METU EE531
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Homework Set 2

Due: Monday, October 21, 2013

1 Let $X_1, X_2, ..., X_{10}$ be independent random variables uniformly distributed on the interval [900 1100]. Define the new random variable $X_k = \frac{1}{k}\sum_{i=1}^{k} X_i$. Compute an approximation for the probability $P(950 < X_k < 1050)$ for $k = 5$ and $k = 10$ by applying the Central Limit Theorem. Comment on the results.

2 Let $\Omega = \{a, b, c, d\}$. Assume that $P(\{a\}) = P(\{b\}) = 0.3$, $P(\{c\}) = 0.4$, $P(\{d\}) = 0$. Define a sequence of random variables, $X_n(w), n = 1, 2, ...$ as follows:

$$\begin{cases} X_n(a) = 1 + (1/2)^n \\ X_n(b) = \dfrac{n-1}{n+1} \\ X_n(c) = 1 \\ X_n(d) = (-1)^n \end{cases}$$

Is this random sequence convergent? In which sense (e.g. in probability, almost surely, in the m.s.e sense, in distribution)? Explain your reasoning clearly and specify the limit if it exists.

Now, change the probabilities to $P(\{a\}) = P(\{b\}) = 0.3$, $P(\{c\}) = 0.3$, $P(\{d\}) = 0.1$, and answer the question for this case.

3 (From the text book) Exercise 1.20

4 (From the text book) Exercise 1.29

5 (From the text book) Exercise 1.30

6 (From the text book) Exercise 1.36

7 (From the text book) Exercise 1.43