METU EE531
Prof. Elif Uysal-Biyikoglu   Wednesday, September 25, 2013

Problem Set 1

1 (**The Monty Hall Problem**) You are a contestant in a game show in which a prize is hidden behind one of three curtains. You will win the prize if you select the correct curtain. After you have picked one curtain but before the curtain is lifted, the game show host lifts one of the other curtains, revealing an empty stage, and asks if you would like to switch from your current selection to the remaining curtain. How will your chances change if you switch? (Hint: Examine each strategy separately. In each, let B be the event of winning, and A the event that the initially chosen door has the prize behind it.)
Show that, when you adopt a randomized strategy (you decide whether to switch or not by tossing a fair coin) the probability of winning is 1/2.

2 There are two independent random variables, denoted by X_1 and X_2 and with CDFs F_{X_1} and F_{X_2}.
Define a new random variable: $U = max\{a_1X_1, a_2X_2 + b\}$,
where $a_1 \neq 0$, $a_2 \neq 0$, b are fixed and known scalars.

(a) Find the CDF of U.
(b) Now, assuming that X_1 and X_2 are continuous random variables, determine the pdf of U (in terms of the derivatives of F_{X_1} and F_{X_2}.)
(c) Find the probability that $U = a_1X_1$, when X_1 and X_2 are both Uniform random variables $U(0,1)$.

3 **Negative evidence**. Suppose that the evidence of an event B increases the probability of a criminal's guilt; that is, if A is the event that the criminal is guilty, then $P(A|B) \geq P(A)$. Does the absence of the event B decrease the criminal's probability of being guilty? In other words, is $P(A|B^c) \leq P(A)$? Prove or provide a counterexample.

4 (from the textbook) Exercise 1.2 parts a, b

5 (from the textbook) Exercise 1.4

6 (from the textbook) Exercise 1.6