Preface

This text has evolved over some 20 years, starting as lecture notes for two first-year graduate subjects at M.I.T., namely, *Discrete Stochastic Processes* (6.262) and *Random Processes, Detection, and Estimation* (6.432). The two sets of notes are closely related and have been integrated into one text. Instructors and students can pick and choose the topics that meet their needs, and suggestions for doing this follow this preface.

These subjects originally had an application emphasis, the first on queueing and congestion in data networks and the second on modulation and detection of signals in the presence of noise. As the notes have evolved, it has become increasingly clear that the mathematical development (with minor enhancements) is applicable to a much broader set of applications in engineering, operations research, physics, biology, economics, finance, statistics, etc.

The field of stochastic processes is essentially a branch of probability theory, treating probabilistic models that evolve in time. It is best viewed as a branch of mathematics, starting with the axioms of probability and containing a rich and fascinating set of results following from those axioms. Although the results are applicable to many areas, they are best understood initially in terms of their mathematical structure and interrelationships.

Applying axiomatic probability results to a real-world area requires creating a probabiity model for the given area. Mathematically precise results can then be derived within the model and translated back to the real world. If the model fits the area sufficiently well, real problems can be solved by analysis within the model. However, since models are almost always simplified approximations of reality, precise results within the model become approximations in the real world.

Choosing an appropriate probability model is an essential part of this process. Sometimes an application area will have customary choices of models, or at least structured ways of selecting them. For example, there is a well developed taxonomy of queueing models. A sound knowledge of the application area, combined with a sound knowledge of the behavior of these queueing models, often lets one choose a suitable model for a given issue within the application area. In other cases, one can start with a particularly simple model and use the behavior of that model to gain insight about the application, and use this to iteratively guide the selection of more general models.

An important aspect of choosing a probability model for a real-world area is that a prospective choice depends heavily on prior understanding, at both an intuitive and mathematical level, of results from the range of mathematical models that might be involved. This partly explains the title of the text — Theory for applications. The aim is to guide the reader in both the mathematical and intuitive understanding necessary in developing and using stochastic process models in studying application areas.

Application-oriented students often ask why it is important to understand axioms, theorems, and proofs in mathematical models when the precise results in the model become approximations in the real-world system being modeled. One answer is that a deeper understanding of the mathematics leads to the required intuition for understanding the differences between model and reality. Another answer is that theorems are transferable between applications, and thus enable insights from one application area to be transferred to another.

Given the need for precision in the theory, however, why is an axiomatic approach needed? Engineering and science students learn to use calculus, linear algebra and undergraduate probability effectively without axioms or rigor. Why doesn't this work for more advanced probability and stochastic processes?

Probability theory has more than its share of apparent paradoxes, and these show up in very elementary arguments. Undergraduates are content with this, since they can postpone these questions to later study. For the more complex issues in graduate work, however, reasoning without a foundation becomes increasingly frustrating, and the axioms provide the foundation needed for sound reasoning without paradoxes.

I have tried to avoid the concise and formal proofs of pure mathematics, and instead use explanations that are longer but more intuitive while still being precise. This is partly to help students with limited exposure to pure math, and partly because intuition is vital when going back and forth between a mathematical model and a real-world problem. In doing research, we grope toward results, and successful groping requires both a strong intuition and precise reasoning.

The text neither uses nor develops measure theory. Measure theory is undoubtedly important in understanding probability at a deep level, but most of the topics useful in many applications can be understood without measure theory. I believe that the level of precision here provides a good background for a later study of measure theory.

The text does require some background in probability at an undergraduate level. Chapter 1 presents this background material as review, but it is too concentrated and deep for most students without prior background. Some exposure to linear algebra and analysis (especially concrete topics like vectors, matrices, and limits) is helpful, but the text develops the necessary results. The most important prerequisite is the mathematical maturity and patience to couple precise reasoning with intuition.

The organization of the text, after the review in Chapter 1 is as follows: Chapters 2, 3, and 4 treat three of the simplest and most important classes of stochastic processes, first Poisson processes, next Gaussian processes, and finally finite-state Markov chains. These are beautiful processes where almost everything is known, and they contribute insights, examples, and initial approaches for almost all other processes. Chapter 5 then treats renewal processes, which generalize Poisson processes and provide the foundation for the rest of the text.

Chapters 6 and 7 use renewal theory to generalize Markov chains to countable state spaces and continuous time. Chapters 8 and 10 then study decision making and estimation, which in a sense gets us out of the world of theory and back to using the theory. Finally Chapter 9 treats random walks, large deviations, and martingales and illustrates many of their applications.

Most results here are quite old and well established, so I have not made any effort to attribute results to investigators. My treatment of the material is indebted to the texts by Bertsekas and Tsitsiklis [2], Sheldon Ross [22] and William Feller [8] and [9].

Contents