8th International Conference on Behavior of Steel Structures in Seismic Areas
Shanghai, China, July 1-3, 2015

AN ACCURATE MODELING APPROACH FOR CALCULATING THE VIBRATION CHARACTERISTICS OF STEEL FRAMED STRUCTURES WITH SEMI-RIGID CONNECTIONS

Halil F. Ozel* and Afsin Saritas**

* PhD. Student at Middle East Technical University
e-mail: firat.ozel@metu.edu.tr

** Associate Professor at Middle East Technical University
e-mail: asaritas@metu.edu.tr

Keywords: Steel framed structures, Finite element modeling, Semi-rigid connection, Vibration characteristics.

Abstract. *Vibration characteristics of steel framed structures are affected by the presence of semi-rigid connections. In this study, a mixed formulation frame finite element is developed from force method, where the variational form of the element bases on the use of three-fields Hu-Washizu-Barr principle. Consistent mass matrix of the element is obtained such that determination of vibration frequencies of members with varying geometry and material distribution as well as the presence of semi-rigid connections at any section on the element is accurately captured without any need for specification of different displacement shape functions for each individual case. The element response does not necessitate further discretization due to the presence of semi-rigid connections. Benchmark numerical examples for a steel I-beam verifies the accuracy of proposed element with and without semi-rigid connections.*

1 INTRODUCTION

Researchers have studied the effects of dynamic behaviors of steel structures in the last decades and found out that the effect of the semi-rigid connections on the behavior of steel structures is especially important under dynamic loadings. Better match with the laboratory test results by [1] and [2] were observed when introducing the flexible joint effects in the numerical studies. Analysis studies by [3], [4], and [5] provided the inclusion of connection stiffness in the dynamic analysis of steel structures, where the effects of the semi-rigid connections has been observed to have deteriorating role on the systems. Sophianapoulos [6] compared vibration properties for some benchmark examples with Eurocode 3 approach for semi-rigid steel frame structures. Results showed that for the fundamental modes, the results of closed form solutions are very close to Eurocode 3; however, for the higher modes, the influence of the flexible joints resulted in differences between the two.

In order to carry out an accurate dynamic analysis of steel framed structures, vibration characteristics of typical steel beams with I-section containing semi-rigid connections should be studied. For this purpose, a mixed formulation frame finite element is developed from three fields Hu-Washizu-Barr functional through the use of force-based interpolation functions. The proposed approach allows determination of vibration frequencies of members with altering geometry and material distribution with semi-rigid connections placed at any location on the element without further specification of different displacement shape functions. Vibration frequencies and mode shapes for both the rigid and semi-rigid connection cases are verified through numerical examples solved with proposed approach and in ANSYS [7] and SAP2000 [8].

2 FRAME ELEMENT FORMULATION

2.1 Kinematic Relations

Displacements on a material point on the section of a beam that deforms in xy-plane can be obtained by calculating Timoshenko beam theory as follows;

$$\begin{Bmatrix} u_x(x,y) \\ u_y(x,y) \end{Bmatrix} = \begin{Bmatrix} u(x) - y\theta(x) \\ v(x) \end{Bmatrix} \tag{1}$$

where ux(x,y) and uy(x,y) are the displacements in x and y directions, respectively of any point in the section. u(x) is the displacement of the point (x,0) along x-axis. v(x) is the transverse deflections of the point (x,0) from x-axis in y direction. θ(x) is the small rotation of the beam cross section around z-axis. The non-zero strain components $\boldsymbol{\varepsilon}$ include the normal strain in the x direction and shear strain with xy component, where these are calculated from section deformations as follows;

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u'(x) - y\theta'(x) \\ -\theta(x) + v'(x) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa(x) \\ \gamma(x) \end{Bmatrix} = \mathbf{a}_s(y,z)\,\mathbf{e}(x) \tag{2}$$

where $\mathbf{e}(x)$ is the section deformation vector given as follows;

$$\mathbf{e}(x) = \begin{bmatrix} \varepsilon_a(x) & \gamma(x) & \kappa(x) \end{bmatrix}^{\mathrm{T}} \tag{3}$$

In Equation (3), $\varepsilon_a(x)$ is the axial strain of the reference axis, $\gamma(x)$ is the shear deformation along *y*-axis and κ is the curvature about *z*-axis. Section deformations can be easily calculated from section reference displacements as clearly visible from a one to one comparison of the terms of Equation (2). Furthermore, section compatibility matrix, $\mathbf{a}_s(y,z)$ introduced in Equation (2) is written as follows;

$$\mathbf{a}_s(y,z) = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & 0 \end{bmatrix} \tag{4}$$

2.2 Basic System without Rigid Body Modes and Force Interpolation Functions

Element formulation considers two end nodes and relies on a transformation from complete system to basic system. The basic system shown in Figure 1 is prescribed for the purpose of removing rigid body modes of motion.

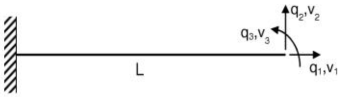

Figure 1. Cantilever basic system forces and deformations

The transformation matrix, **a** for an element with length *L* is used to relate element end forces in complete system to basic element forces as follows;

$$\mathbf{p} = \mathbf{a}^{\mathrm{T}}\mathbf{q}; \quad \text{where} \quad \mathbf{a} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -L & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

It is also possible to relate basic element deformation vector **v** to displacements in complete system by separating 3 rigid body modes and keeping only the basic deformation modes for the element. By this way, it is feasible to derive flexibility matrix that would have been impossible to get in the complete system because of the singularity caused by rigid body modes. Basic element deformations $\mathbf{v}$ can be calculated from nodal displacements $\mathbf{u}$ in complete system as follows;

$$\mathbf{v} = \mathbf{a}\,\mathbf{u} \tag{6}$$

Basic element forces at free end, $\mathbf{q}$ are shown in Figure 1 and given in Equation (5). These forces can be related to internal section forces, $\mathbf{s}(x)$ by using the force interpolation matrix $\mathbf{b}(x,L)$ for the cantilever beam configuration as follows;

$$\begin{gathered}\mathbf{s}(x) = \begin{bmatrix} N(x) & V(x) & M(x)\end{bmatrix}^{\mathrm{T}} = \mathbf{b}(x,L)\,\mathbf{q} + \mathbf{s}_p(x) \\ \mathbf{b}(x,L) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & (L-x) & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{s}_p(x) = \begin{bmatrix} L-x & 0 \\ 0 & L-x \\ 0 & (L-x)^2/2 \end{bmatrix} \begin{Bmatrix} w_x \\ w_y \end{Bmatrix}\end{gathered} \tag{7}$$

By using Equation (7), it is possible to attain exact equilibrium between the forces at free end of the element and forces at any section that is x units away from the fixed end. Section forces are axial force $N(x)$, shear force in y direction $V(x)$, and moment about z-axis $M(x)$. In above equation $\mathbf{s}_p(x)$ is the particular solution for uniformly distributed loads in the axial and transverse directions, i.e. w_x and w_y, respectively. By the way, with this approach, it is easy to calculate the particular solution under arbitrary inter element loads that are concentrated or distributed.

2.3 Variational Base and Finite Element Formulation of the Element

Variational form of the element is written by using three-fields Hu-Washizu functional and implemented as part of beam finite elements by Taylor et al. [9] and Saritas and Filippou [10]. Extension to dynamic case is achieved through introduction of inertial forces $\mathbf{m}\ddot{\mathbf{u}}$ acting at nodes by considering D'Alembert's principle to get the following variational form of the element

$$\begin{aligned}\delta\Pi_{\mathrm{HW}} &= \int_0^L \delta\mathbf{e}^{\mathrm{T}}\left(\hat{\mathbf{s}}(\mathbf{e}(x)) - \mathbf{b}(x,L)\,\mathbf{q} - \mathbf{s}_p(x)\right)dx - \delta\mathbf{q}^{\mathrm{T}}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{e}(x)\,dx + \delta\mathbf{q}^{\mathrm{T}}\mathbf{a}_g\mathbf{u} \\ &+ \delta\mathbf{u}^{\mathrm{T}}\mathbf{a}_g^{\mathrm{T}}\mathbf{q} + \delta\mathbf{u}^{\mathrm{T}}\mathbf{m}\ddot{\mathbf{u}} - \delta\mathbf{u}^{\mathrm{T}}\mathbf{p}_{app} = 0\end{aligned} \tag{8}$$

where independent fields of the functional are element nodal displacements $\mathbf{u}$, element basic forces $\mathbf{q}$, and section deformations $\mathbf{e}$. Above equation can also be obtained by considering the general Hu-Washizu variational form with extension to dynamic case by Barr [11]. Equation (8) should hold for arbitrary $\delta\mathbf{u}$, $\delta\mathbf{q}$ and $\delta\mathbf{e}$, thus the following three equations should be satisfied in order for the Hu-Washizu-Barr variational to be zero.

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{p} \equiv \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{p} = \mathbf{a}_{\mathrm{g}}^{\mathrm{T}}\mathbf{q} \tag{9}$$

$$\mathbf{v} \equiv \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\,\mathbf{e}(x)\,dx; \quad \text{where} \quad \mathbf{v} = \mathbf{a}_{\mathrm{g}}\mathbf{u} \tag{10}$$

$$\hat{\mathbf{s}}(\mathbf{e}(x)) \equiv \mathbf{b}(x,L)\mathbf{q} + \mathbf{s}_p(x) \tag{11}$$

Equation (9) is the equation of motion that holds for linear or nonlinear material response, and this equation can be collected for each element to get structure's equation of motion. A numerical time integration scheme can be employed to get a solution. Consequence of viscous damping can be simply achieved by adding $\mathbf{c}\dot{\mathbf{u}}$ to the left hand side of the equation, where $\mathbf{c}$ is the damping matrix. It is also possible to determine resisting forces $\mathbf{p}$ not only in terms of displacements $\mathbf{u}$ but also as a function of velocities $\dot{\mathbf{u}}$ through the use of a material model that considers time-dependent effects, such as viscoelastic or viscoplastic material models.

For linear elastic material response, section deformations can be calculated as $\mathbf{e}=\mathbf{k}_s^{-1}\hat{\mathbf{s}}$ to obtain the section deformations from section forces through the use of section stiffness matrix $\mathbf{k}_s$. Substitution of section deformations $\mathbf{e}$ to Equation (10) gives:

$$\mathbf{a}_g\mathbf{u} = \mathbf{v} = \mathbf{f}\ \mathbf{q}; \quad \text{where} \quad \mathbf{f} = \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{f}_s\ (x)\ \mathbf{b}(x,L)\, dx \tag{12}$$

In above equation $\mathbf{f}$ is the flexibility matrix of the element in the basic system. $\mathbf{f}_s$ is the section flexibility matrix that can be calculated from the inversion of the section stiffness matrix $\mathbf{k}_s$. Further substitution of above equation for linear elastic response in Equation (9) results in

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{k} = \mathbf{a}^{\mathrm{T}}\mathbf{f}^{-1}\mathbf{a} \tag{13}$$

where $\mathbf{k}$ is the 6×6 element stiffness matrix in the complete system. At this point in the element formulation, presence of semi-rigid connections will be introduced through the following extended version of above equation for the calculation of element end deformations:

$$\begin{gathered}\mathbf{v} = \mathbf{v}_{\text{Frame}} + \mathbf{v}_{\text{Con}}; \quad \text{where} \quad \mathbf{v}_{\text{Frame}} = \int_L \mathbf{b}^{\mathrm{T}}(x)\,\mathbf{e}(x)\,dx; \quad \mathbf{v}_{\text{Con}} = \sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\,\mathbf{\Delta}_{SC,i} \\ \text{and} \quad \mathbf{\Delta}_{SC} = \begin{bmatrix}\delta_{SC}^{axial} & \theta_{SC} & \delta_{SC}^{shear}\end{bmatrix}^{\mathrm{T}}\end{gathered} \tag{14}$$

The first integral along the length of the frame element can be numerically calculated by using a quadrature rule to capture spread of inelastic behavior and nSC is the total number of semi-rigid connections discreetly located along element length; Δ_{SC} is the vector of semi-rigid connection deformations. Introduction of semi-rigid connections along element length in Figure 1 does not alter the force field under small deformations. Element flexibility matrix is similarly discretized as follows:

$$\mathbf{f} = \mathbf{f}_{\text{Frame}} + \mathbf{f}_{\text{Con}}; \quad \text{where} \quad \mathbf{f}_{\text{Frame}} = \int_L \mathbf{b}^{\mathrm{T}}(x)\,\mathbf{f}_s(x)\mathbf{b}(x)dx; \quad \text{and} \quad \mathbf{f}_{\text{Con}} = \sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\,\mathbf{f}_{SC,i}\mathbf{b}(x_i) \tag{15}$$

As a remark, Equations (10) and (11) are related to the element state determination, i.e. these equations can be solved independent of Equation (9), and then the solution can be condensed out into the first equation such that the equations of motion can be assembled for all elements. This process was demonstrated above for the linear elastic case. In general, state determination of the element requires an iterative solution in the case of nonlinear behavior, where Equations (9) to (11) are needed to be solved. This solution requires also the calculation of element flexibility matrix $\mathbf{f}$ under nonlinear response, where taking derivative of element deformations $\mathbf{v}$ in Equation (10) with respect to element forces $\mathbf{q}$ results into the same flexibility integration expression given in Equation (15), but this time the section stiffness will be nonlinear, as well.

2.4 Section Response

Section response can be obtained by the basic assumption that plane sections before deformation remain plane after deformation along the length of the beam by the use of following section compatibility matrix $\mathbf{a}_s$ as given in Equation (2), where the section compatibility matrix now contains the shear correction factor κ_s as follows

$$\mathbf{a}_s = \mathbf{a}_s(y) = \begin{bmatrix} 1 & 0 & -y \\ 0 & \kappa_s & 0 \end{bmatrix} \tag{16}$$

Shear correction factor κ_s is taken as the inverse of the form factor suggested by Charney et al. [12] for I-section:

$$\kappa_s = 1/\kappa; \quad \text{where} \quad \kappa = 0.85 + 2.32\,\frac{b_f t_f}{d\, t_w} \tag{17}$$

The section forces are obtained by integration of the stresses that satisfy the material constitutive relations $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$ according to

$$\mathbf{s} = \int_A \mathbf{a}_s^{\mathrm{T}} \boldsymbol{\sigma}\, dA; \quad \text{where} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix} \tag{18}$$

The derivative of section forces from (18) with respect to the section deformations results in the section tangent stiffness matrix

$$\mathbf{k}_s = \frac{\partial \mathbf{s}}{\partial \mathbf{e}} = \int_A \mathbf{a}_s^{\mathrm{T}} \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \mathbf{e}}\, dA = \int_A \mathbf{a}_s^{\mathrm{T}} \mathbf{k}_m \mathbf{a}_s\, dA \tag{19}$$

The material tangent modulus km is obtained from the stress-strain relation according to $\mathbf{k}_m = \partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon}) / \partial \boldsymbol{\varepsilon}$. Gauss-quadrature, the midpoint or the trapezoidal rule can be used for the numerical evaluation of the integrals in (18) and (19). While Gauss-quadrature gives better results for smooth strain distributions and stress-strain relations, the midpoint rule is preferable for strain distributions and stress-strain relations with discontinuous slope.

2.5 Force-Based Consistent Mass Matrix

The derivation of the consistent mass matrix requires the determination of the section mass matrix, where the mass is considered like a distributed load along the length of the beam in cantilever basic system. The section mass matrix is easily computed by following equation through the use of section compatibility matrix $\mathbf{a}_s$ given in Equation (4):

$$\mathbf{m}_s(x) = \int_A \mathbf{a}_s^{\mathrm{T}} \rho(x, y)\, \mathbf{a}_s\, dA; \tag{20}$$

Mass matrix of the force-based element, which will be used in Equation (9), is written by the method provided by [13] and [14] as follows:

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_{00} & \mathbf{m}_{0L} \\ \mathbf{m}_{L0} & \mathbf{m}_{LL} \end{bmatrix} \tag{21}$$

where the components of element mass matrix are calculated from sub-matrices

$$\begin{aligned}
\mathbf{m}_{LL} &= \mathbf{f}^{-1}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{k}_s^{-1}(x)\left(\int_x^L \mathbf{b}^{\mathrm{T}}(x,\xi)\mathbf{m}_s(\xi)\mathbf{f}_p(\xi)\mathbf{f}^{-1}d\xi\right)dx \\
\mathbf{m}_{L0} &= \mathbf{f}^{-1}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{k}_s^{-1}(x)\left(\int_x^L \mathbf{b}^{\mathrm{T}}(x,\xi)\mathbf{m}_s(\xi)\left(\mathbf{b}^{\mathrm{T}}(0,\xi)-\mathbf{f}_p(\xi)\mathbf{f}^{-1}\mathbf{b}^{\mathrm{T}}(0,L)\right)d\xi\right)dx \\
\mathbf{m}_{0L} &= \mathbf{m}_{L0} = -\mathbf{b}(0,L)\mathbf{m}_{LL} + \int_0^L \mathbf{b}(0,x)\mathbf{m}_s(x)\mathbf{f}_p(x)\mathbf{f}^{-1}dx \\
\mathbf{m}_{00} &= -\mathbf{b}(0,L)\mathbf{m}_{L0} + \int_0^L \mathbf{b}(0,x)\mathbf{m}_s(x)\left(\mathbf{b}^{\mathrm{T}}(0,x)-\mathbf{f}_p(x)\mathbf{f}^{-1}\mathbf{b}^{\mathrm{T}}(0,L)\right)dx
\end{aligned} \tag{22}$$

In above equations, element flexibility matrix $\mathbf{f}$ is obtained as given in Equation (15). The partial flexibility matrix $\mathbf{f}_p$ is calculated as follows:

$$\mathbf{f}_p(x) = \int_0^x \mathbf{b}^{\mathrm{T}}(\xi,x)\mathbf{k}_s^{-1}(x)\mathbf{b}(\xi,x)\,d\xi \tag{23}$$

where above equation should be supplemented by the presence of localized connection flexibility as done in Equation (15) for the element flexibility matrix.

3 NUMERICAL EXAMPLES

A cantilever beam with IPE270 section is considered with and without the presence of semi-rigid connection at fixed support. Length to depth (L/d) ratio of the beam is taken as 10, 5 and 2. Elasticity modulus, Poisson's ratio and density of steel are 200 GPa, 0.3 and 7832 kg/m^3, respectively. Connection stiffness ratio λ for semi-rigid case is taken as 2, 11 and 20, where λ is the ratio of connection stiffness to flexural rigidity *EI/L* of beam. Proposed model is first assessed for the rigid connection case with ANSYS and SAP2000, and then semi-rigid connection case results are compared with SAP2000.

3.1 Cantilever Beam with Rigid Connection

Modal analyses results obtained from ANSYS for the cantilever beam with rigid connection are given in Table 1. Analyses in ANSYS is carried out by the use of 3d brick finite elements with a fine mesh discretization, and the first two bending modes and the first axial mode are reported only for comparison purposes with frame finite element solution.

Table 1. ANSYS Results for Cantilever I-Beam with Rigid Connection

L/d	1st Bending (Hz)	2nd Bending (Hz)	1st Axial (Hz)
10	42.154	227.13	468.35
5	155.66	649.25	937.13
2	671.28	-	2358.7

Modal analyses results obtained by the use of frame finite elements in SAP2000 and through the use of proposed element are given in Table 2. Results obtained with the use of 1 element in SAP2000 gave gross errors, since SAP2000 employs lumped mass matrix; thus, only 4 and 32 element results are given for SAP2000. On the other hand, the proposed element employs a force-based consistent mass matrix and furthermore accurately calculates the stiffness matrix of a beam with I-section with the shear correction factor suggested by Charney [12]. For this reason, 1, 4 and 32 element results are provided for the proposed element solutions. It is evident that the proposed element closely estimates the first bending and axial modes for L/d=10 and 5. It is furthermore observed that the use of 4 elements gives close results for the proposed element with ANSYS results for given vibration modes for all aspect ratios while the same cannot be said at all for SAP2000 solutions.

Table 2. SAP2000 and Proposed Model Results for Cantilever I-Beam with Rigid Connection

L/d	Mode Type	SAP 2000		Proposed Model		
		N_{el} =4	N_{el} =32	N_{el} =1	N_{el} = 4	N_{el} = 32
10	1st Bending (Hz)	41.3104	42.4160	42.4325	42.1005	42.0848
	2nd Bending (Hz)	213.8580	232.2880	388.6018	229.2061	226.9499
	1st Axial (Hz)	464.9001	467.9457	515.9358	470.9140	467.9490
5	1st Bending (Hz)	154.2258	157.8283	157.9986	155.5062	155.3225
	2nd Bending (Hz)	643.5006	692.5208	1031.8716	671.8748	659.1511
	1st Axial (Hz)	930.2326	935.4537	1202.4073	941.8281	935.8980
2	1st Bending (Hz)	692.5208	702.2472	718.4257	686.5626	684.1043
	2nd Bending (Hz)	2136.7521	2277.9043	2579.6789	2080.1213	2030.6245
	1st Axial (Hz)	2325.5814	2341.9204	3118.2153	2354.5701	2339.7449

3.2 Cantilever Beam with Semi-Rigid Connection

Modal analyses results obtained for the cantilever beam with semi-rigid connection at base for SAP2000 and proposed element solutions are given in Tables 3 and 4, where B1, B2 and A1 represent 1st and 2nd bending and 1st axial modes, and λ=2 and 20 are the lower and upper ranges to describe a beam to column connection to fall into semi-rigid connection classification. The error of the results obtained for the proposed element with single element discretization is more significant as the connection stiffness becomes 2. Comparison of 32 element results of SAP2000 and proposed model solutions for both the long and short beam cases demonstrate that the results from proposed approach provide the accurate and reliable vibration frequencies since the same level of difference was also present in Table 2 for the rigid case results.

Table 3. SAP2000 Results for Cantilever I-Beam with Semi-Rigid Connection

	L/d	10			5			2		
Nel	Mode	λ=2	λ=11	λ=20	λ=2	λ=11	λ=20	λ=2	λ=11	λ=20
4	B1(Hz)	24.29	35.68	37.92	94.66	135.34	142.94	508.13	644.33	664.89
	B2(Hz)	178.86	197.90	203.54	596.66	624.61	631.71	2136.75	2136.75	2136.75
	A1(Hz)	444.44	459.14	464.25	930.23	930.23	930.23	2325.58	2325.58	2325.58
32	B1(Hz)	24.75	36.52	38.86	96.39	138.22	146.11	515.20	653.17	673.86
	B2(Hz)	192.75	214.04	220.46	640.21	671.14	678.89	2277.90	2277.90	2277.90
	A1(Hz)	467.95	467.95	467.95	935.45	935.45	935.45	2341.92	2341.92	2341.92

Table 4. Proposed Model Results for Cantilever I-Beam with Semi-Rigid Connection

	L/d	10			5			2		
N_{el}	Mode	λ=2	λ=11	λ=20	λ=2	λ=11	λ=20	λ=2	λ=11	λ=20
1	B1(Hz)	41.43	42.05	42.19	153.81	156.51	157.09	699.33	712.83	715.15
	B2(Hz)	352.54	373.32	378.90	1031.87	1031.87	1031.87	2579.68	2579.68	2579.68
	A1(Hz)	515.94	515.94	515.94	1149.86	1182.72	1190.30	3137.40	3123.86	3121.53
4	B1(Hz)	25.10	36.54	38.76	97.03	137.22	144.62	505.54	639.38	659.39
	B2(Hz)	199.00	215.61	220.42	637.96	658.35	663.43	2062.07	2074.72	2076.94
	A1(Hz)	470.91	470.91	470.91	941.83	941.83	941.83	2354.57	2354.57	2354.57
32	B1(Hz)	24.99	36.49	38.72	96.51	136.91	144.36	501.29	636.50	656.70
	B2(Hz)	189.09	209.88	215.95	610.68	640.00	647.23	2020.76	2027.91	2029.05
	A1(Hz)	467.95	467.95	467.95	935.90	935.90	935.90	2339.75	2339.75	2339.75

4 CONCLUSION

The proposed element is able to determine the vibration characteristics for an I-beam with great accuracy when compared with ANSYS results, while the same cannot be said for SAP2000 solutions. Proposed element accurately captures stiffness and mass distribution without the need for the description of the displacement field. Accuracy obtained from proposed approach is furthermore enhanced by the use of an appropriate shear correction factor for I-section steel beams. The results obtained from proposed element with semi-rigid connection also show that the element is able to consider the inclusion of semi-rigid connection behavior accurately.

ACKNOWLEDGMENT

The authors thank for the support provided by Scientific and Technological Research Council of Turkey (TUBITAK) under Project No: 113M223.

REFERENCES

[1] Chui, P.P.T. and S.L. Chan, "Vibration and deflection characteristics of semi-rigid jointed frames", *Engineering Structures*. 19(12): 1001-1010, 1997.

[2] Nader, M.N. and A. Astaneh-Asl, "Shaking table tests of rigid, semirigid, and flexible steel frames", *Journal of Structural Engineering*. 122: 589-596, 1996.

[3] Galvao, A.S., A.R.D. Silva, R.A.M. Silveira, and P.B. Goncalves, "Nonlinear dynamic behavior and instability of slender frames with semi-rigid connections", *International Journal of Mechanical Sciences*. 52: 1547-1562, 2010.

[4] Da Silva, J.G.S., L.R.O. De Lima, P.C.G. Da S. Vellasco, S.A.L. De Andrade, and R.A. De Castro, "Nonlinear dynamic analysis of steel portal frames with semi-rigid connections", *Engineering Structures*. 30: 2566-2579, 2008.

[5] Al-Aasam, H.S. and P. Mandal, "Simplified procedure to calculate by hand the natural periods of semirigid steel frames", *Journal of Structural Engineering*. 139: 1082-1087, 2013.

[6] Sophianopoulos, D.S., "The effect of joint flexibility on the free elastic vibration characteristics of steel plane frames", *Journal of Constructional Steel Research*. 59: 995-1008, 2003.

[7] ANSYS, "Workbench 2.0 Framework", 2011.

[8] SAP2000, "Structural Analysis Program SAP 2000", CSI Berkeley, CA, 2011.

[9] Taylor, R.L., F.C. Filippou, A. Saritas, and F. Auricchio, "Mixed finite element method for beam and frame problems", *Computational Mechanics*. 31(1-2): 192-203, 2003.

[10] Saritas, A. and F.C. Filippou, "Inelastic axial-flexure–shear coupling in a mixed formulation beam finite element", *International Journal of Non-Linear Mechanics*. 44(8): 913-922, 2009.

[11] Barr, A.D.S., "An Extension of the Hu-Washizu variational principle in linear elasticity for dynamic problems", *Journal of Applied Mechanics*. June: 465, 1966.

[12] Charney, F.A., H. Iyer, and P.W. Spears, "Computation of major axis shear deformations in wide flange steel girders and columns", *Journal of Constructional Steel Research* 61: 1525–1558, 2005.

[13] Molins, C., P. Roca, and A.H. Barbat, "Flexibility-based linear dynamic analysis of complex structures with curved-3D members", *Earthquake Engineering & Structural Dynamics*. 27(7): 731-747, 1998.

[14] Soydas, O., "A three dimensional mixed formulation nonlinear frame finite element based on Hu-Washizu functional", in *Civil Engineering*, METU: Ankara, 2013.