SELECTED WORKS OF Otto H. KEGEL

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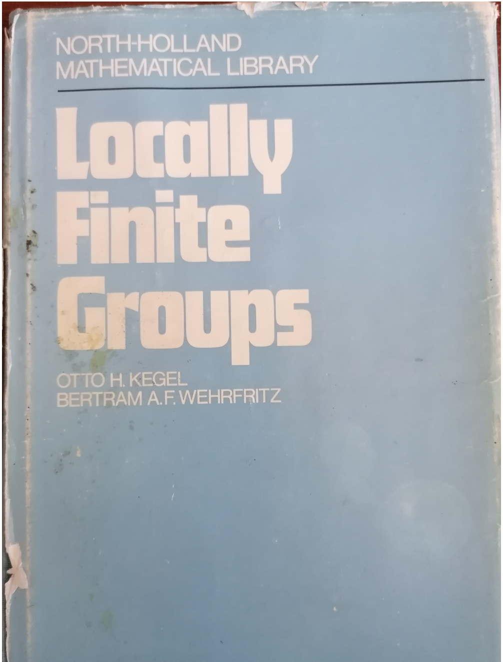
We grew up mathematically learning locally finite groups from O. H. Kegel and B. A. F. Wehrfritz 's book "Locally Finite Groups (1973)" during my graduate studies in Ankara around (1982). I have taken several courses from this book. The book was very famous in our department of that time.

Otto H. Kegel, 1973



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The infinite group G is simple if and only if it has a local system Σ consisting of countably infinite simple subgroups of G.

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The above Theorem is attributed to P. Hall by Kegel in his paper " Four Lectures on Sylow theory in locally finite groups ".

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Theorem 2 (Theorem 4.5(Kegel))

If G is a countably infinite, locally finite, simple group, then there exists a strictly ascending sequence G_n of finite subgroups of G with $G = \bigcup_{n=1}^{\infty} G_n$ such that for each natural number n there exists a maximal normal subgroup M_{n+1} of G_{n+1} satisfying $G_n \cap M_{n+1} = \{1\}$.

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The sequence $\mathcal{K} = (G_i, M_i)_{i \in \mathbb{N}}$ is called a **Kegel Sequence** of G. The maximal normal subgroups M_i are called **Kegel Kernels** and the simple groups G_i/M_i are called **Kegel simple sections.**

By this theorem we may be able to use classification of finite simple groups.

One may apply this theorem to solve whether there exists a simple locally finite minimal non-FC-groups?

This was a question in Kourovka Notebook Unsolved Problems in Group Theory Question 5.1.

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The main ingredients of the proof of this result were Theorem of P. Hall and Kegel.

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First by P. Hall's theorem if such a simple locally finite minimal non-FC-group exists then it must be countable.
Then by Theorem of Kegel it must have a Kegel sequence. Then using properties of minimal non-FC groups, the structure of FC-groups, classification of finite simple groups and centralizer of elements in simple groups, it was possible to decide such a simple group does not exist.

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Theorem 3 (Theorem 5.8, Shunkov)

For the locally finite group G the following properties are equivalent:

1. *G is a Chernikov group;*
2. *G satisfies min;*
3. *The centralizer of every non-identity element of G satisfies min;*
4. *Every abelian subgroup of G satisfies min.*

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The special case of this result where 2-subgroups of G are all finite is contained in O. H. Kegel-B. A. F. Wehfritz, Strong finiteness conditions in locally finite groups, Math. Z. **117** (1970) , 309-324.

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Kegel's Paper " Four Lectures on Sylow theory in locally finite groups " is another important paper to mention here.

Question. Can we extend the Sylow theorems to locally finite groups by ignoring the numerical parts of Sylow's theorems?

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One of the important example of a countable locally finite simple groups is P. Hall's Universal locally finite group which is constructed as a direct limit of right regular representations of finite symmetric groups G_i into $Sym(G_i)$ for $i \in \mathbb{N}$.

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One of the important example of a countable locally finite simple groups is P. Hall's Universal locally finite group which is constructed as a direct limit of right regular representations of finite symmetric groups G_i into $Sym(G_i)$ for $i \in \mathbb{N}$.

K. Hickin proved in Universal locally finite central extensions of groups proc. London Math. Soc. (3) **52**, 53-72 (1986).
For every prime p, every countably infinite locally finite p-group can be embedded into Hall's universal group U as a maximal p-subgroup of U.

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So conjugacy of Sylow subgroups is not true for Hall's universal group U.

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In particular isomorphic copy of infinite elementary abelian p-group and isomorphic copy of the locally cyclic p-group C_{p^∞} are maximal p-subgroups of U.

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A locally finite group G satisfies Sylow theorem for the prime p, if all maximal p-subgroups of G are conjugate in G. Moreover G satisfies strong Sylow theorem for p, if all its subgroups also satisfies Sylow theorem for p.
So locally finite groups satisfying strong Sylow theorem for p must have have restricted properties.
One theorem from this paper:

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Theorem 4

If for the prime $p \geq 5$ the locally finite group G satisfies the strong Sylow theorem, then there is a finite series of normal subgroups N_i of G with

$$G = N_0 \supseteq N_1 \ldots \supseteq N_i \supseteq N_{i+1} \supseteq N_k = \langle 1 \rangle$$

such that the factors N_i/N_{i+1} of this series are either a direct product of finitely many linear simple groups or locally p-soluble.

There is a notion of the Gruenberg-Kegel graph of a finite group.
Thanks to Natalia Moaslova, She will give a talk on this subject.