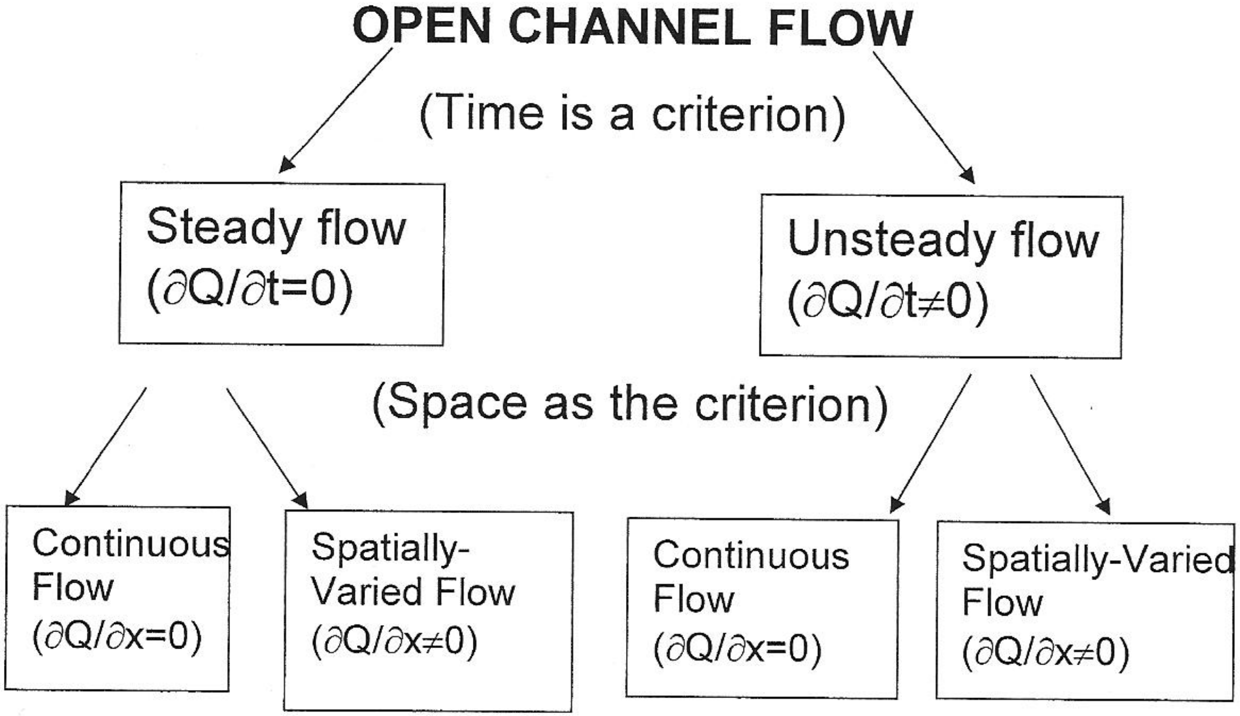
R.V.F. – Bore

GVF: Gradually varied flow
RVF: Rapidly varied flow

General Characteristics of Open Channel Flow

3.1.1 Classification of Open Channel Flows

According to the **change in discharge** with respect to time and space



OPEN CHANNEL FLOW

(Time is a criterion)

Steady flow ($\partial Q/\partial t=0$)

Unsteady flow ($\partial Q/\partial t\neq 0$)

(Space as the criterion)

Continuous Flow ($\partial Q/\partial x=0$)

Spatially-Varied Flow ($\partial Q/\partial x\neq 0$)

Continuous Flow ($\partial Q/\partial x=0$)

Spatially-Varied Flow ($\partial Q/\partial x\neq 0$)

General Characteristics of Open Channel Flow

3.1.1 Classification of Open Channel Flows

State of Flow

Effect of Viscosity

According to the **Reynolds Number**, $Re = (VR)/\nu$

$$\text{IF}\quad \begin{cases} Re < 500 & \rightarrow \text{Laminar Flow} \\ 500 < Re < 1000 & \rightarrow \text{Transitional Flow} \\ Re > 1000 & \rightarrow \text{Turbulent Flow} \end{cases}$$

General Characteristics of Open Channel Flow

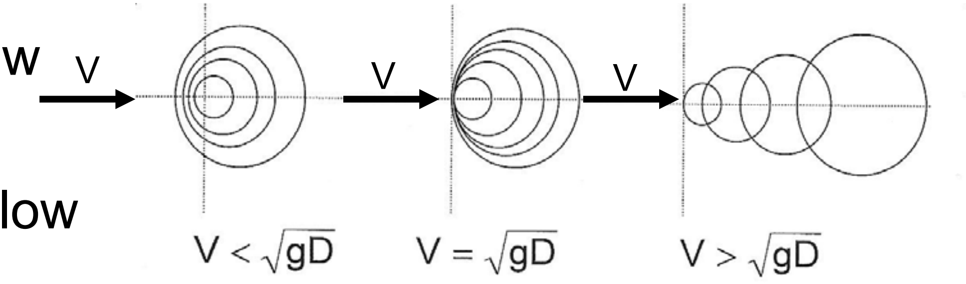
3.1.1 Classification of Open Channel Flows

State of Flow

Effect of Gravity

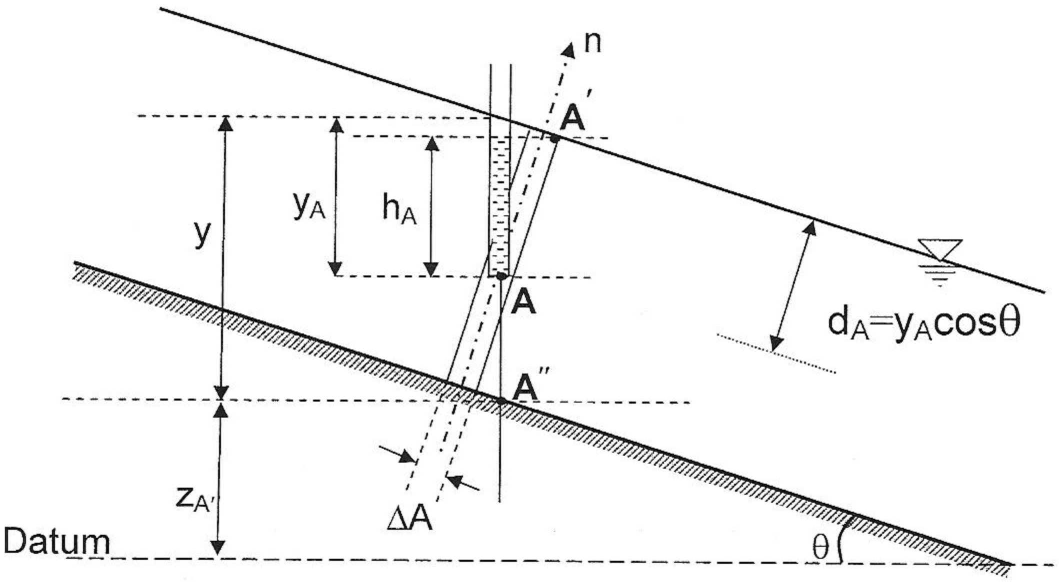
According to the **Froude Number**, Fr= $V/(gD)^{1/2}$

IF
- $Fr < 1$ → Subcritical Flow
- $Fr = 1$ → Critical Flow
- $Fr > 1$ → Supercritical Flow



General Characteristics of Open Channel Flow

3.1.2 Pressure Distribution in Open Channel Flows

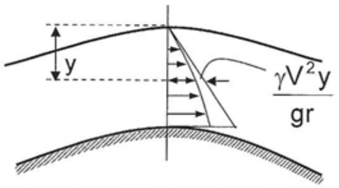


$$\sum Fn = 0, \qquad p_A \Delta A - \gamma d_A \Delta A \cos\theta = 0$$

$$p_A = \gamma d_A \cos\theta, \qquad d_A = y_A \cos\theta$$

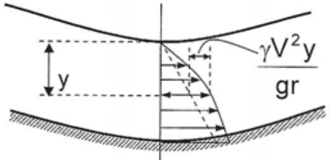
$$p_A = \gamma y_A \cos^2\theta, \qquad \text{If } \theta \text{ is small} \Rightarrow p = \gamma y$$

Hydrostatic Pressure Distribution



$$p = \gamma y\left(1 - \frac{V^2}{gr}\right)$$

Convex Flow



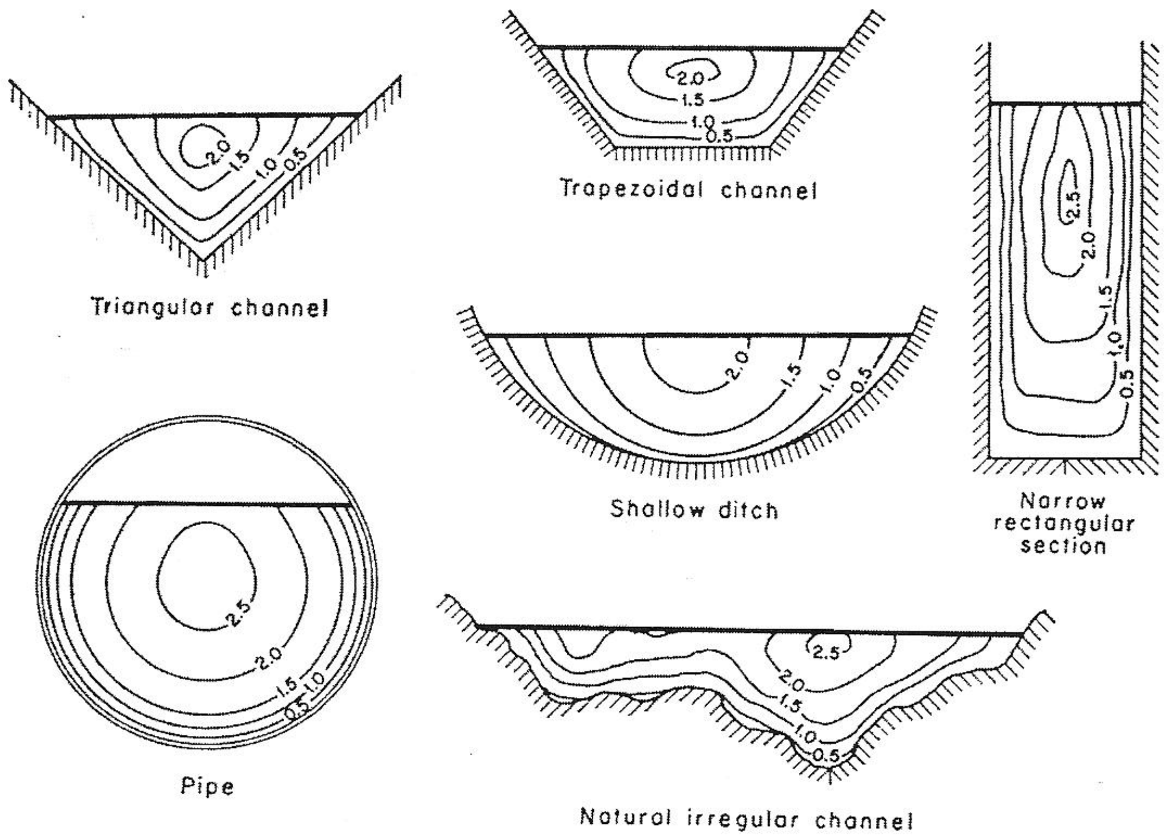
$$p = \gamma y\left(1 + \frac{V^2}{gr}\right)$$

Concave Flow

r: radius of curvature

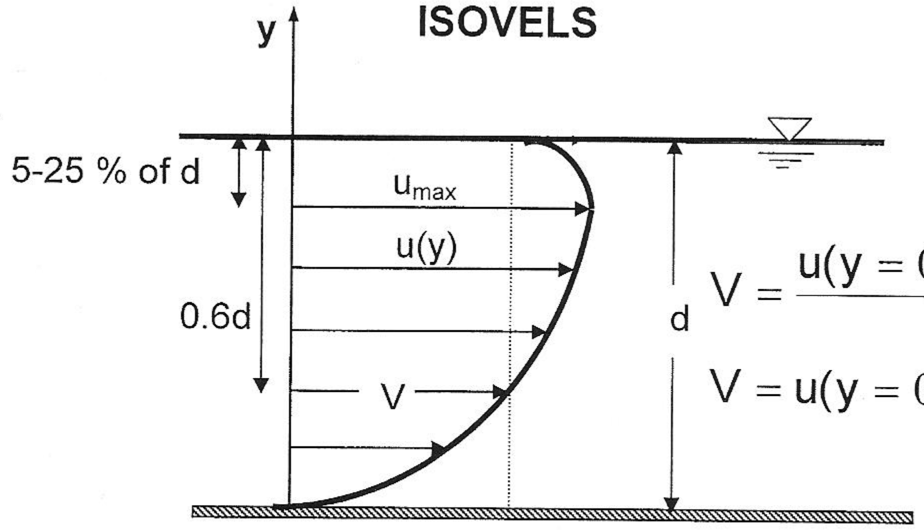
General Characteristics of Open Channel Flow

3.1.3 Velocity Distribution in Open Channel Flows



General Characteristics of Open Channel Flow

3.1.3 Velocity Distribution in Open Channel Flows



$$V = \frac{u(y = 0.2d) + u(y = 0.8d)}{2}$$

$$V = u(y = 0.6d)$$

$$\alpha = \frac{\int_A u^3 dA}{u^3 A} \approx \frac{\sum u^3 \Delta A}{u^3 A}, \qquad \beta = \frac{\int_A u^2 dA}{u^2 A} \approx \frac{\sum u^2 \Delta A}{u^2 A}$$

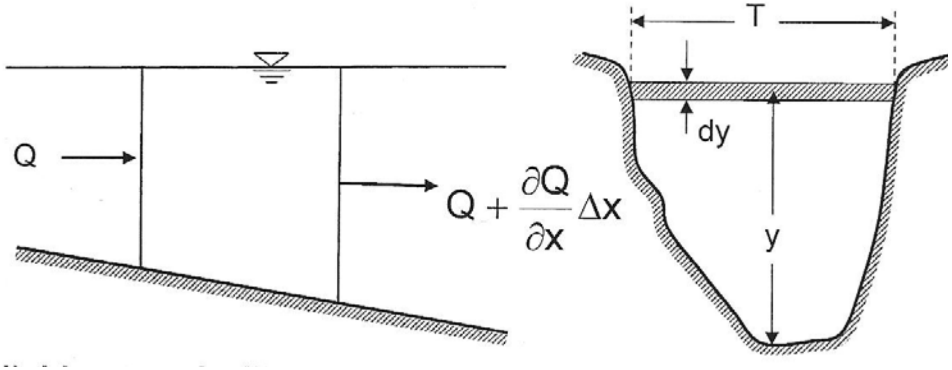
α: energy correction factor β: momentum correction factor

General Characteristics of Open Channel Flow

3.1.3 Velocity Distribution in Open Channel Flows

Equation of Continuity

$$\int_{cs} \rho \vec{u} \cdot d\vec{A} = -\frac{\partial}{\partial t} \int_{cv} \rho \, d\forall$$



i) Unsteady Flow

$$\rho \int_{cs} \vec{u} \cdot d\vec{A} = -\rho \frac{\partial}{\partial t} \int_{cv} d\forall \quad \Rightarrow \quad \int_{cs} \vec{u} \cdot d\vec{A} = -\frac{\partial}{\partial t} \forall_{cv}$$

(incompressible flow)

$$-Q + \left(Q + \frac{\partial Q}{\partial x}\Delta x\right) = -\frac{\partial}{\partial t}(A \cdot \Delta x) \quad \Rightarrow \quad \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$A = A(y), y = y(x,t)$$

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial y}\frac{\partial y}{\partial t} \qquad dA = T dy \qquad \frac{dA}{dy} = T$$

$$\frac{\partial A}{\partial t} = T\frac{\partial y}{\partial t} \qquad \Rightarrow \boxed{T\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = 0}$$

ii) Steady Flow

$$\frac{\partial y}{\partial t} = 0 \quad \Rightarrow \frac{\partial Q}{\partial x} = 0$$

$$\boxed{Q_1 = Q_2 \quad \Rightarrow (uA)_1 = (uA)_2}$$

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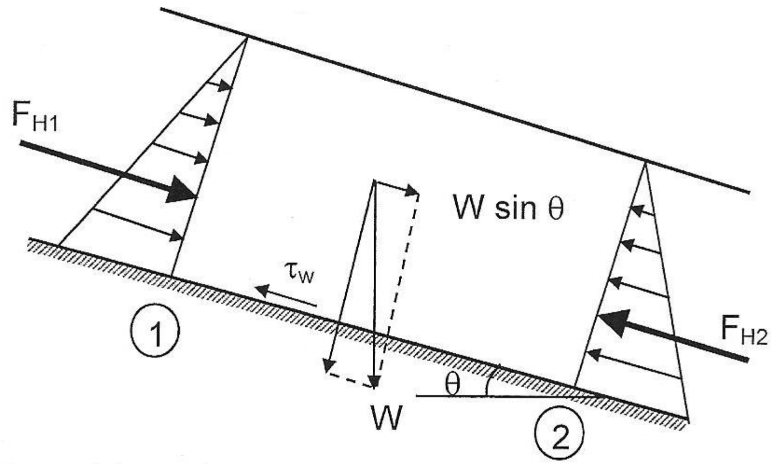
Uniform Flow

3.2.1 Resistance in Open Channel Flow



Momentum Eqn.

$$F_{H1} - F_{H2} - \tau_w PL + W\sin\theta = \rho\, Q\,(V_2 - V_1)$$

$$F_{H1} = F_{H2}\,;\; V_2 = V_1$$

$$W\sin\theta = \tau_w PL$$

$$W = \gamma AL \text{ and } \sin\theta \cong S_0$$

$$\boxed{\tau_w = \gamma R S_0}$$ Resistance formula for uniform flow

τ_w is the resistance to the flow (shear stress) and

assume $\tau_w = \frac{f}{8}\rho V^2$

$$V = \sqrt{\frac{8\gamma}{f\rho}}\sqrt{RS_0}$$

$$\boxed{V = C\sqrt{RS_0}} \text{ where } C = \sqrt{\frac{8g}{f}}$$

- Water depth is constant
- Discharge is constant
- Steady flow
- $S_o = S_w = S_e$

y_n: normal (uniform) depth

y_n: The depth associated with uniform flow.

Uniform Flow

3.2.2 Uniform Flow Equations

Chézy Equation : $V = C\sqrt{R\,S_0}$

Manning Equation : $V = \dfrac{1}{n} R^{2/3} \sqrt{S_0}$

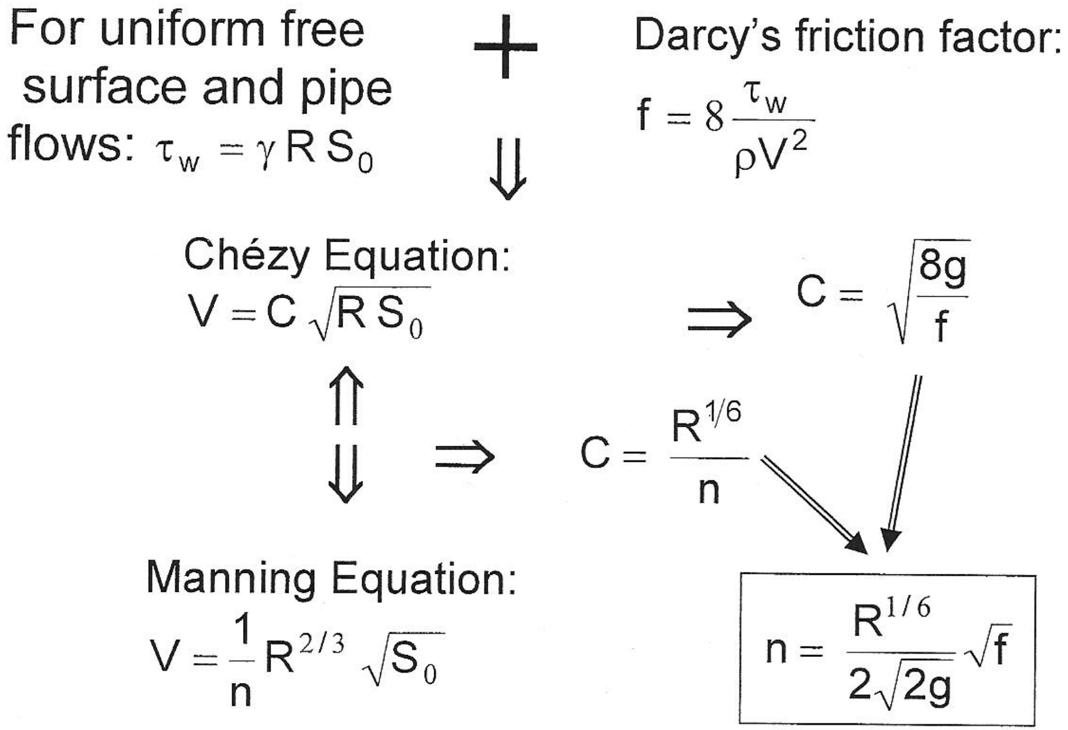
(R is in m, V is in m/s)

C: Chezy coefficient.
n: Manning’s roughness coefficient

Uniform Flow

3.2.2 Uniform Flow Equations

Resistance Coefficients



For uniform free surface and pipe flows: $\tau_w = \gamma R S_0$

$+$

Darcy's friction factor: $f = 8\frac{\tau_w}{\rho V^2}$

$\Downarrow$

Chézy Equation: $V = C\sqrt{R S_0}$ $\Rightarrow$ $C = \sqrt{\frac{8g}{f}}$

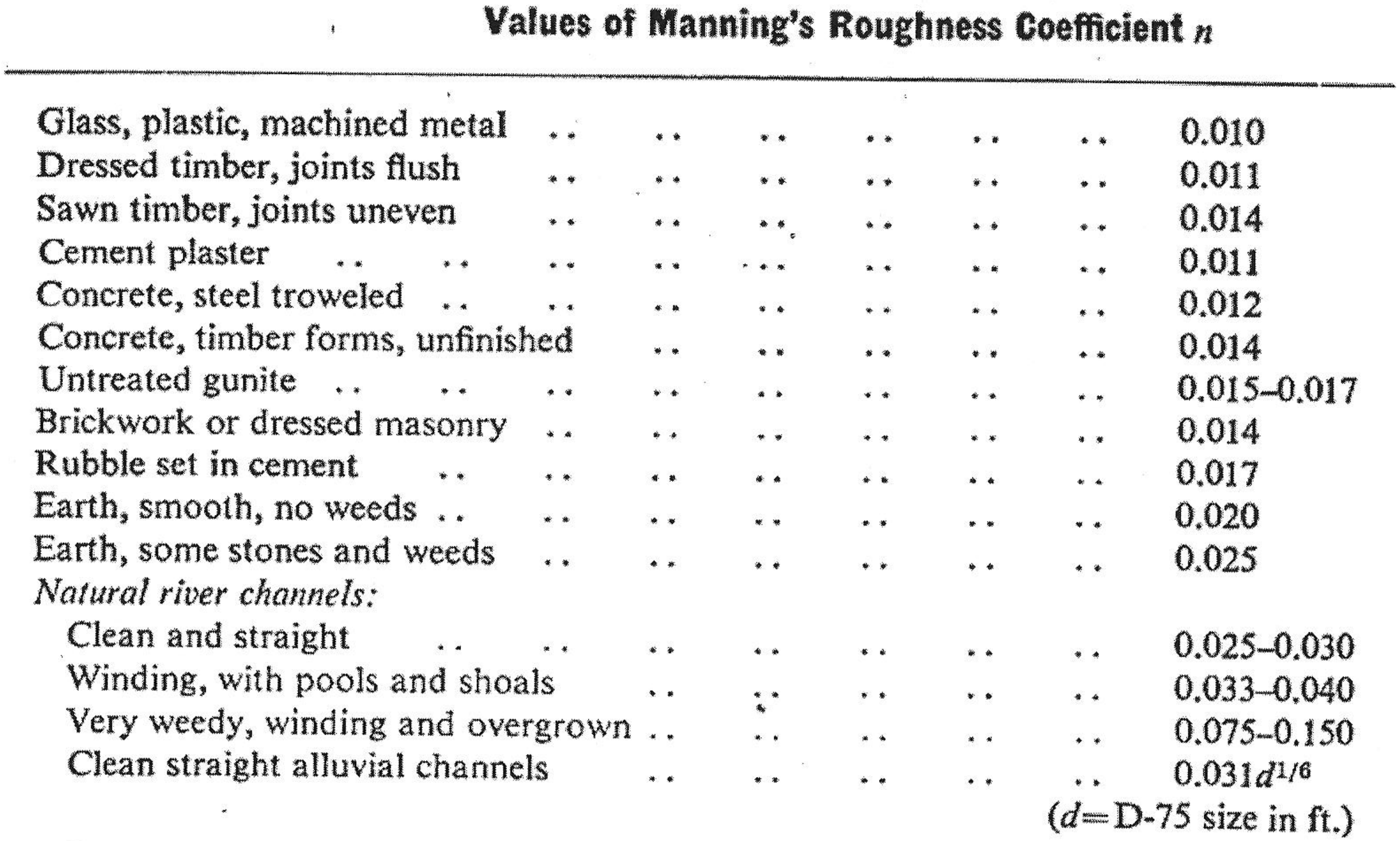
$\Updownarrow$ $\Rightarrow$ $C = \frac{R^{1/6}}{n}$

Manning Equation: $V = \frac{1}{n} R^{2/3} \sqrt{S_0}$

$$\boxed{n = \frac{R^{1/6}}{2\sqrt{2g}}\sqrt{f}}$$

Uniform Flow

3.2.2 Uniform Flow Equations



Values of Manning's Roughness Coefficient n

Glass, plastic, machined metal	0.010
Dressed timber, joints flush	0.011
Sawn timber, joints uneven	0.014
Cement plaster	0.011
Concrete, steel troweled	0.012
Concrete, timber forms, unfinished	0.014
Untreated gunite	0.015–0.017
Brickwork or dressed masonry	0.014
Rubble set in cement	0.017
Earth, smooth, no weeds	0.020
Earth, some stones and weeds	0.025
Natural river channels:	
Clean and straight	0.025–0.030
Winding, with pools and shoals	0.033–0.040
Very weedy, winding and overgrown	0.075–0.150
Clean straight alluvial channels	$0.031d^{1/6}$ (d=D-75 size in ft.)

Uniform Flow

3.2.2 Uniform Flow Equations

Factors Affecting Manning's Roughness Coefficient

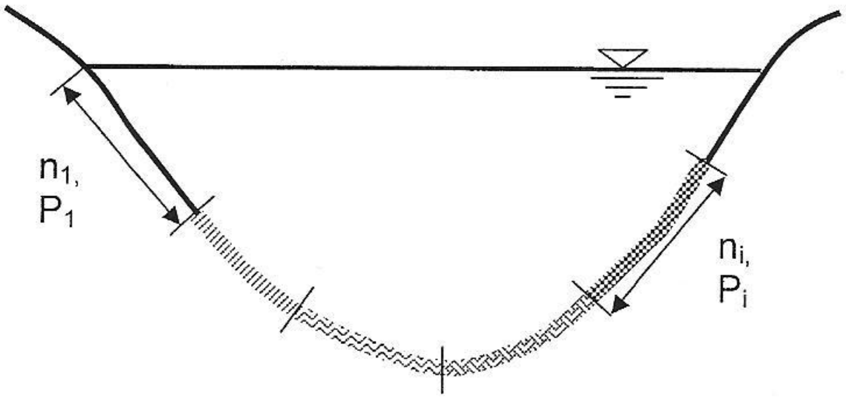
- Surface roughness
- Vegetation
- Channel irregularity
- Channel alignment
- Silting and scouring
- Obstructions
- Size and shape of channel
- Suspended material, bed load.

Uniform Flow

3.2.3 Composite and Compound Sections

Composite Section:

A channel section, which is composed of different roughness along the wetted perimeter.



$$n_{eq} = \sqrt{\frac{\sum n_i^2 P_i}{\sum P_i}}, \quad \left(\begin{array}{c} \text{Pavlovski's eq.} \\ F = \sum_{i=1}^{n} F_i \end{array} \right), \quad Q = \frac{A}{n_{eq}} R^{2/3} \sqrt{S_0}$$

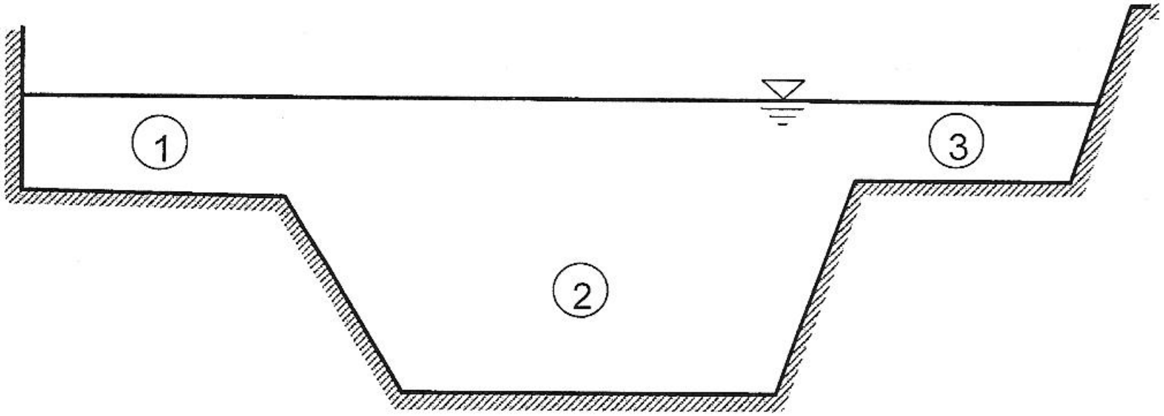
F: Drag force

Uniform Flow

3.2.3 Composite and Compound Sections

Compound Section:

A channel section, which the cross section is composed of several distinct subsections.



$$Q = \sum Q_i = \sum \frac{A_i}{n_i} R_i^{2/3} \sqrt{S_0}$$

Uniform Flow

3.2.3 Composite and Compound Sections

Example 1: A trapezoidal channel has a base width b = 6 m and side slopes 1H:1V. The channel bottom slope is S_o=0.0002 and the Manning roughness coefficient is n=0.014. compute

a) the depth of uniform flow if Q = 12.1 m3/s

b) the state of flow

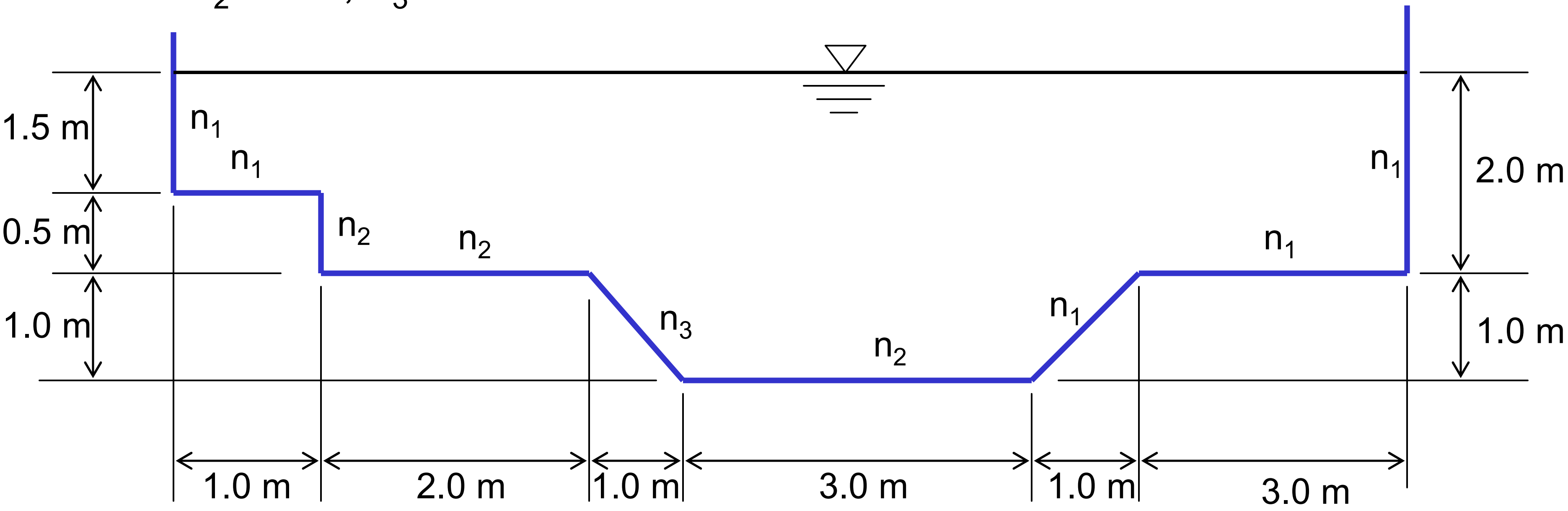
c) the average wall-shear stress along the wetted perimeter.

Solved in the class

Uniform Flow

3.2.3 Composite and Compound Sections

Example 2: For the compound channel given below, determine the total discharge and the state of the flow, if $S_0=0.0009$, $n_1=0.014$, $n_2=0.02$, $n_3=0.025$

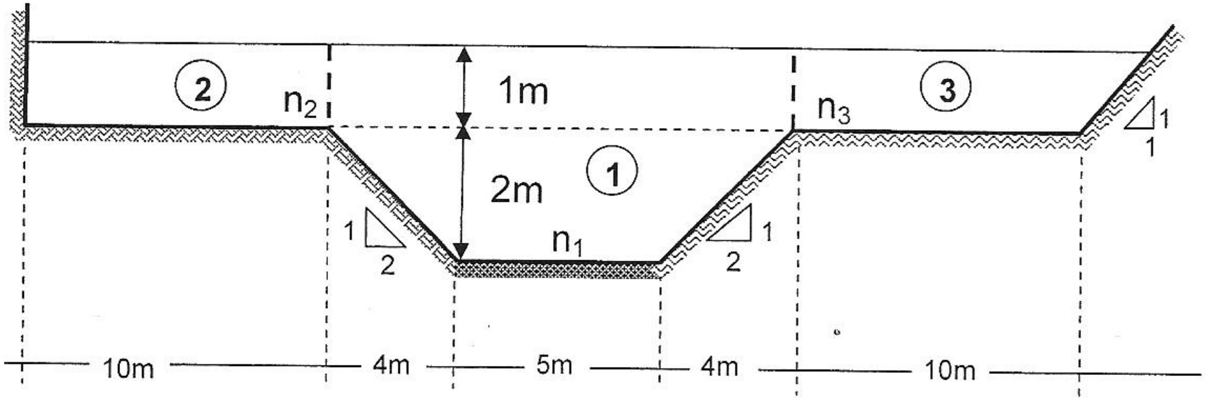


Solved in the class

Uniform Flow

3.2.3 Composite and Compound Sections

Exercise 1: Determine the discharge passing through the cross section of the compound channel shown below. The Manning roughness coefficients are $n_1 = 0.02$, $n_2 = 0.03$ and, $n_3 = 0.04$. The channel bed slope for the whole channel is $S_o = 0.008$.



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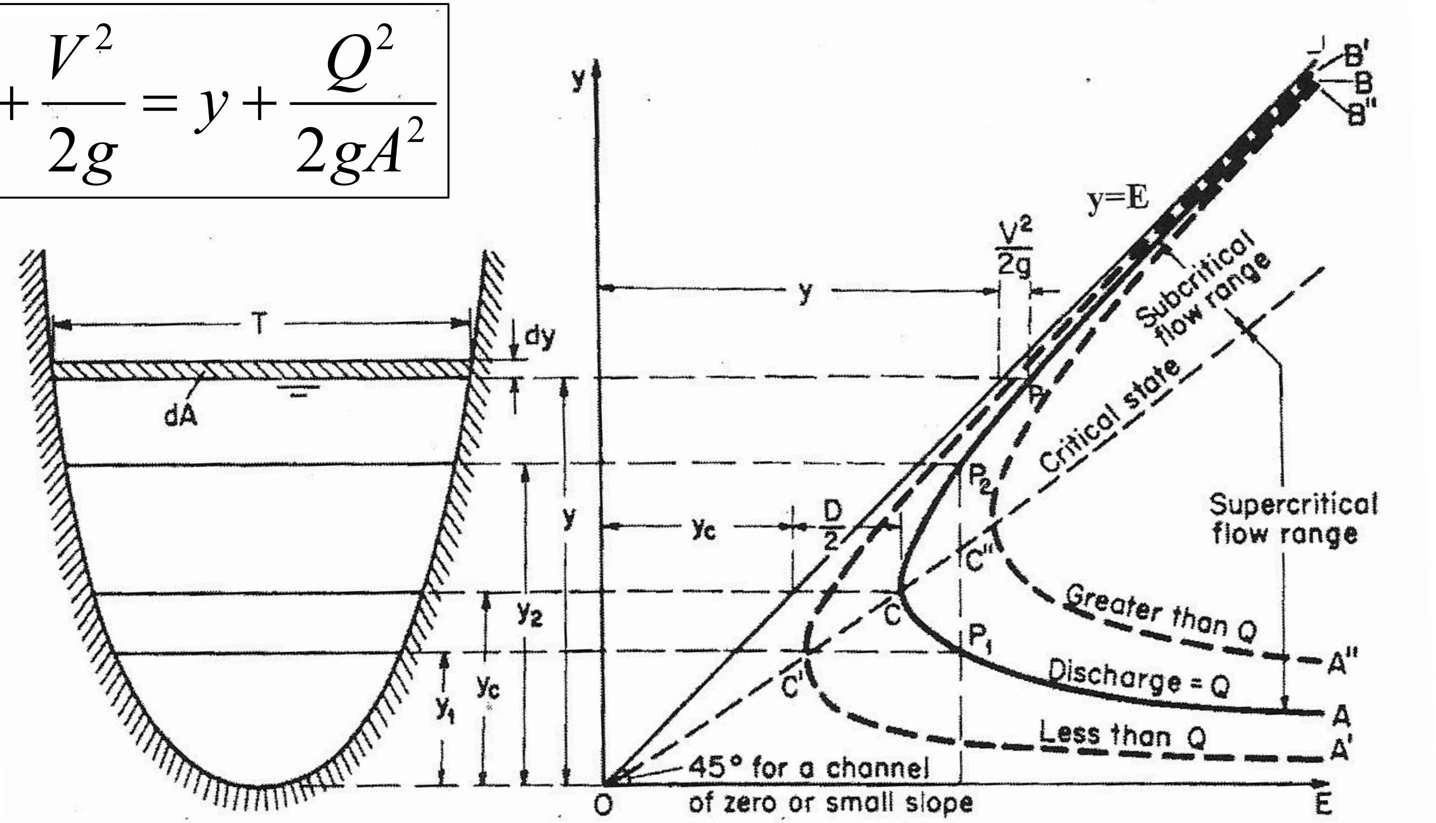
Specific Energy Concept

3.3.1 Specific Energy and Alternate Depth

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

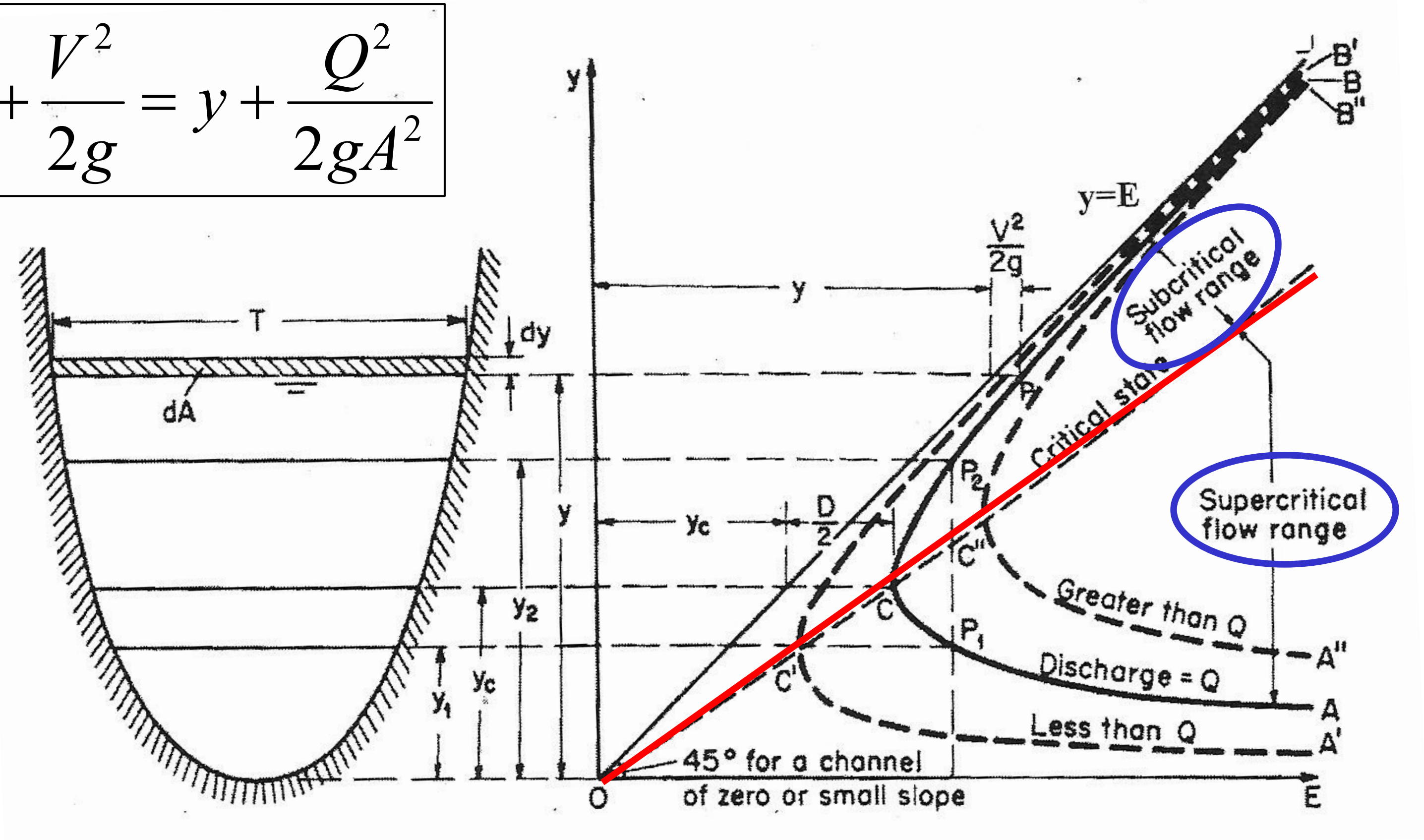


Specific Energy Curve

Specific Energy Concept

3.3.2 Critical Flow

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$



Specific Energy Curve

Specific Energy Concept

3.3.2 Critical Flow

Arbitrary Cross Section	Rectangular Cross Section
$F_r = 1 \rightarrow \frac{Q^2}{g} = \frac{A_c^3}{T_c}$	$F_r = 1 \quad q^2 = g y_c^3 \quad y_c = \sqrt[3]{\frac{q^2}{g}}$
$\frac{V_c^2}{2g} = \frac{D_c}{2} \rightarrow E_c = y_c + \frac{D_c}{2}$	$\frac{V_c^2}{2g} = \frac{y_c}{2} \rightarrow E_c = \frac{3}{2} y_c$
For a given Q, $E = E_{min}$	For a given q, $E = E_{min}$
For a given specific energy, E_o, $Q = Q_{max}$	For a given specific energy E_o, $q = q_{max}$

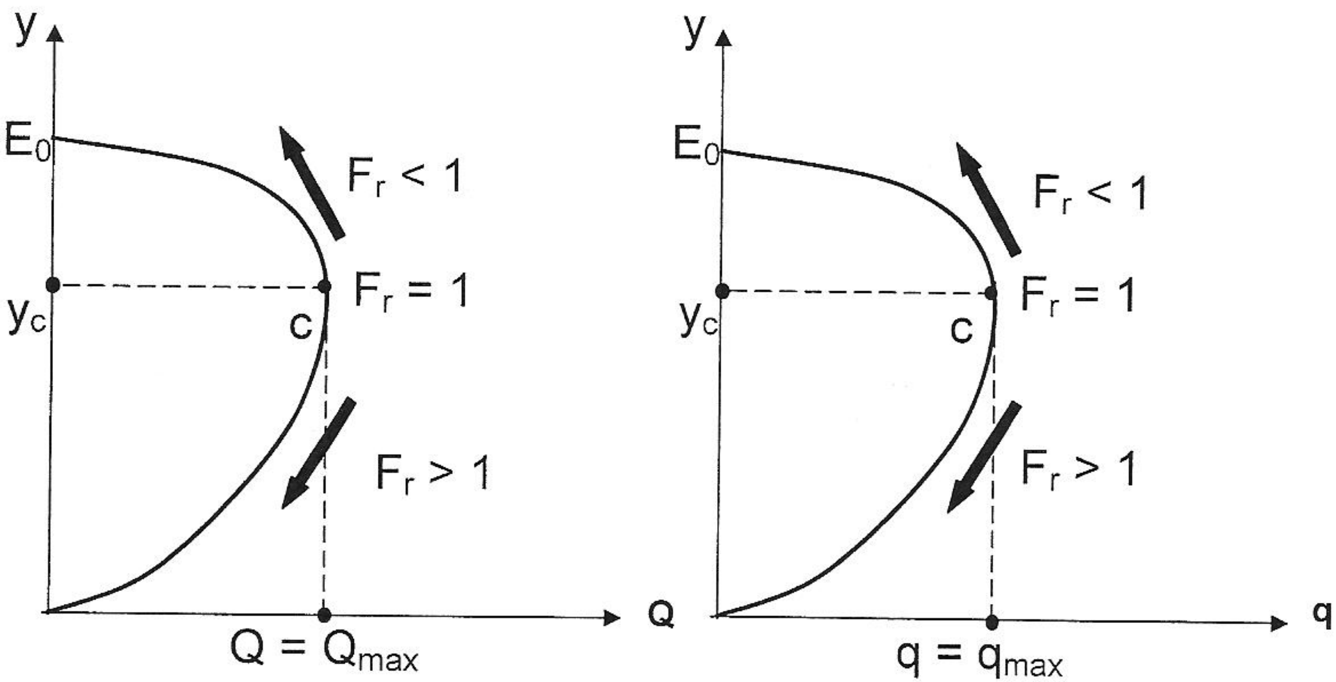
q=Q/T (discharge per unit width)

Specific Energy Concept

3.3.2 Critical Flow

For a given E, $Q = [2gA^2(E-y)]^{1/2}$,

Q=Q(y) [q=q(y) for rectangular channel



The plot of Q vs y (q vs y) gives Koch Parabola

Specific Energy Concept

3.3.2 Critical Flow

Example 3:

A trapezoidal channel is given with b=4 m, n=0.0143, and z=0.5.

a) What is the depth and state of the flow to carry Q=15m^3/s with S_0=0.0009

Specific Energy Concept

3.3.2 Critical Flow

Example 4:

Discharge in an open channel of arbitrary cross-section is $Q=1m^3/s$. Determine:

a) The critical depth for a triangular channel having side slopes of 1:1

b) The critical depth for a semicircular channel

c) The critical depth for a rectangular channel with b=2 m

d) The alternate depth for part c if y=1 m

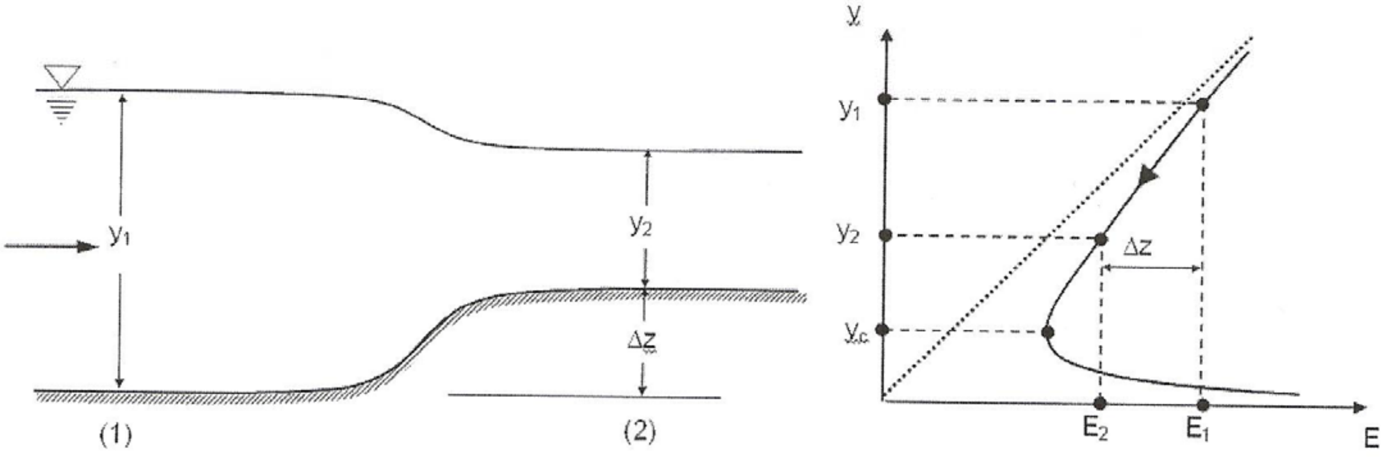
Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

CHANNEL TRANSITIONS

A.) Upward step (constant width)

1) Subcritical Flow

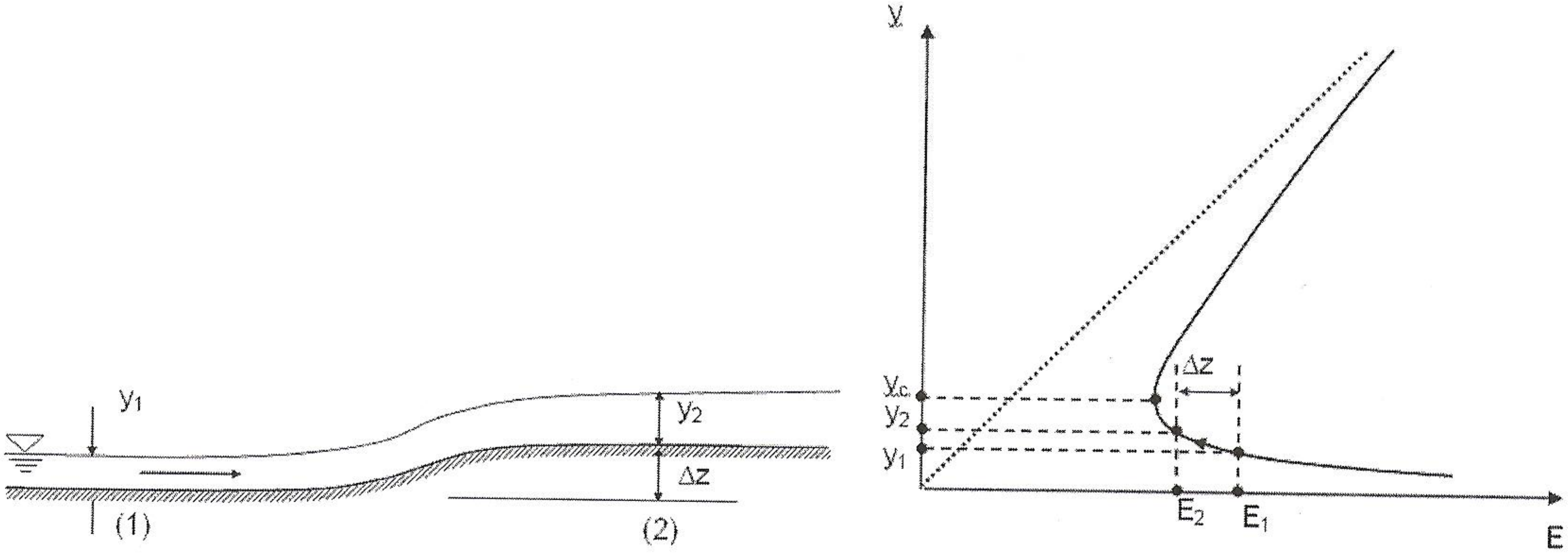


$E_1 = \Delta z + E_2$

Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

2) Supercritical Flow



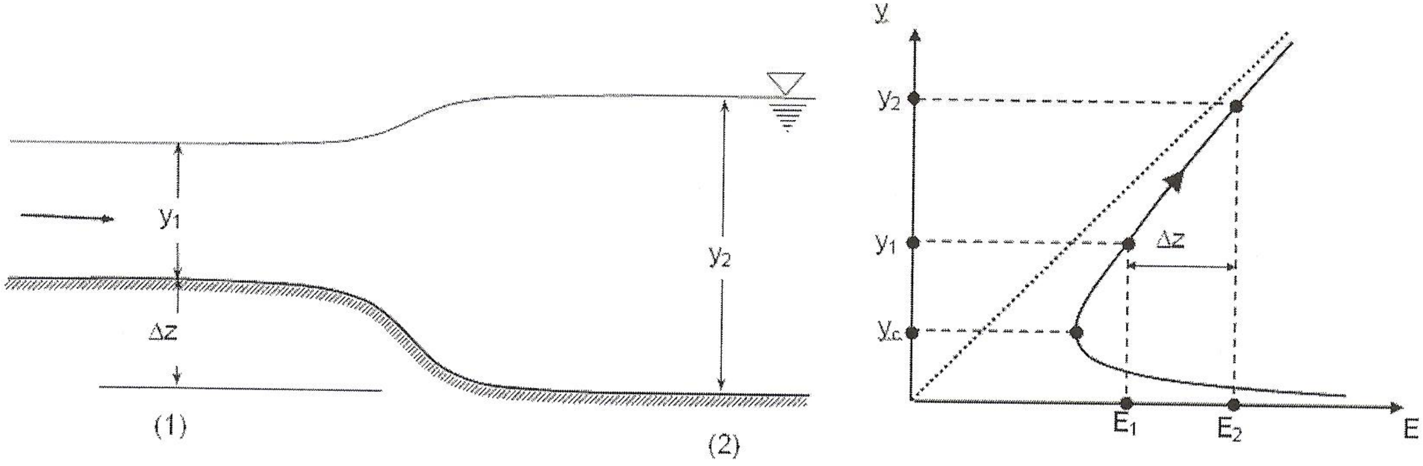
$E_1 = \Delta z + E_2$

Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

B.) Downward Step (Constant Width)

1) Subcritical Flow

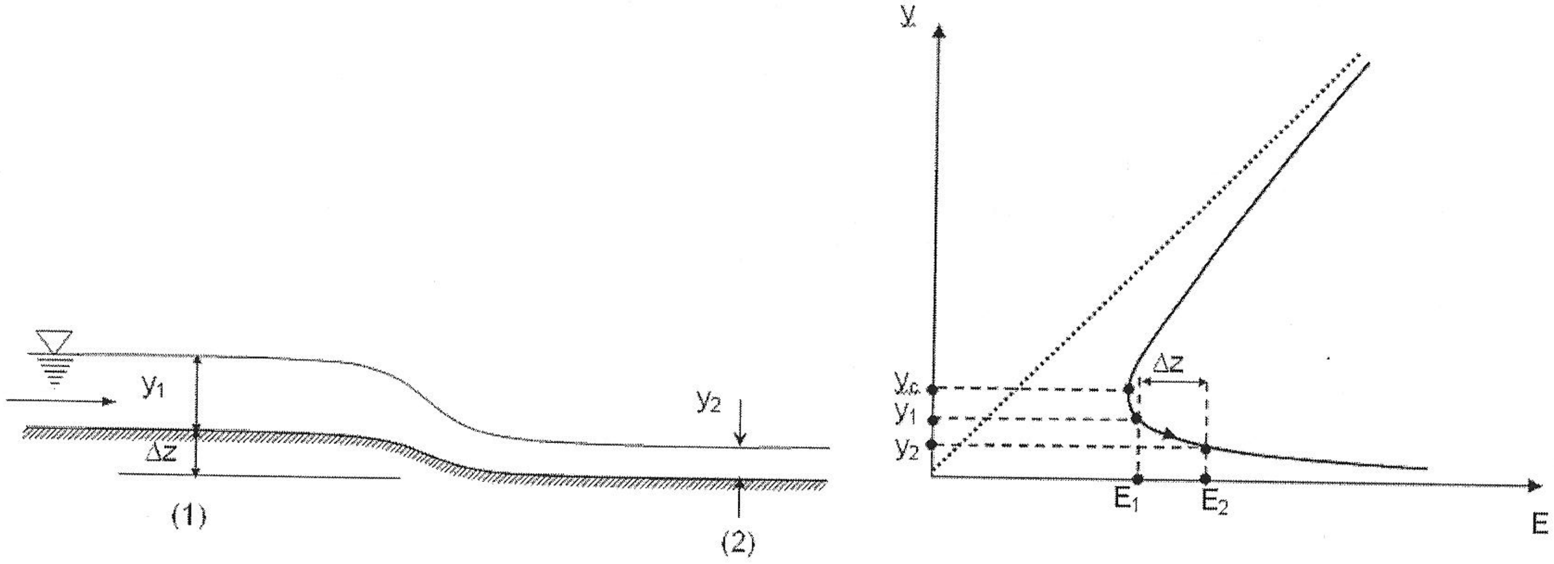


$E_1+\Delta z=E_2$

Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

2) Supercritical Flow

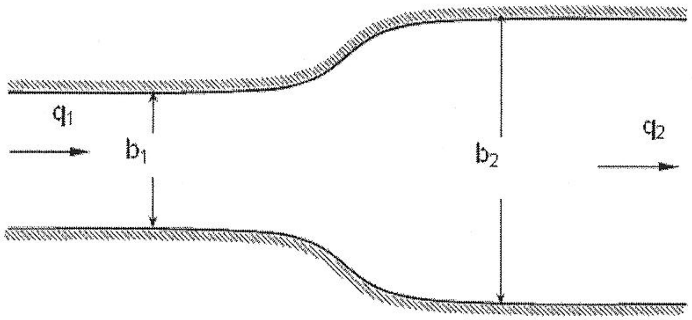


$E_1+\Delta z=E_2$

Specific Energy Concept

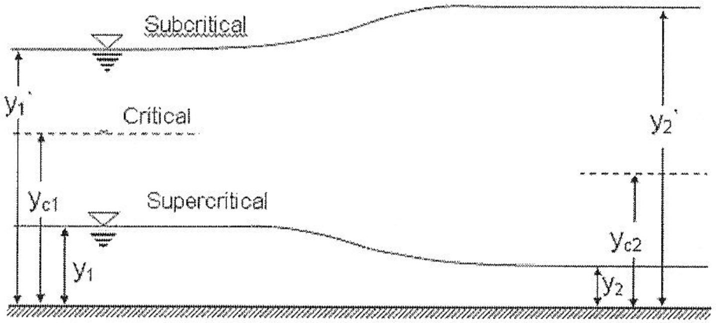
3.3.3 Channel Transition and Choking Problems

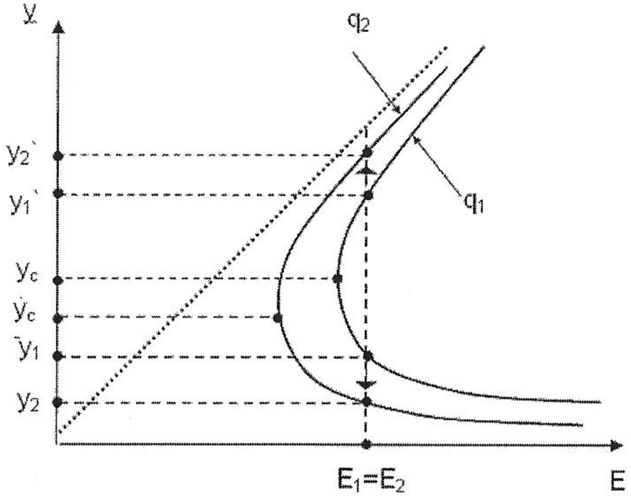
C.) Channel Expansion (Constant Bed Elevation)



$q_1 = Q/b_1$

$q_2 = Q/b_2$

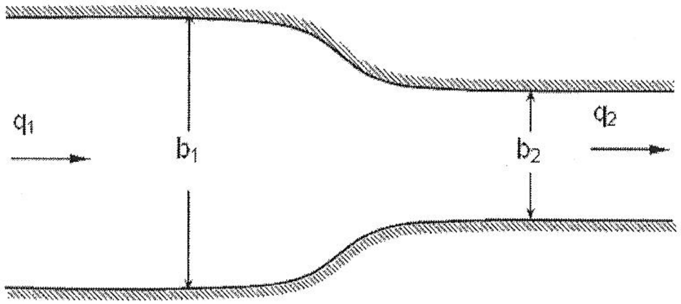




Specific Energy Concept

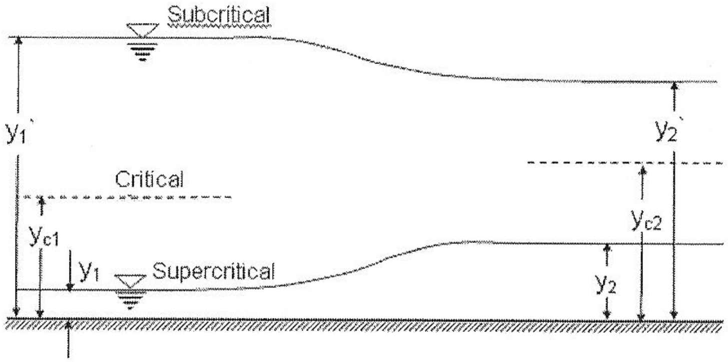
3.3.3 Channel Transition and Choking Problems

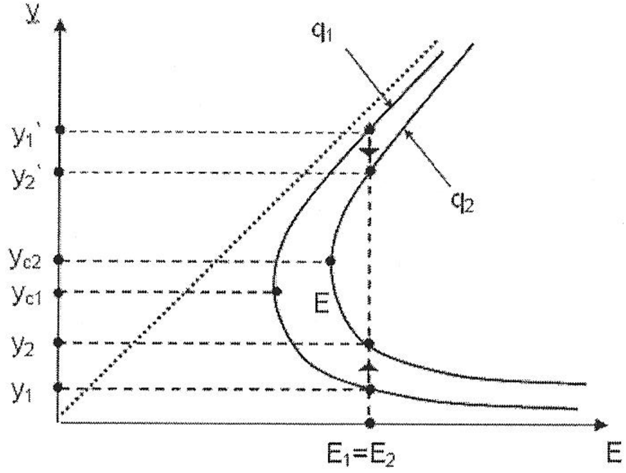
D.) Channel Contraction (Constant Bed Elevation)



$q_1 = Q/b_1$

$q_2 = Q/b_2$



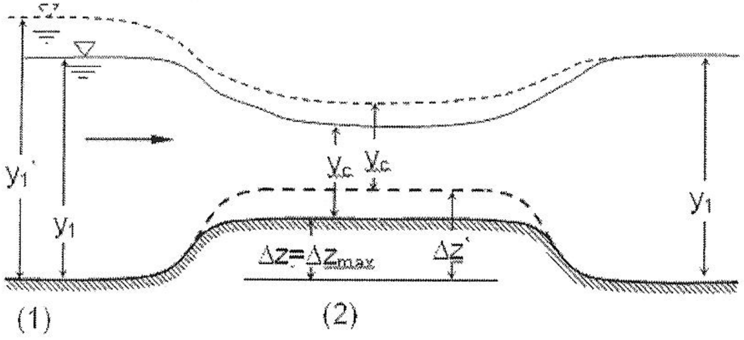


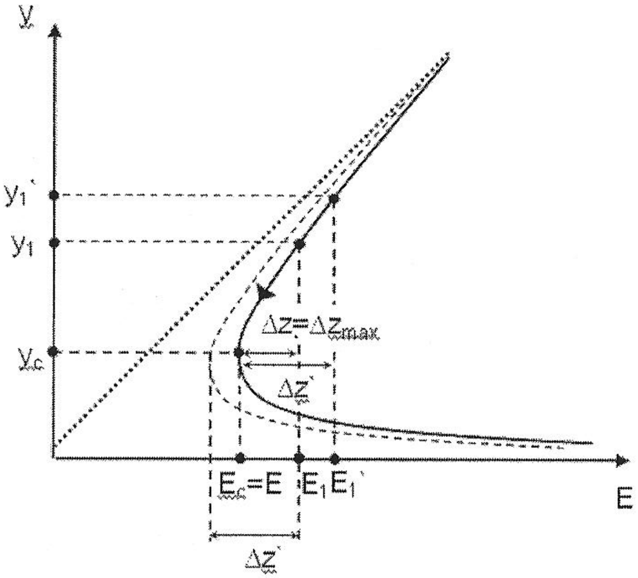
Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

CHOKING

1) Upward step





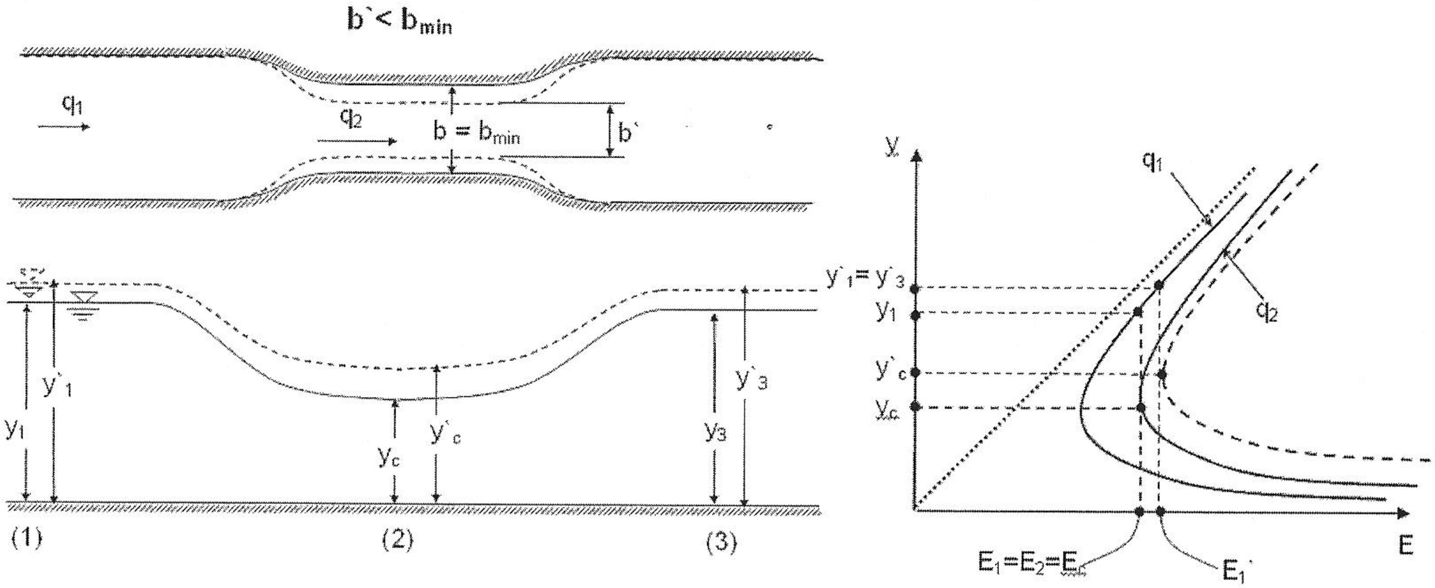
$E_1=\Delta z_{max}+E_c$, $\Delta z` > \Delta z_{max}$

$E_1`=\Delta z`+E_c$

Specific Energy Concept

3.3.3 Channel Transition and Choking Problems

2) Contraction



$$E_1 = (E_c)_2 = E_3 \quad \text{if} \quad b = b_{min}, \quad (E_c)_2 = \frac{3}{2} y_c$$

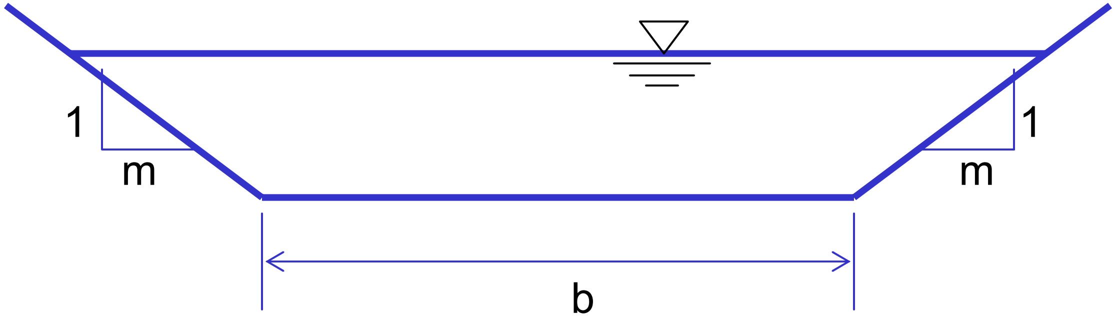
$$E_1' = (E_c)_2' = E_3' \quad \text{if} \quad b < b_{min}, \quad (E_c)_2' = \frac{3}{2} y'$$

Specific Energy Concept

Example 5:

a) What will be the normal depth corresponding to 11 m^3/s discharge in a trapezoidal channel for b=7.5 m, m=3, S_0=0.00001, and n=0.025.

b) Compute the critical depth and the min. specific energy.



Solved in the class

Specific Energy Concept

Solved in the class

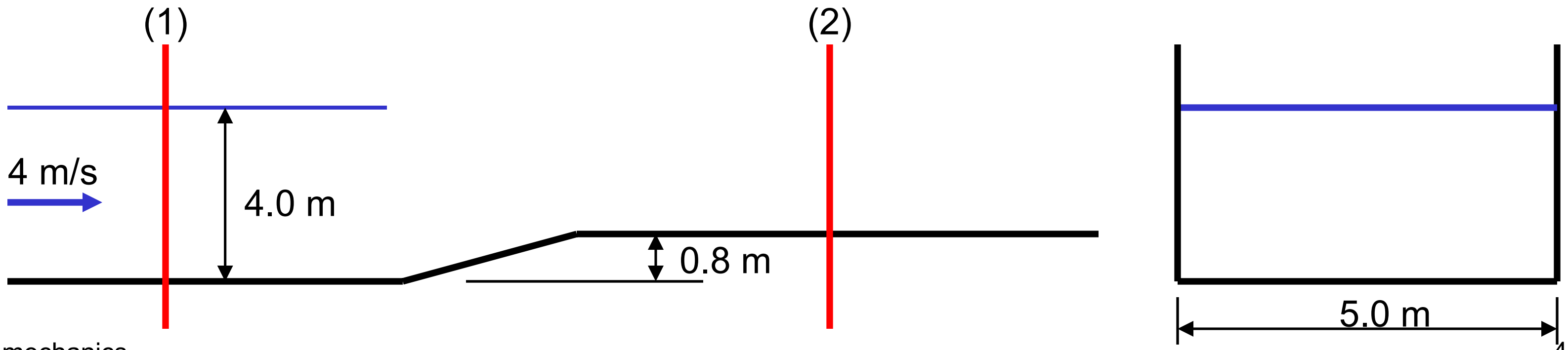
Example 6:

Water is flowing at a velocity of 4 m/s and a depth of 4 m in a channel of rectangular section of 5 m in width. There is then a smooth upward step of 0.8 m.

a) Determine if there is choking.

b) If there is choking, determine the decrease in discharge assuming energy at a section 1 remains constant.

c) If the discharge is kept to be the same as in (a), what should be the increase in specific energy and depth at section 1.



Specific Energy Concept

Example 7:

Water is flowing at a velocity of 4 m/s and a depth of 3 m in a channel of rectangular cross-section 6 m in width. Find the change in depth and absolute water level produced simultaneously by a downward step of 0.4 m and 2 m contraction in width.

Solved in the class