METU EE

EE749
Communication Network Analysis
Homework 1
Feb. 19, 2014

Problem 1. Consider a set of independent nonnegative random variables $\mathbf{X_i}$, i=1,2,...,n with the common cumulative distribution function (CDF) $\mathbf{P(X_i \leq x) = g(x)}$. Random variables **U** and **Y** are defined as:

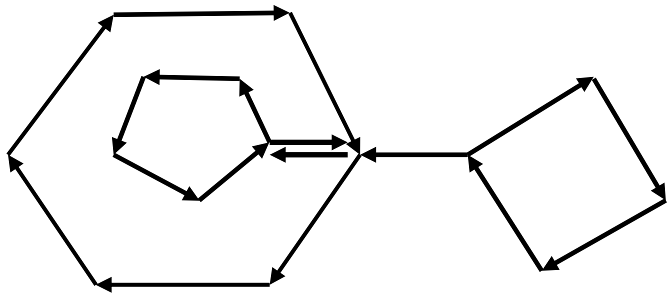
$$\mathbf{U}=\min(\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_n}), \qquad \mathbf{Y}=\max(\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_n})$$

(a) Determine the CDFs $F_U(u)$ and $F_Y(y)$ of the random variables **U** and **Y,** in terms of the function g( ).

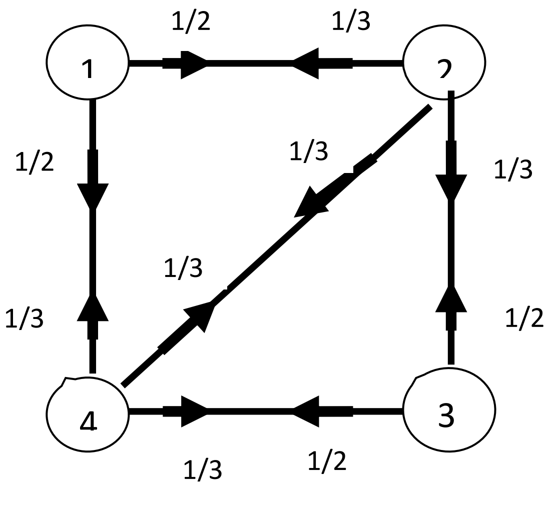
(b) Determine the joint CDF of **U** and **Y** in terms of g( ), by going through the following steps: (i) Show that $F_{UY}(u,y)=F_Y(y)-P(\mathbf{U}>u,\mathbf{Y}\leq y)$. (ii) Express the second term on the right in terms of the $\mathbf{X_i}$'s.

(c) In the special case when $\mathbf{X_i}$'s have the exponential distribution, i.e. $f_{Xi}(x) = \lambda e^{-\lambda x},\ x \geq 0$, obtain the expectation of **U** by integrating the complementary CDF. Find the limit of E[**U]** as m goes to ∞**.**

Problem 2. For the Markov chain shown below identify the classes, determine the period of each class and specify if each class is recurrent or transient. (Your answers should contain sufficient explanation.)



a) For the Markov chain shown below, determine the stationary distribution of states and identify if the chain reversible. Calculate the mean first passage time from state 1 to state 3.



Problem 3. Consider a communication channnel that has a good state and a bad state. In the good state, the noise is Gaussian with mean 0 and variance α^2, in the bad state, the signal noise is again Gaussian with mean 0 and variance β^2, s.t. $0<\alpha<\beta$. The channel has memory: when the state is good, it will remain good in the next time slot with probability p, and switch to the bad state with probability 1-p. Similarly, when the channel is in the bad state, it will stay in the bad state with probability q in the next time slot, and switch to the good state with probability 1-q. However, at any time, with probability r, a sudden system malfunction may occur (that lasts for exactly one slot) during which noise variance is $\alpha^2 + \beta^2$. Directly after the deep fade, the channel goes into the good state.

Compute the PDF of the noise in this system at steady-state.