It is pleasure for me to give a talk in a conference dedicated to the *memories of two fine mathematicians Narain Gupta and James* Wiegold. I haven't met them, but I have a correspondence with James Wiegold, now I would like to mention about it this correspondence. He played an impressive role in my academic life. I am deeply indebted to him. I will tell you why. Back in 1983 when I was a new graduate student in Ankara, TURKEY my supervisor wanted me to study various papers among which there was one written in Russian. In those times Russian was not an accessible language in Turkey and I could think of nothing else, but to go to the library, find the reviewer from Mathematical Reviews and write a letter to the reviewer saying that, since you reviewed the article you might have an English translation, if you have one, please send me a copy. The reviewer was James Wiegold and here is his reply. (Slayt)

Citations
From References: 7
From Reviews: 3

MR611367 (82k:20063) 20F24 (20E34)
Beljaev, V. V. [Belyaev, V. V.]
Minimal non-FC-groups. (Russian)
Sixth All-Union Symposium on Group Theory (*Čerkassy*, 1978) (*Russian*), *pp.* 97–102, 221, ''*Naukova Dumka*'', *Kiev*, 1980.

Author's summary: "It is proved that a nonperfect minimal non-FC-group is a Miller-Moreno group (that is, the commutator subgroup of every proper subgroup is finite). It is also shown that if there is a perfect locally finite minimal non-FC-group, then it must be a p-group, or the central factor-group is simple."
{For the entire collection see MR0611359 (82d:20005)}

Reviewed by *J. Wiegold*

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Dear Mr Kuzucuoglu, Thankyou for your letter of 11 Dec 1983.

Here is the paper you wanted. I didn't have a translation but I've quickly written one out — I hope you find it useful. Warning: I have not checked my translation.

Yours sincerely

James Wiegold

V.V. Belyaev

Minimal non-FC groups

Following [1,2], we call a group with infinite derived group a Miller-Moreno group if the derived group of every proper subgroup is finite. The Miller-Moreno non-perfect groups were completely described in [1]. It was proved [2] that a locally finite Miller-Moreno group is not perfect. In that paper we obtained some properties of minimal non-FC groups — a natural generalization of Miller-Moreno groups. In particular, if there exists a minimal non-FC group G equal to G', then either G is 2-generator and $G/Z(G)$ is simple, or else G is a locally finite group and, for any non-central x and y in G, $|C_G(x) : C_G(x) \cap C_G(y)|$ and $|C_G(y) : C_G(x) \cap C_G(y)|$ are finite.

Here we extend the study of minimal non-FC groups. It was proved that a minimal non-FC-group not equal to its derived group is a Miller-Moreno group (Theorem 1), and if there exists a locally finite minimal non-FC group G equal to G', then either $G/Z(G)$ is simple or G is a p-gp (Theorem 2).

§1 Minimal non-FC groups ~~that~~ are not perfect

Here we show that minimal non-perfect non-FC groups are exactly the Miller-Moreno groups described in [1]

LEMMA 1. An FC-group G containing an abelian subgroup of finite index has finite derived group.

Proof Let A be abelian and write $G = \langle A, g_1, \dots, g_n \rangle$. Then $\bigcap_{i=1}^{n} C(g_i) \cap A$ is of finite index and central.

LEMMA 2. If a minimal non-FC group G contains a proper subgroup of finite index, then G is different from G' and all proper subgroups have finite derived groups.

PROOF. Let G be minimal non-FC and K a proper subgroup of finite index. Clearly $H = \{x \in G \mid |G : C_G(x)| < \infty\}$ is a normal subgroup of G containing K. Thus $|G : H| < \infty$. Since G is not FC, $G \neq H$. Choose $a \in G \setminus H$. If $\langle a, H \rangle \neq G$, then $|G : C_G(a)| < \infty$, so that $a \in H$. Thus $\langle a, H \rangle = G$, and $\therefore$ $G' \leq H$. We shall show that $\langle a \rangle \cap H \leq Z(G)$. Indeed, take $x \in \langle a \rangle \cap H$. Then $C_G(x)$ contains a and has finite index in G. If $C_G(x) \neq G$, then $C_G(x)$ is FC and so $C_G(a)$ has finite index in G, contradicting choice of a. Thus $x \in Z(G)$. We show that H is abelian. If H is not abelian, there is a $g \in H$ such that $C_H(g) \neq H$. Since $C_H(g)$ is of finite index in G, $N := \bigcap_{x \in G} x^{-1} C_H(g) x$ is

He says in the letter that ''I hope it is useful''. Indeed it was very useful. It was an opportunity for me together with R. Phillips, in 1989 we solved one of the questions stated in that paper; namely there exists no simple locally finite minimal non-FC-group.

After almost 30 years later, it is a nice feeling to share this wonderful incident with you, I thank the organizers for having given to me, this opportunity.

Mahmut Kuzucuoğlu