| | **Differential Equations**<br>**II. Midterm** | |
|---|---|---|
| Code : *Math 219*<br>Acad.Year : *2007-2008*<br>Semester : *Fall*<br>Date : *9.12.2007*<br>Time : *14:00*<br>Duration : *120 minutes* | Last Name:<br>Name : Student No.:<br>Department: Section:<br>Signature: | |
| | 5 QUESTIONS ON 5 PAGES<br>TOTAL 100 POINTS | |

| 1 | 2 | 3 | 4 | 5 | | |
|---|---|---|---|---|---|---|
| | | | | | | |

**1.** (20 points) Solve

$$y^{(6)} - y'' = 0$$

**2.** (5+15=20 points) Consider the differential equation $y' + 2xy = x$.

(a) What is the radius of convergence of a power series solution of this equation?

(b) Find all solutions using a power series expansion about $x_0 = 0$. Express your answer in closed form.

**3.** (8+8+8=24 points) Find the inverse Laplace transforms of each of the following functions.

(a) $F(s) = \dfrac{e^{-s} - e^{-3s}}{s^2}$

(b) $F(s) = \dfrac{s}{(s+1)(s+2)(s+3)}$

(c)$F(s) = \dfrac{s}{(s+1)(s^2+s+1)}$

**4.** (20 points) Use the Laplace transform to solve the initial value problem

$$y'' + 4y' + 3y = 1 + u_2(t), \qquad y(0) = y'(0) = 0$$

**5.** (8+4+4=16 points) Let a variable $y(t)$ in an undamped forced mechanical system obey the equation $y'' + w_0^2 y = f(t)$, where $w_0 > 0$ and $y(0) = y'(0) = 0$.

(a) Show that $y(t) = -cosw_0 t \int_0^t \frac{\sin w_0 x f(x)}{w_0} dx + \sin w_0 t \int_0^t \frac{\cos w_0 x f(x)}{w_0} dx = \int_0^t \frac{\sin w_0 (t-x) f(x)}{w_0} dx$

(Hint: use variation of parameters or the Laplace transform)

(b) If $|f(t)| \leq w_0$ for all $t \geq 0$, show that $|y(t)| \leq t$ for all $t \geq 0$.

(c) Give an example for $f(t)$ which shows that $\lim_{t\to\infty} \frac{y(t)}{t}$ does not have to be 0 even though $|f(t)| \leq w_0$ for all $t \geq 0$. (Hint: use a function $f(t)$ which causes resonance.)