Math 219 Differential Equations II. Exam 02.12.2008

Last Name : Name : Student No:	Dept./Sec. : Time : 17: 40 Duration : 120 *minutes*	Signature

6 QUESTIONS ON 5 PAGES TOTAL 100 POINTS

1	2	3	4	5	6			

Question 1 (20 pts.) Consider the differential equation

$$y^{(4)} - y = 0.$$

Find a fundamental set of solutions and compute their Wronskian.

Question 2 (20 pts.) Find the solution of the initial value problem

$$y'' + y = \begin{cases} t/2 & \text{if } 0 \le t < 6 \\ 0 & \text{if } t \ge 6, \end{cases} \qquad y(0) = 0,\ y'(0) = 1.$$

Question 3 (10 pts.) Use the Laplace transform and the Convolution Theorem to find the function $y(x)$ which satisfies the equation

$$y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt.$$

Question 4 (10 pts.) Express the solution of the initial value problem

$$y'' + y = g(x) + \delta(x), \;\; y(0) = 1,\, y'(0) = -1$$

as a convolution integral. ($\delta(x)$ is the unit impulse function at $x = 0$.)

Question 5 (20 pts.) Find the power series solution of the differential equation

$$y' = 2xy.$$

Question 6 (20 pts.) Consider the damped spring-mass system with an external forcing function, so that the position $x(t)$ of the object with unit mass obeys

$$x'' + bx + kx = F(t).$$

If b and $F(t)$ are 0, the solution has a minimal period of $\frac{\pi}{2}$. The system is critically damped (i.e. the characterisric equation has a double root). $F(t) = 1$ for $o \leq t < 1$ and for $3 \leq t < 4$, and 0 otherwise.

(a) Show that $k = 16$ and $b = 8$.

(b) Express $F(t)$ as a combination of step functions.

(c) Assuming $x(0) = 0$, $x'(0) = 0$, solve the equation.