Reading and Writing in the Mathematics Classroom

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Imagine the following scenario. Mrs. Smith begins her third period mathematics class by telling the students how much ground they need to cover for the midterm. Typically, Mrs. Smith uses several different methods to teach mathematics, but since time is running short, she feels that she must lecture for the next few weeks. For the next forty minutes she presents new concepts on the board, while the students copy the information furiously. She stops occasionally to ask for questions, and receives only a few responses. At the end of the period, Mrs. Smith assigns the homework for the night before dismissing the students.

Later that night, Angie, who is one of the better students in Mrs. Smith's class, sits down to do her homework. She feels that she did not get much from the day's lesson, since she had to concentrate on reading and copying the notes on the board instead of listening to the oral explanations of the concepts. While reviewing her notes, Angie realizes that she does not understand the concepts, but she hopes by reading the textbook these ideas will become clear to her. She has never been required to read the text, and she finds herself struggling through it. She soon realizes that reading the text is not helping, and so she gives up trying to understand the concepts and moves to the problems. Angie is not exactly sure how they are to be done, but she can follow the examples well enough to get the correct answers. She puts away her homework, satisfied with the work she has done.

The next few weeks pass by in similar fashion and the day of the midterm arrives. As Mrs. Smith sits down to grade the tests, she is wholeheartedly disappointed with her students' performance. They perform the skills and solve the problems well, but their answers to the writing questions are troublesome. The students simply could not communicate their ideas clearly and correctly in writing. They were using incorrect terminology and notation, and their sentence structure was poor. Because of the poor writing, Mrs. Smith could not be sure the students actually understood the concepts assessed by the writing questions. Although Mrs. Smith believes that writing can be an effective assessment and learning tool in mathematics, she doubts she has the time to teach her students writing skills in addition to all the mathematical content. She may have to abandon writing in mathematics as a lost cause.

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This scenario shows that students can struggle with reading and writing in mathematics; skills which are increasing in importance in the mathematics classroom (Grossman et. al, 1993; Noonan, 1990). One of the new goals for students in *the Curriculum and Evaluations Standards* (NCTM, 1989) states that

> the development of a student's power to use mathematics involves learning the signs, symbols and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking (p. 6).

Certainly, school students do not lack training in written communication, since on average they receive 12 school years of training in writing and 6-8 school years of training in reading (Burley-Allen, 1982). However, it is unclear how much of this time is spent working with mathematical text; text which may require students to develop special reading and writing skills. This article will discuss the nature of reading and writing in mathematics, list some problems that students may encounter while reading or writing mathematics, and suggest ways of implementing and improving reading and writing in the mathematics classroom.

Reading

Reading can be considered a two part process. First, it is the transfer of encoded information from written text to the reader. Several cognitive psychologists (Gough, 1972; Rumelhart, 1977; Chall, 1983) suggest theories on how this might occur. Second, and of more importance to the paper, reading is the comprehension of the information by the reader. Rosenblatt (1978), in her transactional theory of reading, maintains that reading comprehension occurs as the reader actively shapes what is being read by bringing background knowledge, cultural orientation, personal reading history, beliefs and feelings to bear on the text. Thus, as the reader or the context in which the text is read changes, the comprehension of the text changes. This implies that a student with a strong mathematical background will most likely understand a mathematical text better than a student with a limited background. A student who comes from a household where mathematics and

reading are valued, will most likely understand mathematical text better and take more from it than would a student who does not come from such a home. A reader with higher level reading skills will most likely be able to comprehend mathematical text better, and a student with few mathematical anxieties will most likely read mathematical text easier and with greater comprehension.

In order to comprehend the texts of certain disciplines, readers must often develop reading skills peculiar to those disciplines. In particular, students must learn to read mathematically. But what does this mean? Noonan (1990) writes:

> To read a mathematics text means to take the global meaning from the page, not just to be able to read a few sentences. It means to appreciate the structure of a question and how the various graphs, diagrams and pictures relate to it. Reading mathematics texts requires different skills and knowledge on the part of the readers to achieve acceptable levels of reading comprehension ....The reading of a mathematics text is far more complex than simply being able to read the words on the page. It is about comprehending the mathematical ideas being put forward. (p. 79)

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To comprehend mathematical text, readers should be aware of the purpose of the text, the most predominant of which is to explain and transfer mathematical concepts, algorithms and reasoning to the reader (Shuard & Rothery, 1984). Expositional text may include stating and explaining definitions, theorems, and concepts; providing graphs and pictures for clarification; and providing applications of the mathematics. Unfortunately, students are rarely taught how to read mathematical exposition, and so they often struggle with it (Porras, 1994). Another purpose for mathematical text is to provide instruction on how to carry out a task. Procedural text tells the reader the method to be used when carrying out a specific mathematical task or how to write a final answer in a particular form (Shuard & Rothery). Students often fail to realize that the textbook is trying to help them master the procedures, and that it contains rules and hints as to when a procedure or algorithm could be used (Porras). Finally, mathematical text provides example problems that demonstrate procedures to students as well as exercises and problems so that students can implement what they have learned (Shuard & Rothery). Again, many students do not recognize the fact that the examples can serve as models for the practice exercises at the end of the section (Porras, 1994).

Noonan (1990) provides other reasons why mathematical text can be difficult to comprehend. Mathematical vocabulary can present problems in many ways. Students must learn new terms that can only be found in mathematics, and they must come to understand the concepts behind these terms. Many common English words can take on drastically different meanings in mathematics. For example, the word *mean*. Outside of mathematics it describes a person who is not very nice, but as a mathematical term, it represents the average of a set of numbers. Pictures, charts and graphs, which can be difficult to read and interpret, may be other potential stumbling blocks for students. Mathematical symbols and notation may cause students trouble for at least two reasons. First, each symbol or piece of notation must be learned by the student, in order for that student to be able to decipher many passages in mathematical text. This decoding may become particularly difficult as the notation accumulates. Second, sentences in mathematical text often include formulas or equations, which interrupt the flow of the prose, making it difficult to read. Because reading mathematics has so many potential pitfalls, the mathematical reader must necessarily be active and careful.

Another reason reading mathematics can be difficult is that it requires the reader to recall or find all information, definitions, theorems or notation pertinent to the concept currently being read. For example, if the definition of a square is given as a quadrilateral with four right angles and four congruent sides, then the student must know the meanings of the words quadrilateral, right angle, and congruent. Finding this information may entail looking at previous sections in the book, looking at other mathematical texts, or looking in the texts of disciplines other than mathematics. It may require the reader to scan the page to find any pictures, graphs, tables or charts that accompany the discussion and then be able to understand how they are related to the discussion. Thus, mathematical reading is not linear, and students should not try to read mathematical text as they would a drugstore novel; straight through, cover to cover. Reading mathematical text may require the reader to reread passages several times in order to gain understanding. Students should realize that time and effort are needed to completely comprehend mathematical text, and so they need to exercise patience, concentration and determination.

How can mathematics educators improve mathematical reading comprehension? DiGisi and Yore (1992) offer five suggestions for science teachers that may also be of benefit to mathematics educators. Before the students read, the teacher can use an advanced organizer to prepare the students for any new information they will encounter. An organizer might be a brief statement that reminds the students of previously learned concepts that relate to the topic about to be read, or it might demonstrate how previously learned information is inadequate for solving particular types of problems; a situation that will be remedied by the new information. The organizer allows the students to gather any personal background knowledge related to the subject, and then use it assimilate and understand the new information. While the students are reading, the teacher can help them determine whether the author is writing for exposition, instruction, or student practice. Often, understanding the purpose of the text will help students comprehend an author's message better. After the students have read, the teacher can use a series of conceptual questions to assess what the students have learned from the reading. Some sample questions include: What were the main ideas in the reading? How does this material relate to previous material? What new procedures or algorithms were developed? And: What connections does this material have to areas outside of mathematics? Also, the teacher should address any misconceptions that might have developed from the reading. This can only be done if the teacher is actively engaging the students with what they have read.

Siegel et al. (1996) provide two other strategies for promoting reading comprehension. The first is the "say something" strategy (p. 68). Its underlying principle is that "making sense of text is a social event in which readers talk their way through a piece, sharing their responses, questions, confusions and insights with partners as they go" (p. 67). In other words, students might read a certain section of text and then divide into small groups to discuss what was read. Through discussion and debate, perhaps challenging one another's notions, they work towards understanding of the material. The second method is the "sketch to stretch" method (p. 67). Here, the students draw pictures of a situation which they see as analogous to the mathematical concepts in the reading. They then use the pictures to create understanding of the material. A standard example is the analogy of a function to a machine. Students may draw a picture of a machine, with an intake funnel into which numbers are placed, a crank representing the rule that operates on the numbers, and a chute out of which come the resulting values.

It is important that students practice reading on their own, so teachers ought to assign reading as homework. Students need to value reading as an integral aspect of doing mathematics. Often, students believe they have learned the material solely from the class sessions, and may thus view reading assignments as redundant and unimportant. Mathematics educators must help students realize the importance of reading the text. Students must see that a mathematics text is a resource in which most of the information for the class can be obtained. It is a resource that can be taken home, where often students have no other source for guidance. Students must realize that the mathematics text is much more than a collection of practice exercises; it is a collection of concepts, procedures, and examples designed to help students comprehend mathematics.

Reading mathematics is a complex process that requires readers to develop special skills in order to comprehend what is read. Underdeveloped reading skills can keep our students from realizing their full potential and developing into the mathematical learners they are capable of becoming. As mathematics teachers, we must help our students acquire these skills, so that when we are not around to guide them, they will have enough confidence to pursue learning on their own.

Writing

Because of its potential for helping students to learn, writing is a pedagogical tool that has become of interest to many educators outside of the language arts disciplines (McIntosh, 1991). Emig (1977) maintains that writing is the most powerful and unique mode of learning. In contrast with the other modes, namely listening, talking and reading, writing is unique and powerful in that it originates from the student and is graphically recorded. If learning is viewed as enactive, iconic, and symbolic, then writing encompasses all three modes, and thus involves the fullest possible functioning of the brain. In other words, "the symbolic transformation of experience through the specific symbol system of verbal language is shaped into an icon by the enactive hand" (p. 124).

Further, writing is powerful because it uses both hemispheres of the brain. The right hemisphere is thought to control emotion, and intuition, as well as initiate metaphors and "abstractions occurring as visual or spatial wholes" (p. 125). The left hemisphere is thought to provide the linear thinking that is required to structure the ideas of a paper in a coherent fashion. Hence, one side generates the ideas and the other structures them.

Writing is a powerful tool because it provides a unique form of feedback. As the student writes, information from the process is immediately and visibly available, which allows the learner to review the reasoning for correctness. Moreover, writing clarifies and organizes a student's thoughts. Since writing prepares a product for another

person, the author must make sure that the writing flows and that all points are made clearly. Finally, Emig asserts that the slow pace of writing is conducive to student learning. Since writing is comparatively slow to reading, talking, or listening, it forces thinking to slow down to the pace of the writing. This allows students to reason through thoughts carefully to make sure they are correct and complete before they are stated.

Writing can be beneficial to students in other ways as well. Moore (1993) claims that effective writing can be used to influence opinions of the public and colleagues, which is a crucial skill in a wide variety of occupations. Also, writing is often necessary for occupational success. He notes that many employers both want and expect their employees to be good writers.

Writing may also have beneficial aspects for student learning that are specific to mathematics courses. In a research study of writing in mathematics, Grossman, Smith, and Miller (1993) suggest that

> a student's ability to explain concepts in writing is related to the ability to comprehend and apply mathematical concepts. This is applicable in both short and longer time frames. The findings further suggest that when a student demonstrates the ability to write about concepts this may be viewed as both an expression of comprehension and a product of knowledge. (p. 4)

Shibli (1992) maintains that writing allows the student to see the steps used in problem solving and helps the student draw conclusions and interpretations from the solution. If the solution to a problem has more than one interpretation, then clear writing becomes especially important in avoiding ambiguity as to which interpretation is being presented. Sipka (1990) believes that writing in mathematics may also improve student writing in general and that the structured nature of mathematics may help students impose structure when writing prose in other classes.

Student writing can be beneficial to teachers as well (Drake & Amspaugh, 1994). Written explanations of a student's problem solving process allow the teacher to understand and assess the student's thinking and comprehension of material in a way that computational steps alone may not provide. Teachers can diagnose and address errors in a student's thinking or knowledge of procedure more effectively. The depth of a student's misunderstanding can be determined by the teacher, and thus provide insight for an instructional starting point. A teacher may not always need to start from the beginning of a process in order to clear up a misconception. Having a student write may provide the instructor with "evidence of where or why a student has failed to make connections between strands of the mathematics curriculum" (p. 45). Finally, writing may provide clues as to why a student is unable to complete an assignment by oneself.

Teachers can assign writing tasks that may reveal attitudes, anxieties and beliefs about mathematics that might be interfering with a student's learning. Through such an assignment, the student can express to the teacher, in a private and direct way, concerns about learning the material (Dodd, 1992). A quick and caring response to these concerns may increase the motivation of the student and strengthen the student-teacher relationship. Dodd maintains that this may be of particular importance in disciplines such as mathematics, where there tends to be a large amount of student anxiety.

Written explanations of a student's problem solving process allow the teacher to understand and assess the student's thinking and comprehension.

When writing mathematics, students may encounter many of the same problems when they read mathematics. They may have difficulty using mathematical notation and vocabulary properly. They may have difficulty expressing mathematical situations in graphical or tabular form, or explaining how the graphs and tables relate to the situation being discussed. While the difficulties in reading and writing mathematics are similar, it may be that writing mathematics is a more troublesome task. Reading only asks the student to understand the message of the author, whereas in writing, the student must not only understand the message being written, but must try to generate understanding in another person. Thus, writing may require the student to have a greater understanding of the content and a better ability to communicate than does reading.

Sipka (1990) provides several ways of using writing in the mathematics classroom. He classifies them into two categories, *informal* and *formal*. Informal writing activities use content as the main criterion for judgment of the paper. They are especially useful for helping students to understand material, and may include free writing, mathematics autobiographies, journals, and reading logs. Formal writing activities are evaluated with respect to content and quality of writing. These assignments include letters to authors, proofs, formal lecture notes written by the students, and research papers.

One of the more non-standard informal writing assignments is the *free write* (Sipka, 1990), which is, essentially, "stream of consciousness" writing. It need not have arisen from a plan or have any organization; students simply

write down every thought that occurs to them. Other than providing students with general writing topics, such as favorite mathematical topics or expectations for the course, the teacher must be careful not to impose any restrictions. This form of writing is useful for capturing random thoughts and generating ideas. Since it is non-stop writing, the length of writing time should be kept to five minutes or less.

Another unusual informal writing assignment is the mathematics autobiography (Sipka, 1990). In this assignment students describe their previous mathematical experiences and classes. For example, they might describe one successful and one not-so-successful mathematical experience. The teacher can thus become acquainted with the students, in particular their attitudes and beliefs about mathematics. Appropriate teacher responses to these assignments can convey care for individual students and can establish positive teacher-student relationships. Sipka maintains most students enjoy such an assignment, since it allows them to vent their grievances and concerns.

Journals are another medium for informal writing. Waywood (1992) claims that journal entries generally fall into one of three categories: a restatement of information, a summary of information, or a mode of dialogue with the teacher. As a mode of dialogue, the teacher should encourage students to ask probing questions and give deep explanations of the material covered in class. The dialogue can also provide the students a way to regularly communicate concerns, misunderstandings and difficulties to the teacher. Waywood also gives advice to teachers who are interested in using journals. Students need to have a clear understanding of what is intended by keeping a journal, and teachers must allow class and homework time for journal writing, so that it is valued by the students as a means of assessment. Finally, journals must be used over the course of several years to be effective, and this requires that they be implemented as school or departmental policy.

Writing letters to teachers, authors and fellow students is an unusual, formal writing assignment that can be used in the mathematics classroom (Sipka, 1990). Sample assignments might include: write a letter to the instructor regarding which topics you find troublesome and why; write a letter expressing your feelings about your last test performance; write a letter to the textbook's author describing good and bad points; and write a letter to future students explaining this course.

Proof writing can be another effective writing tool in the mathematics classroom (Sipka, 1990). Since proofs generally proceed in a series of logical steps, proof writing may learn improve the logical structure of students' writing. Since students may not write a complete or correct proof the first time, proof writing can be used to stress the need for rewriting.

Another interesting idea is to have the students rewrite the lecture notes. (Sipka, 1990). Each student takes a different day of class on which he or she is responsible for producing a polished set of lecture notes. The notes are checked for accuracy by the teacher and then distributed to the whole class. Because students know they are writing for an audience, this activity may improve the clarity and structure of the students writing. Rewriting the class notes provides a review of the material, and furnishes the class with a complete set of correct notes.

Although the importance of writing has long been recognized by many teachers in fields other than English, its use as a learning tool in mathematics has met with resistance from many mathematics teachers (McIntosh, 1991). McIntosh suggests that many mathematics teachers believe that writing is an activity that is simply not done in mathematics. Many mathematics teachers believe that mathematics revolves around numbers and formulas, not words. Mathematics teachers might also fear that they will have to teach writing skills, something they may feel they are not trained to do. Shibli (1992) claims that students may not be adequately prepared to write mathematics and so teachers will need to train them with practice, patience and feedback. A study by Moore (1993), suggests that if students receive no training or feedback on how to write effectively, their learning will not be affected by the experience. Although mathematics teachers may find implementing writing in their classrooms initially difficult, the benefits of writing should make the struggle worth while.

Connections between reading and writing

To this point, reading and writing have been considered separately, but they are processes that often occur simultaneously. In fact, many of the skills involved in reading mathematics are very similar to those in writing mathematics, and so reading and writing in mathematics may be mutually beneficial.

Taking notes while reading, may help students improve their reading skills in a variety of reasons. Note taking slows reading down and makes the student an active reader: it causes the student to immediately review what was just read, and often the more a concept is used, stated, or discussed, the better the idea is retained. Writing allows the student to express comments, concerns or questions about what is being read. It gives the student the privilege of expressing the concepts in a personal way that might be easier to understand, and it allows the student to organize the topics in a way that makes sense. By writing short notes in the margins, students can easily return to recall what was read, make connections to material located in other areas of the book, and state their thoughts on the topic. Unfortunately, students are seldom allowed to write in textbooks.

Reading mathematical text may help mathematical writing by providing a guideline for students. While students read, they can learn how the author uses certain symbols and notation. They can learn how and in what context the author uses certain words. Students can learn how logical progressions of ideas and concepts are developed and how the author justifies mathematical conclusions. While reading and mimicking a good example of mathematical writing does not guarantee that students will write mathematics well, it does provide them with a good place to start.

The importance of reading and writing cannot be overstated. Throughout their entire lives, students will be using their ability to read and write to learn and to communicate. Certain disciplines, such as mathematics, may require students to develop special skills in order to read and write effectively in that discipline. It should be important for the teachers in each discipline to educate their students to learn and communicate in that discipline. Otherwise, we may be failing to prepare the students to survive in a society where communication and learning on one's own are increasing in importance.

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