M E T U
Northern Cyprus Campus

Math 219 Differential Equations Final Exam 12.01.2009		
Last Name: Name : Student No:	Dept./Sec.: Time : 16: 00 Duration : 160 *minutes*	Signature
9 QUESTIONS ON 6 PAGES		TOTAL 100 POINTS

1	2	3	4	5	6	7	8	9			

Question 1 (10 pts.) Using the definition show that the functions $f(x) = 1$, $g(x) = |x|$, $h(x) = x + 1$ are linearly independent on the interval $[-1, 1]$.

Question 2 (10 pts.) Consider the initial value problem $(y')^3 = f(y, t)$, $y(0) = 0$ where $f(y, t) = 9y^2$. The functions $f(y, t)$ and $\frac{\partial f}{\partial y}$ are continuous everywhere but both of $y_1(t) = \frac{t^3}{3}$ and $y_2(t) = 0$ satisfy the given initial value problem. Why does this not contradict the Existence-Uniqueness Theorem?

Question 3 (20 pts.) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues and the corresponding eigenvectors of the matrix A.

(b) Find the general solution of the system $\mathbf{x}' = A\mathbf{x}$.

Question 4 (10 pts.) Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}.$$

Question 5 (10 pts.) Sketch a phase portrait for the system $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Question 6 (16 pts.) Find the solution of the initial value problem

$$y'' + 4y = 3\sin 2t$$

$$y(0) = 2,\ y'(0) = -1.$$

Question 7 (8 pts.) Suppose that a solution of the partial differential equation

$$u_t = u_{xx} + u_x$$

has the form $u(x,t) = X(x)T(t)$. Find a pair of ordinary differential equations for the functions X and T.

Question 8 (8 pts.) Find the eigenvalues and the eigenfunctions of the homogeneous two-point boundary value problem

$$y'' + 2y' + \lambda y = 0$$

$$y(0) = 0,\ y(10) = 0.$$

Question 9 (8 pts.) Let

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x < 2. \end{cases}$$

Extend f as an even function of period 4 to all real numbers and sketch its graph. Find its Fourier cosine series.